THE ACOUSTICS AND PSYCHOACOUSTICS OF THE GUITAR

by

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A thesis submitted to
the University of Wales
for the degree of

Doctor of Philosophy

Department of Physics and Astronomy,
University of Wales, College of Cardiff

September 1996

Declarations

This work has not previously been accepted in substance for any degree and is not being concurrently submitted in candidature for any degree.

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All the work presented in this thesis is the result of my own investigations, except where otherwise stated. References are given where other sources are acknowledged. A bibliography is appended.

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Acknowledgements

I would like to thank my supervisor, Dr. B.E. Richardson, for his help and support throughout my period of research. The work presented in this thesis owes much to his enthusiasm and careful guidance.

I am indebted to Professor R.H. Williams for providing research facilities at the Department of Physics and Astronomy at UWCC, and to EPSRC for provision of the grant which funded my research period.

I am also grateful to Professor D. Jones for advice on the preparation and presentation of the sounds for the psychoacoustical tests, and to Dr. M. Greenhough for allowing me to use the recording equipment and facilities at UWCC for both work and pleasure.

I would also like to thank all who volunteered for the psychoacoustical tests. It is your perceptions that have provided some of the most important results. I am grateful to Ed Burges, Matt Nisbet and Dan Jones, guitarists from the Welsh College of Music and Drama, who took part in the psychoacoustical work and provided useful feedback from the guitarist's perspective.

I thank all the friends and colleagues who have shared in the wonderful times I have spent in Cardiff. To Ali and Jon, it was a pleasure to live with such caring and fun-loving flatmates. Many thanks for the good times we shared at number 11. Special thanks goes to Maria, Simon and Ian, joint founders of the Bedford Street Musical Acoustics Fan Club, and friends with whom I have shared much music, laughter and alcohol. Many thanks to Mario Antonioletti who was generous with his time in helping me to overcome a variety of computing problems. Thanks also to Oona and family for their friendship and support, and to all who sailed with me on 'the good ship Bracken Hill' for helping me to make it through my first degree.

Finally, thanks to my brother Gordon who helped to stimulate my interest in music and guitars many years ago, and to my parents for their love and support for the last 25 years.

There is no such thing as music divorced from the listener. Music, as such, is unfulfilled until it has penetrated our ears.

Yehudi Menuhin.

Summary

The work presented in this thesis is concerned with the relationships between the perceived tone quality of classical guitars and the vibrational behaviour of the guitar body. A numerical model is described which calculates the sound pressure response of a guitar when a sinusoidal force is applied to one of its strings. The response of the body is described in terms of its modes of vibration, each mode being characterised by four parameters: a resonance frequency, an effective mass, an effective monopole area and a Q-value. Coupling between the string, top plate and fundamental modes of the back plate and air cavity is included.

The output of the model represents the sound of a plucked note as heard by a listener at a given distance in front of the guitar. Using notes synthesised from the model, psychoacoustical listening tests are performed which examine the effect on tone quality of a variety of changes to the mode parameters.

The thesis is divided into nine chapters. Chapter 1 outlines the aims and methods of the research. Chapter 2 reviews the literature relating to stringed musical instruments. Chapter 3 presents a description of the processes that occur in the guitar during sound production. Chapter 4 outlines the theory for the numerical model. Chapter 5 describes experimental measurements of the frequency responses of two guitars and the curve-fitting techniques used to obtain values of the four mode parameters for a number of body modes. Chapter 6 describes experimental measurements of the coupling between string and body. Chapter 7 describes the four psychoacoustical listening tests. Chapter 8 discusses the results of the listening tests, and establishes connections between the properties of the body modes and certain characteristics of tone quality. Chapter 9 presents a brief summary of the conclusions reached, and also outlines topics for future work.

Contents

Chapter 1

Introduction

1.1 Aims of the research

One of the main goals in musical acoustics research is to link measurable, physical properties of a musical instrument with subjective assessments of its tone quality. A better understanding of the relationships between tone quality and the vibrational response of the instrument will allow instrument makers greater powers to adjust the tonal characteristics of an instrument by altering the materials used and the pattern of construction followed. Valuable information and experience can be gained from of a variety of people including instrument makers, musicians and psychologists. The techniques, backgrounds and terminology of these groups will be very different. We must try to take ideas and experience from all these people and link them together in a scientific framework.

In undertaking research to link physical characteristics of an instrument to subjective judgements of its tone quality, many problems are encountered. The psychoacoustical evaluation of the tone quality of an instrument presents some of the greatest challenges. Judgements of an instrument's sound are, of course, highly subjective. A group of professional musicians, when presented with a number of different instruments, would be unlikely to agree on which instrument possessed the most desirable tone quality. Each player has his or her own preferences in terms of sound quality and would choose an instrument that best suited his or her technique and repertoire. To obtain meaningful evaluations of perceived tone quality, it is more productive to avoid phrasing questions in terms of preference, and to concentrate on measurements of particular timbral attributes. A group of musicians are more likely to reach agreement on whether one instrument was perceived as 'louder' or 'brighter' than another.

Additional complications arise when psychological factors are considered. A musical instrument that is unattractive to the eye is likely to be considered inferior to another instrument, yet it may possess very desirable musical qualities. A player is likely to make judgements of an instrument in terms of how comfortable it feels when it is held and how responsive it is to the touch. These are all important judgements for the musician, but if we are primarily interested in assessing the sound quality of an instrument we must take great care to minimise the influence of other factors which may affect perceptions of an instrument's 'quality'.

This thesis is exclusively concerned with the physical properties and tone quality of the classical (nylon-strung) guitar. Some of the results and conclusions will be of relevance to related instruments, such as the steel-strung acoustic guitar, as well as members of the violin family of instruments. In this chapter I will first outline the methods that can be used to investigate the relationships between the physical properties of the classical guitar and its sound quality. I will then cover the anatomy of the guitar and the materials commonly used for its construction. Aspects of the 'quality' of an instrument will be discussed, and I will finish with an overview of the important physical processes involved in sound production from the guitar.

1.2 Methods

Obtaining a meaningful evaluation of the sound quality of an instrument is difficult and must be carefully planned. Certain instruments are well suited to certain styles of music; a well-chosen selection of musical pieces will help to achieve a balanced judgement. Precautions may also be necessary to prevent the player from altering his playing technique in response to the appearance of the instrument. An instrument that is attractive to player may cause him to 'try harder' to obtain a pleasing tone. Care should be taken to ensure that all those who are judging the instruments are listening to the same sound. The tone quality perceived at different locations in a room may differ significantly enough to necessitate the use of recorded musical extracts which can be played over headphones. In this case additional precautions must be taken to ensure that the volume at which the different sounds are recorded and played back to the listener remains constant. The acoustics of the room in which the recordings are made, and the position of the microphone used should all be carefully considered.

Once the sound quality of a number of instruments has been properly evaluated, the problems of linking these sound-quality judgements with measured physical properties of the instruments must be faced. When comparing instruments made by different instrument makers, or luthiers, the design, materials and methods used by each are likely to vary considerably. Experimental measurements will reveal large variations in the instruments' vibrational behaviour. Detailed work is likely to uncover a great many differences between properties of individual body modes of different instruments. Statistical techniques are often the only way to make sense of the vast amount of information such a study will provide. Statistical correlations may be calculated, which indicate the likelihood of a link between certain physical features and judgements of sound quality, but it is more useful to use other techniques so that firm, causal links may be established between physical features of an instrument and its tone quality.

The use of experimental techniques on real instruments to investigate links between construction and sound quality presents several difficulties. Many of the physical and acoustical properties of the wood are inter-related. It is therefore virtually impossible to vary a single physical parameter independently of all others in order to measure its influence on tone quality. Similarly, the modal properties of the instrument cannot be altered independently. It is also difficult to subject a guitar to a series of controlled physical alterations. Research has been performed (Meyer, 1983b) in which a series of changes to the guitar's construction were made, and their effects on the physical behaviour of the instrument were measured. Work of this kind presents a number of risks. Repeated removal and regluing of the guitar bridge, for example, is likely to inflict some damage to the top plate. Changes to the strutting arrangement or thickness of the top plate are difficult to achieve without inadvertently causing small changes to the rest of the guitar. The application of such methods to hand-made guitars of high quality would present an unacceptably high risk of damage to the instrument.

Another approach is to make measurements on a series of instruments built to the same basic design but with single constructional details varied. For example, the top plate thickness could be varied. Analysis of a number of these instruments might reveal differences in sound quality which could be related directly to the differing plate thicknesses. This approach is, however, unlikely to succeed. Obtaining suitable wood for the instrument parts is one of the luthiers greatest problems. To obtain three, four or five sets of virtually identical wood pieces with which to build the set of instruments would be impossible. Wood taken from different parts of the same tree will have significant variations in material properties. The variations in the materials used cannot be overcome and it would not be possible to build instruments in which individual construction parameters are systematically varied.

Many of the problems associated with experiments on real instruments can be avoided by building computer models from which sounds can be synthesised and evaluated, as discussed in the next section.

1.3 Numerical methods

The ever-increasing power and speed of computers has given scientists new ways of solving a variety of complex problems. Models of biological, economic or physical systems can be built on computers and experiments run on them which would be impossible to perform in reality due to constraints of time or money. One of the most promising methods of solving problems in musical acoustics is the use of computer models that simulate the behaviour of real instruments.

In the field of musical acoustics, numerical models can be used to obtain complete and independent control of all physical parameters relating to the instrument. The construction and material properties of the guitar can be altered by changing the input data for the model. For example, it might be interesting to listen to the change in the sound quality of a guitar whose back plate was removed and replaced with one made from a different type of wood. Performing the operation on a real instrument would be a highly delicate and risky operation. Using numerical models, the experiment could be performed many times by changing the input data relating to the physical properties of the back plate and listening to the resulting changes in the sounds synthesised from the model. Numerical modelling gives the user complete and independent control over all parameters in the model. Values of individual material constants, thicknesses of wood and dimensions can all be changed at will.

It would be misleading, however, to suggest that every detail of an instrument's behaviour can be modelled to perfection. Approximations and simplifications are an important first step in building numerical models. Indeed, it is by making simplifications that a greater understanding of the problem is eventually achieved. The highly simplified case can be solved first, and the detail can then be added in stages until a more realistic working model is established. McIntyre and Woodhouse (1978) emphasise the importance of breaking down the complex physical behaviour of musical instruments into simpler components in order that these may be studied and modelled in simpler interactions. For stringed instruments they suggest a division into three parts: the behaviour of the string, the response of the body, and the radiation of sound, though they are careful to point out that these three cannot be entirely separated due to important interactions occurring between them.

One of the simplifications that must often be made when modelling musical instruments is the removal of the player from the system being studied. When a musical instrument is played, the interaction between player and instrument undoubtedly has important effects on the sound quality produced. The perceptual information that the player uses to judge the response of the instrument, and adapt his or her technique accordingly, is rather difficult to determine. Visual and audio cues, as well as vibrations sensed through the fingers and body, are all likely to play a part. Many numerical models omit the player completely so that the behaviour of the instrument itself can be studied with fewer complications. This omission must be considered when discussing the quality of an instrument. Features which are important to the player, such as the 'playability' of an instrument, cannot be totally ignored.

Other factors that limit the success of a numerical model are the available computing resources. A 'simple' model of a musical instrument may still require a relatively large amount of processing time on a powerful computer. Sacrifices in the scope of the model may have to be made in order to accommodate the available computing power. However, advances in computer technology over the last 20 years have brought reasonably powerful computers within the financial means of many researchers. Numerical models are being used more frequently as an investigative tool in musical acoustics research.

Care must be taken when the 'success' of a numerical model of a musical instrument is judged. Since we are dealing with objects that produce musical sounds, the final judge of a model must be the ear-brain system of a listener. We have a limited understanding of the features of a sound that the ear judges to be important, and so we must not assume that small physical changes necessarily produce small perceptual effects (McIntyre and Woodhouse, 1978). Proper psychoacoustical work should be performed so that the perceptions of tone quality associated with the modelled physical behaviour of the instrument can be examined and quantified.

1.4 Anatomy and evolution of the classical guitar

I will now deal with the anatomy of the guitar and the materials commonly used in its construction. Figure 1.1 shows the main features of the modern instrument. Although there is now relatively little variation in appearance between guitars built by different guitar makers, in its early development the guitar passed through many different evolutionary stages. The early guitar makers experimented a great deal with the construction and dimensions of the instrument in an attempt to improve its tone quality. In the early 1800s the guitar underwent a period of rapid change. Around this time fixed metal frets replaced tied gut frets, struts were introduced to help strengthen the body, geared metal tuning heads became commonly used and the stringing of the guitar was altered. Prior to this time five pairs of strings was the most common arrangement; six single strings became the new standard. The material used for the strings at this time was gut. Much later in the guitar's development, around the time of Segovia, the nylon string became established as a superior material. It offered greater uniformity in thickness and material properties, yielding notes with smaller fluctuations in harmonicity. More detailed accounts of the history and evolution of the guitar and its construction are given by Huber (1991) and Richardson (1994, 1995).

The number of strings on the guitar has been fixed at six since around 1800. This allows chords to be played in a variety of voicings over a wide frequency range. The strings are usually tuned to the notes E_2 A_2 D_3 G_3 B_3 E_4, the B string being just below middle C. Using conventional tuning, the fundamental frequency of the lowest note is 82.4 Hz and that of the highest note is 1047 Hz. One common alternative tuning is for the bottom E string to be tuned down to a D at 73.4 Hz.

Many of the standard features of the modern classical guitar (its larger size and fan arrangement of struts) are attributed to Antonio de Torres, although it is true to say that such features were not invented by him. These design features evolved during the early 19th century when the instrument underwent its period of accelerated evolution. Torres started his work at the end of this period, absorbing some of the ideas of earlier luthiers, and through

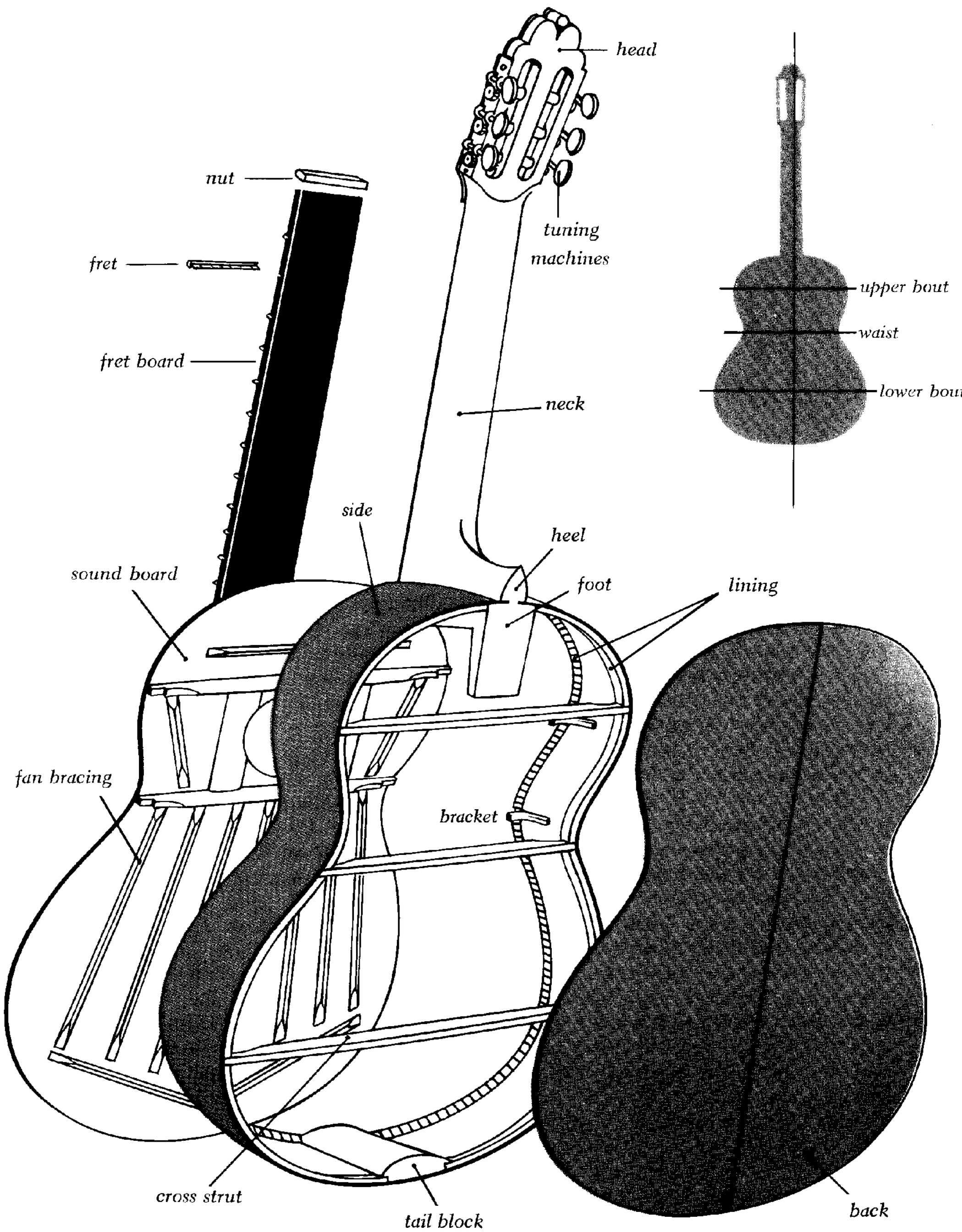


Figure 1.1: Exploded view of the classical guitar, from Sloane (1976)

a combination of skilled designs and his association with influential players such as Tarrega, popularised his guitars. Thus, features such as the larger instrument dimensions and the fan-strutting became the standard of the modern guitar.

The classical guitar has evolved to achieve a balance between many conflicting ideals. It should be large enough to radiate sound efficiently at low frequencies, yet small enough to be carried around and held conveniently. It should be able to play chords as an accompaniment as well as being able to carry a melodic line as a solo voice. It should be both pleasing to the ear and attractive to the eye. Although modern luthiers still experiment with the details of the instrument's construction, the main features remain the same and are described below.

1.4.1 The top plate

The guitar body is made up of a top plate (or soundboard), back plate and sides (or ribs). These enclose an air cavity that communicates with the surrounding air mass via the soundhole which is cut from the top plate. The strings are attached to the top plate via the bridge.

The material chosen for the top plate must be stiff yet lightweight. Light materials will couple more readily to the strings and air to give a louder sound. The top plate must also be stiff enough to withstand the forces at the bridge arising from the string tension. A low value for the internal damping of the material used in the top plate is probably also desirable as this will minimise the amount of vibrational energy lost as heat. The two most common materials chosen for the top plate are spruce and cedar; both woods have a high ratio of stiffness to density. The stiffness of the plate depends strongly on the way in which it is cut from the tree (see Section 1.4.4). The top plate is thinned to make it light, and struts are added on its underside to give it extra stiffness without greatly increasing its mass. A number of strengthening bars are also used on the upper half of the top plate.

1.4.2 The back plate and sides

When a string is plucked on the instrument the whole body will be driven into vibration but the amplitude of vibration for the back will be lower than that of the top plate. The amplitude of vibration of the sides will be lower still. The back and sides radiate less sound than the top plate and have a smaller role to play in governing sound quality than the top plate. For this reason the appearance of the wood used for the back plate is often deemed as important as its acoustical properties. The materials chosen for the sides and back vary more, although hardwoods are nearly always preferred. Common choices include rosewood and mahogany. Struts and bars are also used on the back for increased strength.

1.4.3 The neck and strings

The neck of the guitar is made from mahogany or cedar and has a fingerboard, usually around 5 mm thick and made of ebony or rosewood, attached to it. The frets are made of metal and are fixed in place along the fingerboard allowing the strings to be stopped to produce different vibrating lengths. This permits all notes in the chromatic scale to be played. The neck of the guitar has twelve frets (one octave) extending from the nut to the join of the neck and body but also usually has a further seven or eight frets above the join with body, giving a possible playing range of around four octaves. The joint between the neck and body must be strong enough to withstand the considerable torque exerted by the force of the strings at the nut.

Typical values for the dimensions of the body are given in Table 1.1 (data taken from Huber, 1991 and Sloane, 1976), and the definitions of the body dimensions are given in Figure 1.2. The radiation of low-frequency sound is partially limited by the dimensions of the guitar, larger instruments being stronger radiators at low frequencies. The frequencies and amplitudes of the body modes will also influence the radiation of sound in the low-frequency region.

1.4.4 Materials

The choice of materials for instrument building has a strong influence on the sound quality of the finished product. The more established luthiers often have first refusal on the best pieces of wood, leaving the poorer cuts to the up-and-coming instrument makers. However, there is no 'ideal' piece of wood. Methods must always be adapted to match the needs of the materials used. The art of building good instruments is inextricably linked with the need to assess the materials available and to adapt the construction to compensate for any shortfalls in the wood.

The stiffness of a piece of wood depends critically on the way it is cut from the tree. The maximum cross-grain stiffness is obtained when wood is 'quarter-sawn' from the trunk. This method of cutting produces radial slices from the trunk with the grain making an angle of

	Dimensions (mm)
Body length	470-490
Upper bout width	260-290
Waist width	220-240
Lower bout width	350-370
Soundhole diameter	83-89
Depth	90-110

Table 1.1: Typical dimensions of the classical guitar body

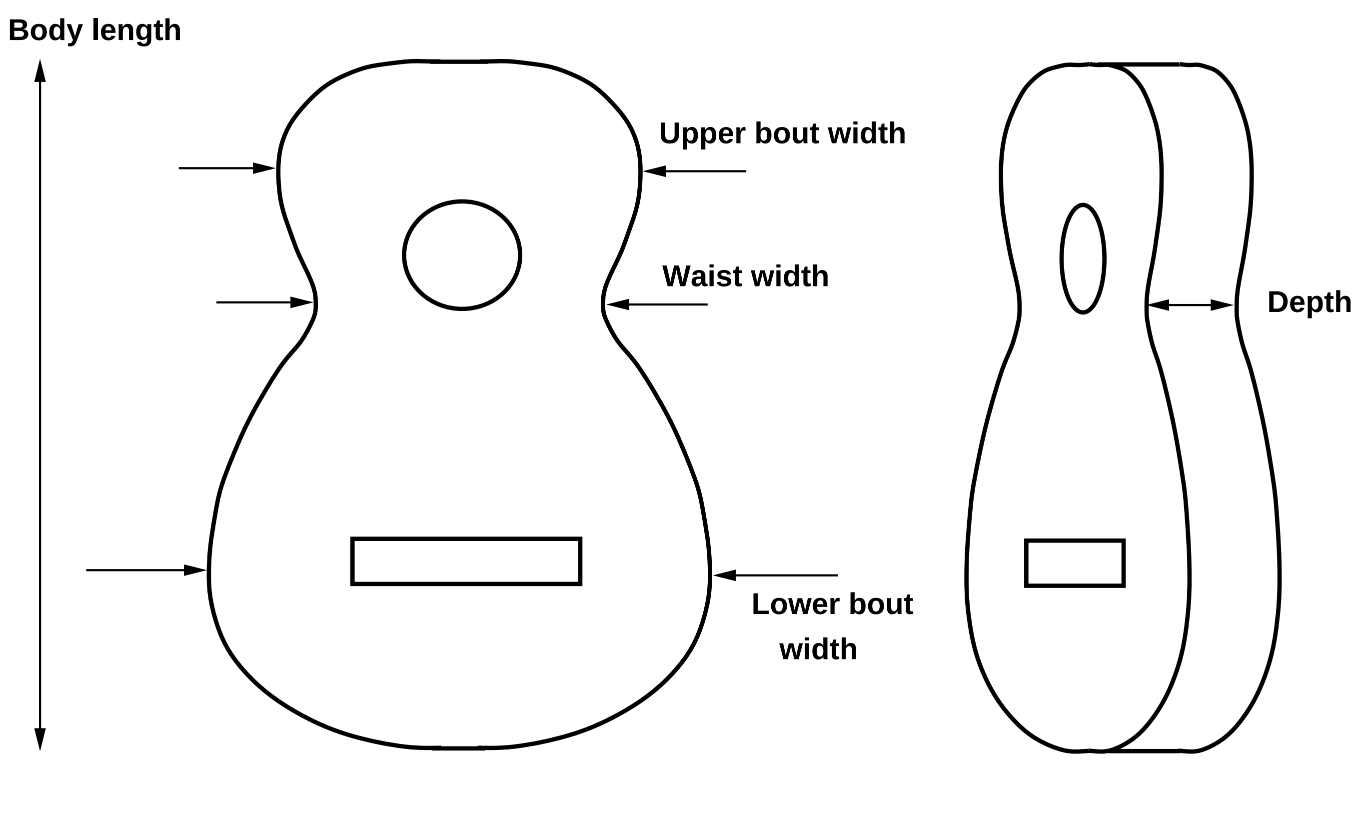


Figure 1.2: Definition of body dimensions

90 degrees with the surface of the wood. Wood cut in this manner tends to produce fewer problems of shrinkage and distortion as well as offering the best cross-grain stiffness. The orientation of the wood fibres affects the stiffness along the grain. Wood that is split rather than sawn ensures the fibres run parallel to the surface and thus maximises the stiffness along the grain.

Commercial techniques and economic considerations often mean that wood is not truly quartered, leading to reduced stiffness. It may not be easy for luthiers to obtain two pieces of wood for the top plate that have adequate cross-grain stiffness and in this case some luthiers (Torres, Romanillos) prefer to use several smaller pieces of wood offering good stiffness to make the top plate.

Luthiers may be able to judge some of the material properties of the wood by listening to the so-called tap tones. The piece of wood is held lightly and tapped with the knuckle or finger to excite some of the modes of the plate. The decay rate of the tap tones gives a measure of the wood's damping and the pitch of the tap tones gives information about the frequencies of the modes. The luthier can adjust the construction of an instrument according to his perception of the tap tones.

1.5 The 'quality' of an instrument

The quality of an instrument can be judged using a number of criteria. The aesthetics of the guitar's appearance are of considerable importance to many players. The quality and finish of the wood, the carving of the guitar head and the inlaid design around the soundhole all contribute to the beauty of the instrument as a work of art. The skill with which the wood has been worked, shaped, and joined is another measure of quality. Good craftsmanship is reflected in strong, neat joints which help to make the guitar robust. The accuracy of the fretting is of vital importance if all notes on the instrument are to be correctly in tune. The weight and size of the instrument, the shape of the neck and the ease and comfort with which the player can hold and play the guitar can also be used to judge its subjective quality.

All these considerations contribute to a person's impression of an instrument's quality. The perceived sound quality of an instrument is something entirely separate. When judging the 'quality' of an instrument one must be clear whether it is the appearance, craftsmanship or sound of the instrument that is being assessed. All are important to the player who is choosing an instrument to buy. This research concentrates only on the quality of the sound made by the instrument.

1.6 Sound production in the classical guitar

The details of the physical processes that occur when a guitar is played will be covered in Chapter 3. In this section I will give a brief outline of the functioning of the guitar as a musical instrument.

The process of sound production starts with the interaction between the player's finger and the string. When the string is set into motion, its vibrations can be described in terms of its normal modes. The frequencies of the string modes have a near-harmonic relationship giving the sound a pleasing musical quality and a sense of pitch. By altering the position at which the string is plucked, the player can excite different string modes and vary the tone quality of the sound produced. On its own, however, the string radiates very little sound as its dimensions are many times smaller than the typical wavelengths of sound involved. The function of the body of the guitar is to convert the energy of the vibrating string into radiated sound energy.

The top plate of the guitar is the primary radiator of sound, although the back plate and air cavity make significant contributions. Like the string, the guitar body's vibrations can be described in terms of its normal modes (see Figure 2.9). The frequencies of the body modes will not be harmonically related, and consequently the impulse response of the body, obtained by damping the strings and tapping the bridge, has a 'noise-like' quality, rather than the more musical 'note-like' quality of the string. Each time a note is plucked, an impulse-like force is transmitted from the string to the top plate, exciting the body modes and creating a 'knocking noise' or body transient. This sound is short-lived, but may help listeners to distinguish between different instruments (Brooke 1992). The frequencies, amplitudes and Q-values of the body modes are unique to each guitar and the body transient can be interpreted as a kind of 'acoustical fingerprint' of the instrument.

The body transient lasts for only a fraction of a second, and so it is the slowly-decaying string modes that dominate the sound of each plucked note. The guitar body, having a large surface area, radiates the sound from the string modes much more effectively than the string itself. The response of the guitar body is strongly dependent on driving frequency, causing some string modes to be radiated more strongly than others. The frequencies and amplitudes of the body modes determine the frequency regions in which the string modes are radiated more strongly. The decay rates of the string modes are also influenced by the body's modal properties; body modes that are driven easily by the string tend to drain the string's vibrational energy rapidly, and will increase the string's rate of decay.

The sound that reaches a listener's ears is determined partly by the efficiency with which the string drives the body modes, and partly by the radiation efficiency of those modes. In the low-frequency region, body modes that generate a net volume displacement will radiate equally in all directions, the efficiency of radiation being determined by the size of the volume displacement. Mode shapes that are antisymmetric produce a zero net volume change, since one half of the mode produces a volume change equal and opposite to the other half. Such modes have a dipole character, radiating poorly to the front of the instrument, but more strongly to the sides. At higher frequencies, the directional nature of sound radiation from the body increases as multipole radiation becomes more efficient. High-frequency body modes tend to have a large number of anti-nodal areas producing radiation fields with a more complicated spatial dependence.

1.7 The guitar, guitar-player and acoustic environment

A schematic diagram showing the various interactions that occur between player, instrument, surroundings and audience is given in Figure 1.3. The interaction processes, marked by the arrows, are generally two-way processes. For example, the coupling between string and top plate causes the behaviour of each one to be modified by the other. Further two-way interactions occurring between the top plate, air cavity and back plate mean that the string is directly affected by the vibrational behaviour of all parts of the body.

Not all of the interactions have been illustrated. The primary interaction between top plate and back plate is the coupling that occurs via changes in the pressure of the air cavity. The coupling between top and back that occurs due to movement of the sides of the body is not indicated on the diagram although this may have consequences for the response of the instrument. Similarly, the contact between the player's body and the the instrument is not indicated.

Figure 1.3 illustrates one of the main difficulties in musical acoustics research: the physical behaviour of the guitar is determined by a complex inter-dependency of its component parts. The vibrational behaviour of the top plate, back plate and sides, when viewed in isolation, differs significantly from their behaviour when assembled in a complete instrument. A great deal can be learned by studying the top plate as a single piece of wood, but we must always take account of the forces that couple together the different elements in the finished instrument. All parts of the guitar vibrate together and the behaviour of one part will have direct consequences for the motion of all other parts. We cannot construct separate models of the string, top plate, air cavity and back plate but we must examine and model the interactions that occur between them.

The interactions occurring between different elements are not confined to the guitar. As Figure 1.3 shows, there are significant processes occurring that involve the relationships between guitar and concert hall, player and guitar as well as player and audience. Sound radiating from the top plate, back plate and soundhole will be reflected from the walls, floor and ceiling of the room in which it is played. These reflected waves will return back and cause the guitar body to vibrate in response and alter the subsequent sound radiation. The audience will also affect these reflected waves, and a large number of people can greatly affect the acoustics of a room or concert hall.

The interaction between player and instrument is more complex. Figure 1.3 shows only the interaction between the player's finger and the strings that occurs during the plucking process. There is also the physical contact between the back and sides of the guitar and the players body and legs. This will tend to increase the damping and slightly lower the frequencies of the guitar's normal modes. In addition there is a range of information about the behaviour of the guitar that the player may sense. This will include vibrations of the body sensed through the legs or body of the player as well as information picked up through the contact between fingertips and strings. These may all provide useful cues to the player and may help him to adjust his playing technique to adapt to the instrument. A player who is introduced to an instrument for the first time may produce a tone quite different from when he has had time to become familiar with the instrument. The sound produced before and after familiarisation

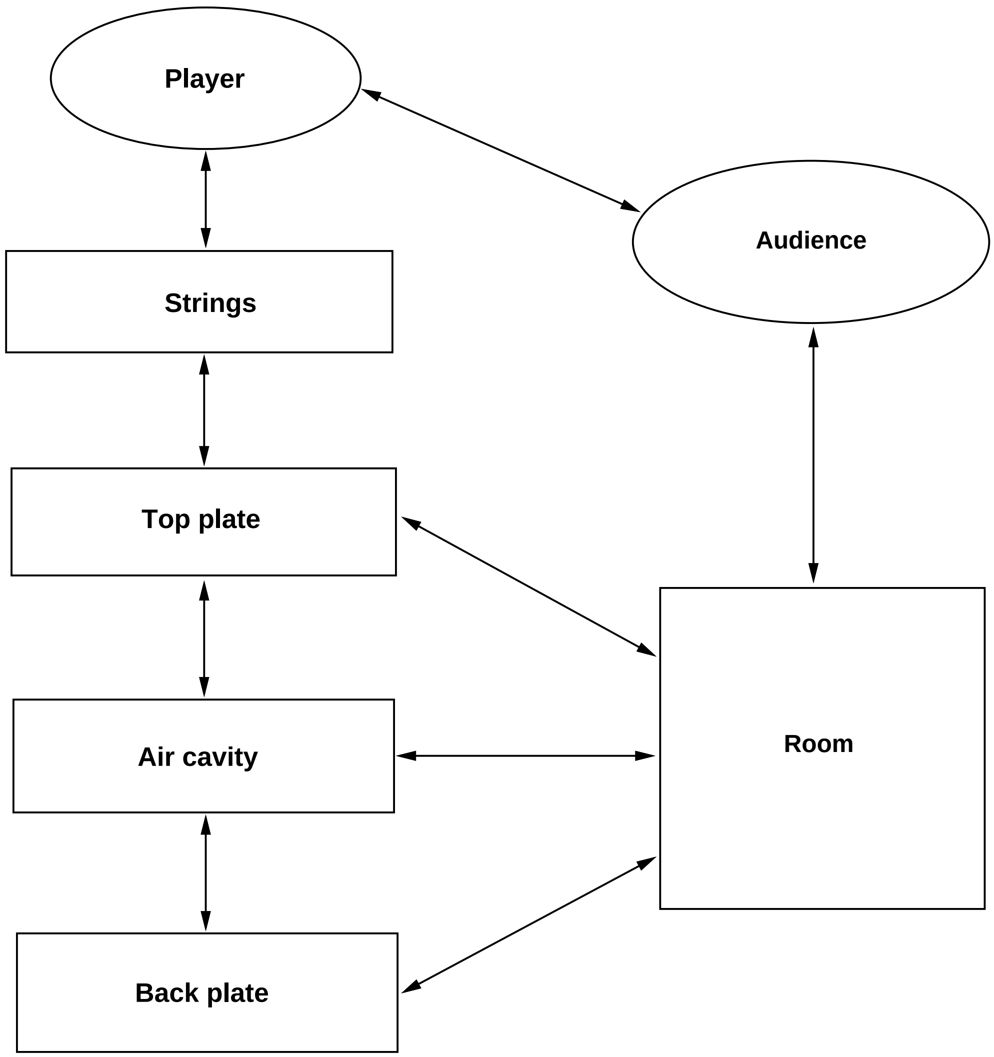


Figure 1.3: Schematic view of the interactions that may occur between guitar, player, listener (audience) and acoustic environment. The player-listener interaction is psychological; other interactions are physical in nature.

with the guitar will be different, but the instrument is the same. The player will simply learn about the strengths and weaknesses of the instrument's sound and adjust his playing technique accordingly.

A good player would be able to pick up a 'bad' instrument and make relatively pleasing sounds with it. Likewise, a poor player could pick up a 'high-quality' instrument and make relatively poor-sounding music. The variations in sound quality produced by different players playing a single guitar may be as large as those produced by one player with a number of instruments. To study the variations in one, we must ensure that the other remains constant.

Other psychological influences may involve a player who responds according to the visual appearance of a guitar. This could lead to changes in playing technique that yield different tone qualities. Additional psychological influences could arise from a player's perception of an audience's reaction to the music.

All of these effects may have important influences on the tone quality of a guitar, but to make progress in understanding the influence of the guitar body on the sound quality it is important that all other effects are carefully controlled. Some are relatively easy to control; experiments can be performed in an anechoic environment to remove the influence of reflections from the walls and ceilings of a room. Other factors, such as the psychological response of a player to the appearance of an instrument, are much harder to remove. By building a numerical model of the guitar these effects can be controlled. The influences of the player and the room are removed and we can concentrate on the acoustical properties of the instrument itself.

1.8 Summary

In this chapter I have described the need for a greater understanding of the links between physical properties of the guitar (materials and construction) and the tone quality produced by the instrument. I have outlined the difficulties of achieving this linkage through experimental measurements on real instruments. Numerical modelling of musical instruments, with appropriate psychoacoustical work, offers one way in which some of these difficulties can be overcome.

The work presented in this thesis describes a numerical model of the classical guitar capable of synthesising the radiated sound of plucked notes. The model makes no attempt to take into consideration the acoustic environment in which the guitar may be played (anechoic conditions are assumed), nor does it include the influence that different players have on the tone quality produced from a particular instrument. The range of tones available to a player is fixed by the modal properties of the instrument. It is the relationship between these modal properties and the perceived tone quality of an instrument that is the focus of the work. Using sounds synthesised from the model, psychoacoustical listening tests have been performed which probe the links between perceived tone quality of the guitar and the properties of its normal modes.

The next chapter presents a review of published work relating to stringed musical instruments. Chapter 3 gives a physical description of the sound-producing processes that occur in the guitar. Chapter 4 presents the theory for the numerical model. Experimental work relating to the guitar body is presented in Chapter 5. Curve-fitting work is also described which allows data relating to individual body modes to be obtained from frequency response curves. Chapter 6 describes experimental measurements of the coupling between the strings and body of the guitar. The psychoacoustical tests are presented in Chapter 7 and the results are discussed in Chapter 8, which outlines links between properties of the body modes and aspects of tone quality. Chapter 9 summarises the conclusions and offers suggestions for future work.

Chapter 2

Previous work on stringed instrument acoustics

In this chapter I will review work relating to the acoustics of stringed musical instruments. Much of the available literature relates to violins rather than guitars. The two instruments are basically similar in design, consisting of strings stretched over the neck, connected to a bridge, with the top plate, back and sides making up the body. The processes involved in sound production from each are also similar and so results obtained from one instrument are often applicable to the other. Important differences between the two include the addition of the soundpost in the violin, and the use of different excitation mechanisms (bowing and plucking).

Research in musical acoustics has been approached by people working in a variety of disciplines including physics, mathematics, engineering, computing and psychology. Research in this area has been aided by the recent development of powerful new investigative techniques such as holographic interferometry and cheap, quick digital Fourier analysis. The areas that have been investigated cover many aspects of musical instruments, from the physical properties of the materials used for the instrument's construction to the psychoacoustical analysis of its radiated sounds.

Numerical models of musical instruments have been used increasingly as a research tool in recent years due to the greater availability and lower cost of computer processing power. Models of whole instruments or of individual components, such as the bowed and plucked string, are leading to a greater understanding of the functioning and interaction of the instrument's parts. So far, however, the majority of the numerical modelling work has not been complemented with appropriate psychoacoustical work, so the implications of the physical behaviour have not yet been linked to subjective aspects of the sound quality. The importance of this step, underlined by McIntyre and Woodhouse (1978), has already been emphasised in Chapter 1.

2.1 Modal analysis

Many of the important physical processes occurring in a musical instrument are related to the properties of the instrument's modes of vibration. Some of the most important early work performed on stringed musical instruments concerned itself with the identification and visualisation of these body modes. Holographic interferometry and Chladni techniques have been used to visualise the mode shapes of free plates and complete instruments. Measurements of an instrument's vibrational or sound pressure response, over a certain frequency range, also yield useful information about the instrument's modes. Peaks in the response curves are associated with the body's normal modes. The properties of the modes can be determined by measuring the amplitudes, frequencies and Q-values of the peaks.

A great deal of work has focussed on the frequencies of the body modes and attempts have been made to relate the subjective quality of an instrument with the mode frequencies. One of the arguments developed later in this thesis is that the mode properties which determine the *amplitude* of the peaks on a response curve are of much greater importance than the mode frequencies.

Conventions for mode labelling

In this thesis, the modes of the guitar body are characterised by the number of different anti-nodal areas visible in the lower bout of the guitar.[1]. Motion of the top plate is usually confined to the lower bout so it is often simpler to ignore the small vibrations that may occur in the upper bout. The modes are labelled with two numbers; the first is the number of anti-nodal regions lying across the top plate (left to right) and the second is the number lying along the plate (top to bottom). Modes of the top and back are distinguished by using the letters T

[1]Some researchers count the number of nodal lines rather than the number of anti-nodal regions.

and B. Figure 2.1 shows the shapes of the first four top-plate modes of the guitar, along with the labels used to identify them. Because of coupling between some of the plate modes and modes of the air cavity, some modes appear more than once. For example, coupling between the fundamental modes of the top plate, back plate and air cavity produces three T(1,1)-type modes. Subscripts will be used to distinguish between multiply-occurring modes (e.g. $T(1,1)_1$ for the lowest-frequency mode of the T(1,1) triplet).

The names used to identify particular body modes sometimes owe more to convenience than to scientific accuracy. The terms 'top-plate mode', 'back-plate mode' and 'air-cavity mode' should strictly be applied to the isolated (uncoupled) modes of the top plate, back plate or air cavity. When the plates are coupled to the rest of the body, they vibrate as part of a larger system resulting in modes in which the top plate, back plate and air cavity and all play a part. The modes of the fully coupled instrument therefore involve motion of the whole body. It is, however, common practice in the literature to refer to coupled modes of the body as 'top-plate modes', 'back-plate modes' or 'air-cavity modes'. For example, the $T(1,1)_2$ mode, which arises due to the coupling of the fundamental top-plate mode with the air cavity, is often referred to simply as the 'fundamental top-plate mode'. The reader should be aware of this language of convenience.

Jansson (1971) was the first to publish holographic interferograms of some of the guitar top-plate modes. In this paper he uses the holographic data to interpret the peaks observed in the measured sound pressure responses of the guitar. A number of sound pressure responses were measured using a variety of driving positions on the top plate and a variety of microphone positions. The differences in the responses thus obtained underline the importance of carefully choosing the position of driver and microphone. The radiation fields of some modes are such that a microphone placed directly in front of the centre of the top plate will not pick up any signal; the peaks corresponding to such modes would be missing from the response. This is particularly true of the symmetrical modes such as the T(2,1) and T(2,2). These produce radiation fields of a dipole character and tend to radiate poorly to the front of the instrument. Jansson did in fact find that some of the so-called symmetrical modes radiated some sound to the front of the instrument indicating that their shapes were not totally symmetric.

Jansson also points out that when the driving position is close to a nodal line of one of the top-plate modes, the mode will be very weakly driven and the peak will again be absent

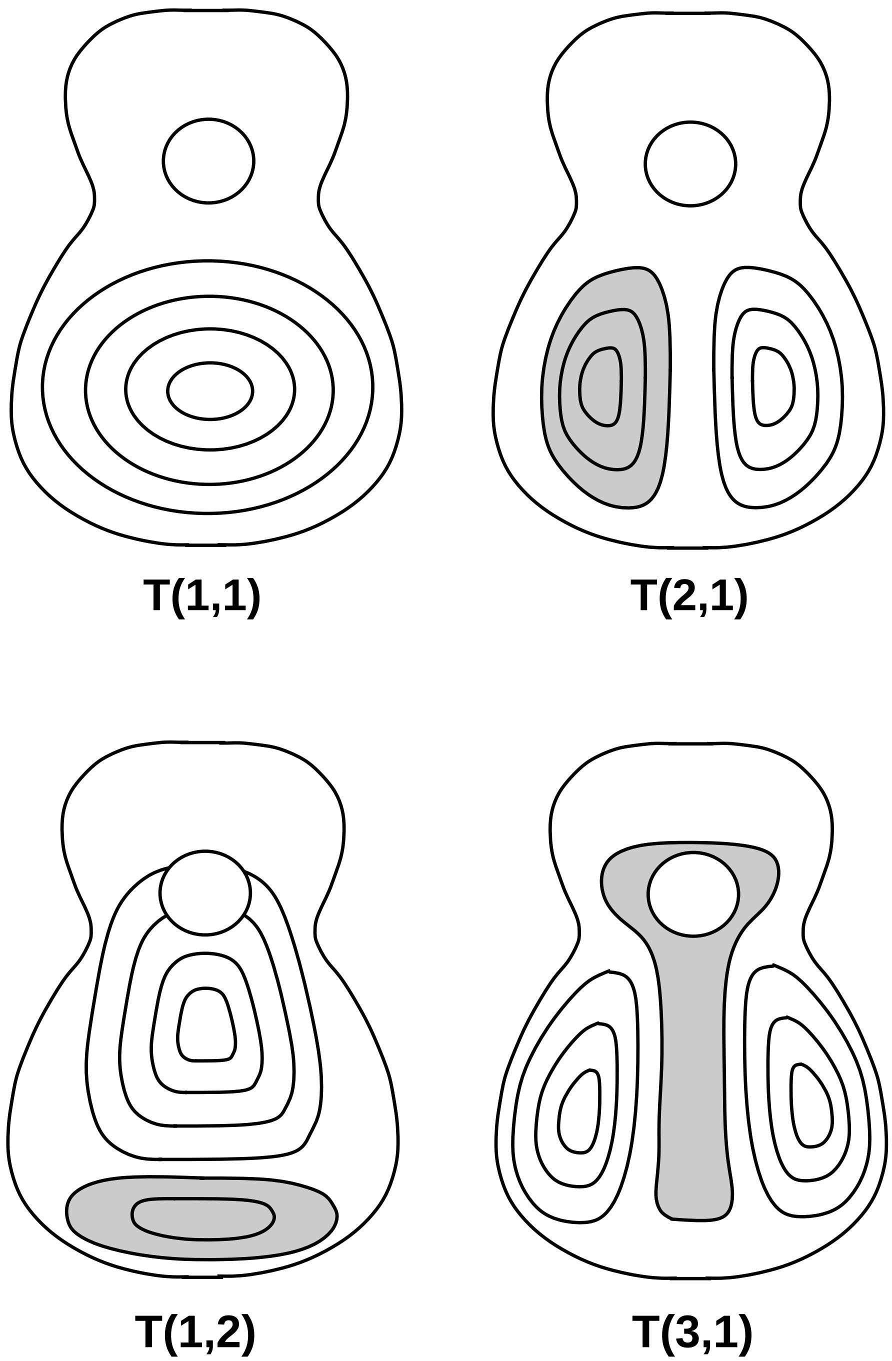


Figure 2.1: Mode shapes of the four lowest top-plate modes and labels used to identify them. Numbers in brackets refer to the number of antinodal regions measured across and along the plate. Contour lines of vibration amplitude are indicated. Areas of the top plate that move with opposite phase are indicated by different shading.

from the response. From the different frequency response measurements he obtained accurate values for the frequencies and Q-values of the lowest five modes and was able to confirm that each mode visualised holographically corresponded to a peak in the sound pressure response.

Moral and Jansson (1982) visualised the body modes of four different violins using TV speckle interferometry which allows the mode shapes to be viewed in real time on a TV screen. Two modes were found in the frequency range 150–300 Hz in which the whole instrument underwent a one-dimensional, bar-like motion. In the region 300–800 Hz three modes were found in which corresponding areas of the top and back vibrated in phase causing the whole body to vibrate in a similar way to a free plate. The amplitude of vibration was found to be a maximum at the edges of the instrument. At higher frequencies (700–2000 Hz), the modes were found to have nodal lines at the ribs so the vibrations tended to be confined to either the top or the back plate.

A complete modal analysis of a violin was performed by Marshall (1985). The instrument was excited using an impact hammer with a medium-hard rubber tip, which was found to give a uniform energy distribution between 10 and 1800 Hz. An accelerometer was used to measure the body vibrations, and the signals from both the impact hammer and accelerometer were fed to a microcomputer for storage and analysis. The violin was mounted using rubber bands attached to a metal framework. Marshall emphasises the importance of carefully checking any resonances associated with the mounting system to ensure that they are well below the lowest resonance of the instrument under investigation.

190 locations on the violin were selected. The accelerometer was fixed in place at one location and the hammer was used to excite the violin at each of the other points in turn. A small number of responses were measured with the accelerometer at two other positions yielding a total of 238 impulse responses. The data was then processed revealing 35 different modes below 1300 Hz. Mode parameters were extracted from the data giving values for the frequency, damping coefficient and amplitude distribution for each mode. These parameters were then used to synthesise a response at a location not previously measured. Experimental measurements were made at the new location and comparisons with the synthesised response showed agreement in amplitude to within 10% in the frequency range 100–1300 Hz, indicating that the violin body is a linear system that is well represented by a summation of real normal modes.

The modes were categorised by Marshall as either bending modes, air modes or plate modes. A number of bending and twisting modes were found below 800 Hz. These are not good radiators of sound but Marshall suggests that they are likely to be important to the player as they give subjective impressions of the 'feel' of an instrument.

Three air modes were identified, the lowest of which is commonly called the Helmholtz mode. In this mode the top and back plate both move in and out together so that the volume of the instrument increases and decreases periodically in a breathing motion. The second air mode was found to interact with a plate mode at similar frequency. A large number of plate modes were identified, all involving motion of both top and back plate although the top plate vibrations were often much larger than those of the back.

A great deal of work relating to the modes of violins has been published by Hutchins (1962, 1981). One of her main interests is the way in which measurements of the modes of the free plates can be used to predict the tone quality of the assembled instrument. She has placed particular emphasis on 'plate tuning' in which the maker carefully removes small amounts of wood from particular areas of the plate in order to adjust the frequency or Q-value of a particular mode. The relationship between the modes of the free plates and the modes of the assembled instrument are, however, complex and still relatively poorly understood.

Modes of the air cavity

Rather less work has been done on air-cavity modes of stringed instruments than their body modes. Jansson (1977) estimated the frequencies, Q-values and density of air cavity modes for guitar and violin-shaped cavities by applying first order perturbation theory to solutions of the wave equation for rectangular and cylindrical cavities. Comparisons with experimental data showed that the resonance densities could be well predicted from data on the volume, wall area and edge length of the cavity. Figure 2.2 shows some of the air-cavity modes measured by Jansson for a guitar-shaped cavity.

Three different sets of cavity-width measurements were used to predict mode frequencies. Firstly, the maximum widths (in the upper bout, waist area and lower bout) were used; secondly, a set of averaged widths was used and finally, perturbations on the first set of width measurements were used. The first and second methods predicted resonance frequencies that tended to be either side of the measured values. The third method, using perturbation theory,

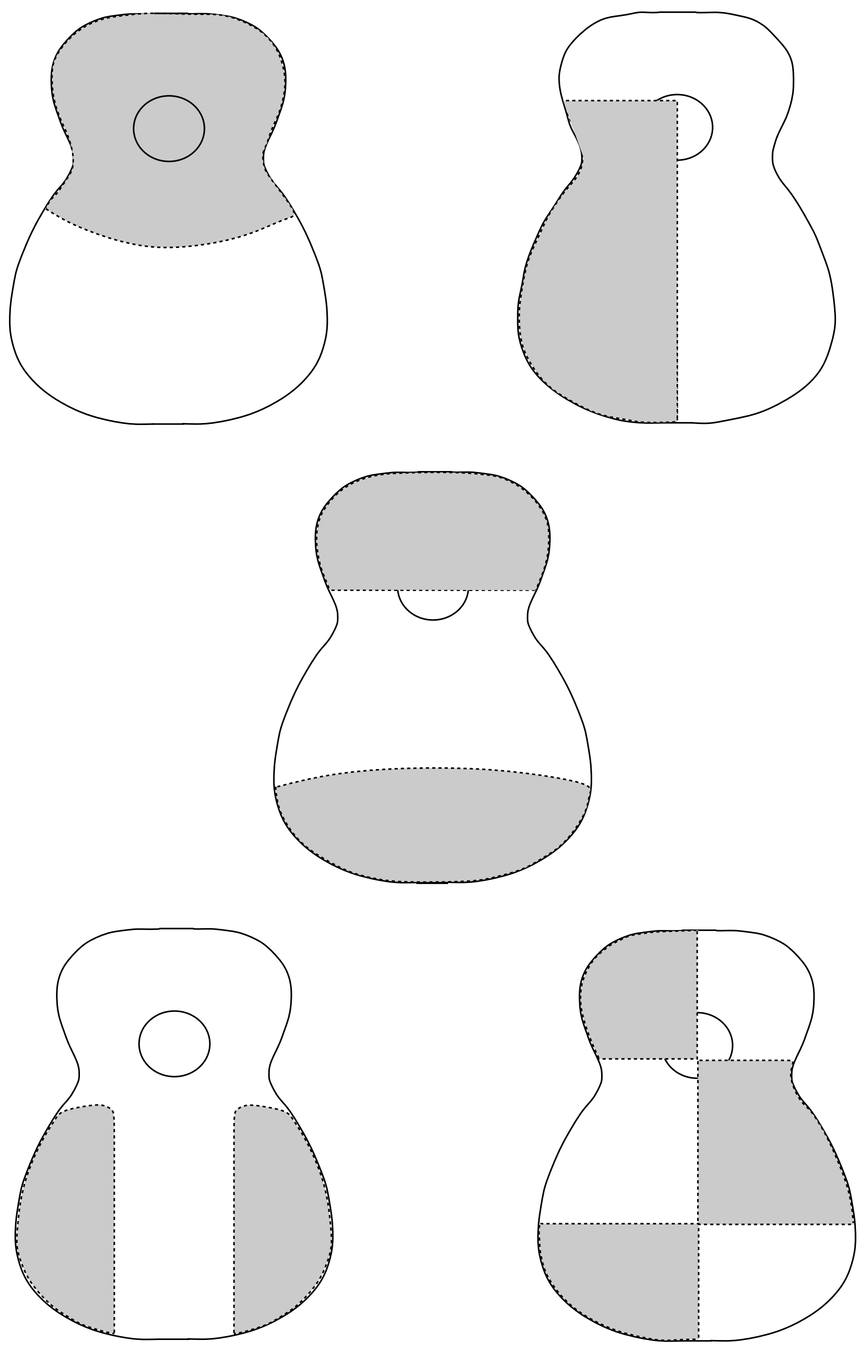


Figure 2.2: First five internal air-cavity modes of a guitar-shaped cavity; areas moving in anti-phase are indicated by different shading. Figure adapted from Jansson (1977).

gave predicted mode frequencies that were no more than 7% away from the measured values. The degree of arching (in the violin) and the size of the soundhole were found to have only a small effect on the frequencies of the air-cavity modes.

Estimates of the Q-values of the modes were made from measurements of the cavity volume, wall area and absorption characteristics of the wall. Jansson predicted that the Q-values of the modes would be largely determined by losses in the walls. Q-value measurements on the five different experimental cavities in fact showed moderate differences between them despite the fact that they had approximately the same values for cavity volume and wall area. The measured Q-values were found to be in reasonable agreement with experimental data only for the modes with frequencies greater than 2 kHz.

2.2 Coupling between top plate and air cavity

Meyer (1982) was the first to provide experimental results that indicated that the lowest two resonance peaks in the guitar's response curve result from the coupling between the fundamental modes of the top plate and air cavity. Sound pressure responses were measured in an anechoic room using an array of six microphones with the guitar being driven at the centre of the bridge. When the guitar body was filled with foam rubber cubes the lowest resonance disappeared and the frequency of the second resonance was slightly lowered. The guitar was then embedded in sand to prevent vibrations of the sides, back and top. The response of the instrument showed only the lower resonance remaining, though at a somewhat higher frequency than when the whole guitar body was free to vibrate. The top plate of the instrument was then freed, the sides and back remaining embedded in the sand. The response showed both of the lower resonance peaks at frequencies almost the same as when the whole guitar body was free to vibrate. These results provided strong evidence for the existence of coupling between the fundamental modes of the top plate and air cavity.

Meyer then conducted a series of experiments in which a variety of top plates were glued to a massive wooden frame whose inside contours matched those of a guitar. A number of top plates with different mode frequencies and strutting arrangements was tried, and the relationships between the mode frequencies of the coupled system and the mode frequencies of the uncoupled top plate and cavity were investigated. Addition of a flexible back plate introduced a third resonance peak and also modified the frequencies of the two lower peaks. Similar work was performed at Cardiff (Richardson et al., 1986) in which the coupling between top plate, back plate and air cavity was investigated using a guitar-shaped frame onto which a variety of top plates could be glued. Only top-plate modes at frequencies below 600 Hz were found to couple significantly to the air cavity and back plate.

Other investigations of the lowest two resonances of the instrument include work by Firth (1977) who used powder pattern (Chladni) techniques as well as holographic interferometry to obtain information on the shapes of the top-plate modes of a guitar. Measurements of the input admittance, its phase relative to the driving force and the radiated sound pressure were made in the frequency range corresponding to the first two resonances (approximately 70–250 Hz). The driving point initially chosen was at the low E string position on the bridge, but this excited both the first and second top-plate modes. To simplify the response, the driving point was moved to the centre of the top plate. This corresponds with the nodal line of the second mode (the T(2,1) resonance) and so the peak corresponding to this mode was absent from subsequent responses, making an investigation of the first two modes simpler.

The frequency response measurements were made with the soundhole open and then with the soundhole sealed. When the soundhole was closed the lowest peak (at around 100 Hz) disappeared from the response and the frequency of the second mode was lowered indicating that the two modes were coupled. At frequencies below the bottom resonance the sound pressures contributed by the top plate and soundhole were found to be almost 180 degrees out of phase causing a rapid decline in sound pressure. Between the two resonances the phase difference between the two sound pressure contributions is small and the overall sound pressure level is increased. Firth describes the action of the guitar at these low frequencies by using an analogous acoustical circuit, similar to those used to describe the bass-reflex action of a loudspeaker. He emphasises that values for the uncoupled resonance frequencies of the fundamental top-plate and air-cavity modes can never be found by direct measurements on real instruments because the coupling between the two can never be totally removed.

Work on the behaviour of the lowest two modes of the guitar was also performed by Caldersmith (1978). Rather than using equivalent circuits to describe the behaviour of the two lowest modes, Caldersmith develops a set of Newtonian equations to solve the problem. He ignores the motion of the back plate as it is measured to be of much smaller amplitude than the top-plate motion at the frequency of the T(1,1) mode. A set of equations is developed which describes the coupling between the fundamental top-plate mode and the motion of the air mass in the soundhole. At these low frequencies the wavelength of sound involved is larger than the dimensions of the guitar, so the radiation in the far field can be approximated to that produced by a simple volume source. The coupling between the two modes is seen to produce a sound pressure response with two peaks. The lower peak is at a frequency somewhat below the uncoupled Helmholtz frequency and the upper peak is at a frequency slightly above its natural resonance.

Examination of Caldersmith's solution for the motion of the coupled top plate and air cavity confirms some of the points already mentioned by Firth (1977). Below the frequency of the lower peak the air mass in the soundhole vibrates out of phase with the top plate; above the lower peak, they vibrate in co-phase. The motion of the air piston is found to be the dominant contribution to the sound pressure at the lowest body resonance. At the higher resonance it is the top-plate motion that contributes most to the sound pressure.

Caldersmith goes on to discuss the interaction of the near-field flows produced by the air piston and the top plate. Solutions to his equations are presented for three cases: negligible overlap between the two near-fields, intermediate overlap and complete overlap. The overlap is found to decrease the amplitudes of the peaks in the sound pressure response; it also alters the frequencies of the two peaks to values closer to their uncoupled frequencies. For the lowest body modes of the guitar, the soundhole is sufficiently far from the position of maximum top plate displacement that only small overlap occurs. This will therefore cause a moderately weaker response at these low frequencies.

Christensen and Vistisen (1980) also used a Newtonian scheme to model the admittance (velocity per unit driving force) and sound pressure response of the guitar in the region 60–300 Hz. Experimental measurements of admittance and sound pressure were made in an anechoic chamber with the guitar driven at the centre of the bridge to avoid excitation of the second top-plate mode. Motion of the back plate was suppressed. Masses were attached to the top plate and the frequencies of the two lowest peaks were decreased. Similarly, when a cardboard collar was inserted into the soundhole to increase the effective length of the air mass in the soundhole, the frequency of both peaks moved down, again indicating the existence of coupling between the fundamental modes of the top plate and air cavity.

The authors created a model with two simple harmonic oscillators with resonance frequency f, Q-value Q and effective mass m, each driving a piston with effective area A. The motion of the top-plate piston causes pressure changes in the air cavity that drive the air piston. Again, the sound pressure radiation in the far-field is approximated to that produced by a simple volume source. All parameters for the two pistons, apart from the Q-values, are obtained from the experimental measurements. The admittance and sound pressure response curves calculated from the model match the experimental responses to within 4 dB over much of the frequency range. Above the second resonance, contributions from other top-plate modes cause slightly larger deviations. The low-frequency response of the guitar is hence seen to be accurately described by this simple two-oscillator model using just four parameters for each oscillator. Values of the effective mass of the fundamental top-plate mode were measured on four guitars and found to be within the range 60 to 110 g. The effective area of the top plate was found to be around 550 cm^2, approximately 50% of the actual area of the lower bout.

This two-oscillator model is extended in a later paper (Christensen 1982) in which a third oscillator is added to account for the motion of the back plate. All three oscillators are coupled together via the common pressure changes in the air cavity, the addition of a flexible back introducing a third peak in the response. When the uncoupled resonance of the back plate fundamental mode is higher than that of the top plate, the frequencies of both of the first two peaks are lowered. If the back plate uncoupled frequency is lower than that of the top plate, the peak corresponding to the Helmholtz resonance is lowered and the peak corresponding to the top-plate mode is raised. Christensen describes the action of the coupling between the three oscillators as a 'repulsive force' which tends to move the peaks away from each other. Measurements were made on nine different guitars to extract the model parameters for the first two oscillators. The influence of the fundamental back-plate mode was found to be significant in only two of the nine guitars.

2.3 The string

Simple treatments of the motion of a rigidly supported, perfectly flexible string yield normal modes that are perfectly harmonic. In other words, the frequency of the nth mode is n times the fundamental frequency. The mode frequencies of real strings will deviate from perfect harmonicity because of yielding supports, finite stiffness or variations in tension caused by large amplitude vibrations. Some of the theoretical treatments of these non-linear effects are given below. Aspects of the non-linear behaviour of the string were studied by Gough (1984). In the small amplitude, linear regime, a string given some angular momentum during plucking will execute a stable, stationary, elliptical orbit of decaying amplitude. With larger amplitudes of vibration, the non-linear behaviour of the string becomes important and causes precession of the elliptical orbit. Gough found that the rate of precession is proportional (to first order) to orbital area and the perturbation of the mode frequency is proportional to the mean square radius vector. Hence the frequency perturbation and rate of precession both fall as the amplitude of vibration falls.

The string displacements, measured in the two transverse directions, show a characteristic rise and fall in amplitude as the major axis of the elliptical orbit rotates and energy is exchanged between the two transverse modes. A number of computer simulations of the string vibrations were performed and found to be in good agreement with experimental measurements.

Fletcher (1964) performed a theoretical analysis of the modes of a stiff piano string for two sets of boundary conditions: pinned and clamped. The following expression for the partial frequencies, valid for both boundary conditions, was obtained

$$f_n = nF(1 + Bn^2)^{\frac{1}{2}} , \tag{2.1}$$

where n is the partial number, and both F and B are constants which can be determined from accurate measurements of two string partial frequencies. From this expression it is clear that the inharmonicity will be greater for higher partials (large n), and Fletcher observed a deviation from true harmonicity of up to two full tones (26%) for the 50th partial of some strings. Calculated frequencies for the string partials were then compared with experimental measurements for two piano strings, and excellent agreement to within around 0.25% was obtained.

An overview of the behaviour of plucked, struck and bowed strings is given by Fletcher and Rossing (1991). Many aspects relevant to strings on musical instruments are discussed. An analysis is given of the different mechanisms through which energy is lost by the string and the relative effects of air damping, internal damping, damping due to sound radiation from the instrument body and damping due to the fingertip are assessed for metal, gut and nylon strings.

2.4 Coupling between string and body

The coupling between the body modes of a guitar and string motions was investigated by Jansson (1973). Static forces were applied to the strings, in the three orthogonal directions, and the resulting body deformations measured. The orthogonal directions x, y, z are defined in Figure 2.3. Coupling between string and body in the x-direction was found to be independent of the string; all forces applied in this direction coupled almost exclusively with the T(2,1) mode. Forces applied in the y-direction coupled with a mixture of modes, the T(1,2) mode being dominant. In the z-direction, coupling occurred almost exclusively with the T(1,1) mode. The source strengths for each of the body modes were estimated from the results of the body deformation measurements.

Jansson goes on to discuss the implications of these results for the guitar maker in terms of bridge design. Increased height of the bridge could be used to increase the coupling between the body and the x and y motion of the string. Reducing the bridge stiffness would tend to increase the excitation of the T(1,1) mode relative to the T(2,1) mode, and an asymmetrical bridge will increase the output from the T(2,1) mode.

One of the consequences of coupling between string and body is the phenomenon of the wolf-note, which occurs in the guitar, violin and viola, but affects the cello particularly strongly. A wolf-note occurs when the frequency of one of the string modes (usually the fundamental) coincides with a strong body resonance, often the fundamental top-plate mode. In the bowed string instruments, coupling between string and body produces an unstable pattern of vibration with a resulting sound that is unsteady and rather unpleasant. The phenomenon has been studied by a variety of authors (Gough, 1980; Benade, 1975; Firth, 1978).

Experimental measurements of the behaviour of a string mounted on a violin were made by Gough (1980). The violin exhibited a pronounced wolf-note around 460 Hz, due to a strong body resonance, when played in high positions on the G-string. The string was driven using a localised, sinusoidal electromagnetic force and its displacement was measured, using

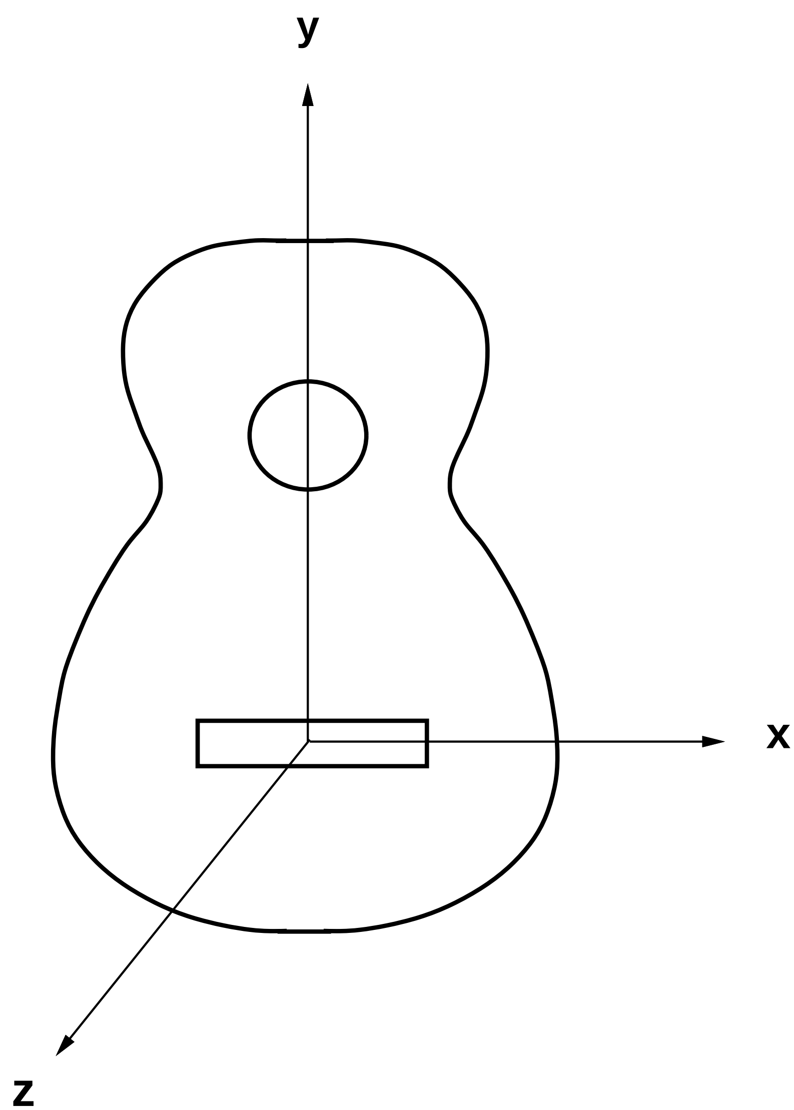


Figure 2.3: Definition of the three orthogonal directions (see Section 2.4).

phototransistors, in the vicinity of this body resonance. The measurements showed a characteristic split resonance, confirming earlier theoretical predictions by Schelleng (1963). The experiments also showed that two independent transverse string modes existed, only one of which coupled strongly to the body.

A theoretical investigation of the resonances of a string mounted on a musical instrument was carried out by Gough (1981). He treats the case of weak coupling between string and body as a small perturbation and finds that the frequencies of the string modes are slightly modified and their damping is increased. The increased damping is the result of energy losses from the string to the body, and the changes in mode frequency can be visualised as changes in the string's effective length, as shown in Figure 2.4. For a string resonance at a lower frequency than that of the body mode, the bridge moves with the same phase as the forces acting on it and the string's effective length is increased, hence the resonance frequency is decreased. For a string mode at a higher frequency than that of the body mode, the bridge moves with opposite phase to that of of the force acting on it thus decreasing the string's effective length and increasing the frequency of the resonance.

For the case of strong coupling between string and body mode, Gough first considers solutions to the equations when the frequencies of the uncoupled string and body mode coincide. For this case he defines a parameter K, given below, to measure the degree of coupling where Q_b is the Q-value of the body mode, n is the number of the string partial, m is the mass of the string and M is the effective mass of the body mode:

$$K = \frac{2Q_b}{n\pi}\left(\frac{2m}{M}\right)^{\frac{1}{2}}. \tag{2.2}$$

For a string mode weakly coupled to the body, where $K < 1$, the coupling does not perturb the frequencies of the modes but their damping is increased. For strong coupling, where $K > 1$, the coupling splits the resonance frequencies symmetrically about the unperturbed frequencies. The damping of the both modes is the same, and approximately equal to $2Q_b$.[2] The low-frequency mode corresponds to the string and bridge moving in phase (with the string's effective length increased, Figure 2.4(a)), the high-frequency mode corresponds to

[2]This represents a change in damping of around one order of magnitude for the string resonance. Typical Q-values for uncoupled string modes are in the range 700–3000; typical Q-values of body modes are in the range 15–60.

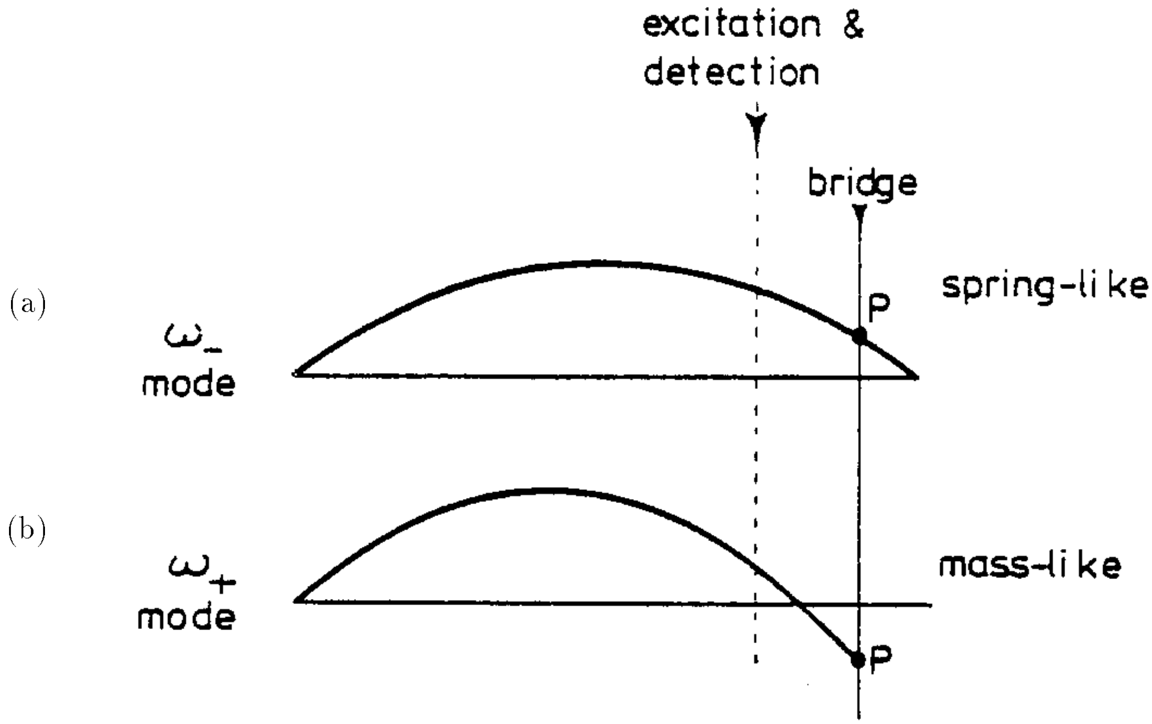


Figure 2.4: Changes to the frequencies of the string modes caused by motion of the end support. A string with a rigid end support will have a node at its end point (position P). A spring-like termination causes the bridge to move in phase with the string, creating an effective node to the right of point P. The effective length of the string is increased and its resonance frequency is decreased. A mass-like termination causes the effective length of the string to be decreased and its resonance frequency is increased. When the string is driven at the excitation point indicated, the lower frequency mode will dominate due to its lower impedance at this excitation point. Figure from Gough (1983).

out-of-phase motion of the bridge and string (with the string's effective length decreased, Figure 2.4(b)). Usually, the string is driven at a point relatively near the bridge, causing the low-frequency mode to be more efficiently driven since the node for this mode is more distant from the plucking position.

An extension of these results for the case where the uncoupled string and body mode frequencies are not coincident is then discussed, showing that for strong coupling, the coupled mode nearest in frequency to the unperturbed string resonance will dominate the response.

Gough (1983) outlines his approach to studying the behaviour of the body of a stringed instrument by measuring the resonances of the string ('string resonance spectroscopy'). Using this technique he shows that the damping imposed on strongly radiating body modes due to fluid loading may account for around half of the total damping. The Q-value of the 'main body mode' of a violin increased from 17 to 30 when the instrument was placed in an evacuated chamber.

2.5 Radiation of sound

Christensen (1984) developed a simple oscillator model to account for the response of several top-plate modes. The sound pressure response up to around 800 Hz was modelled by a superposition of monopole contributions from single oscillators, each oscillator corresponding to a mode of the top plate. As before, each oscillator is defined by four parameters: a resonance frequency f, a Q-value Q, an effective mass m and an effective monopole area A.

Some of the higher top-plate modes have several areas vibrating in antiphase (see Figure 2.1) and so the effective monopole area must be defined as the area which, when moving with the velocity of the driving point, produces the actual net volume displacement of the source. This means that modes above the fundamental top-plate mode may have negative values for the effective area. The combination of several top-plate modes with both positive and negative piston areas allows considerable variability in the sound pressure response, as shown in Figure 2.5.

The sound pressure responses of five hand-made guitars were measured in an anechoic chamber and curve-fitting was used to extract values of f, Q and the ratio A/m for several modes of each guitar. Coupling to the Helmholtz air-cavity mode was not included. Figure 2.6

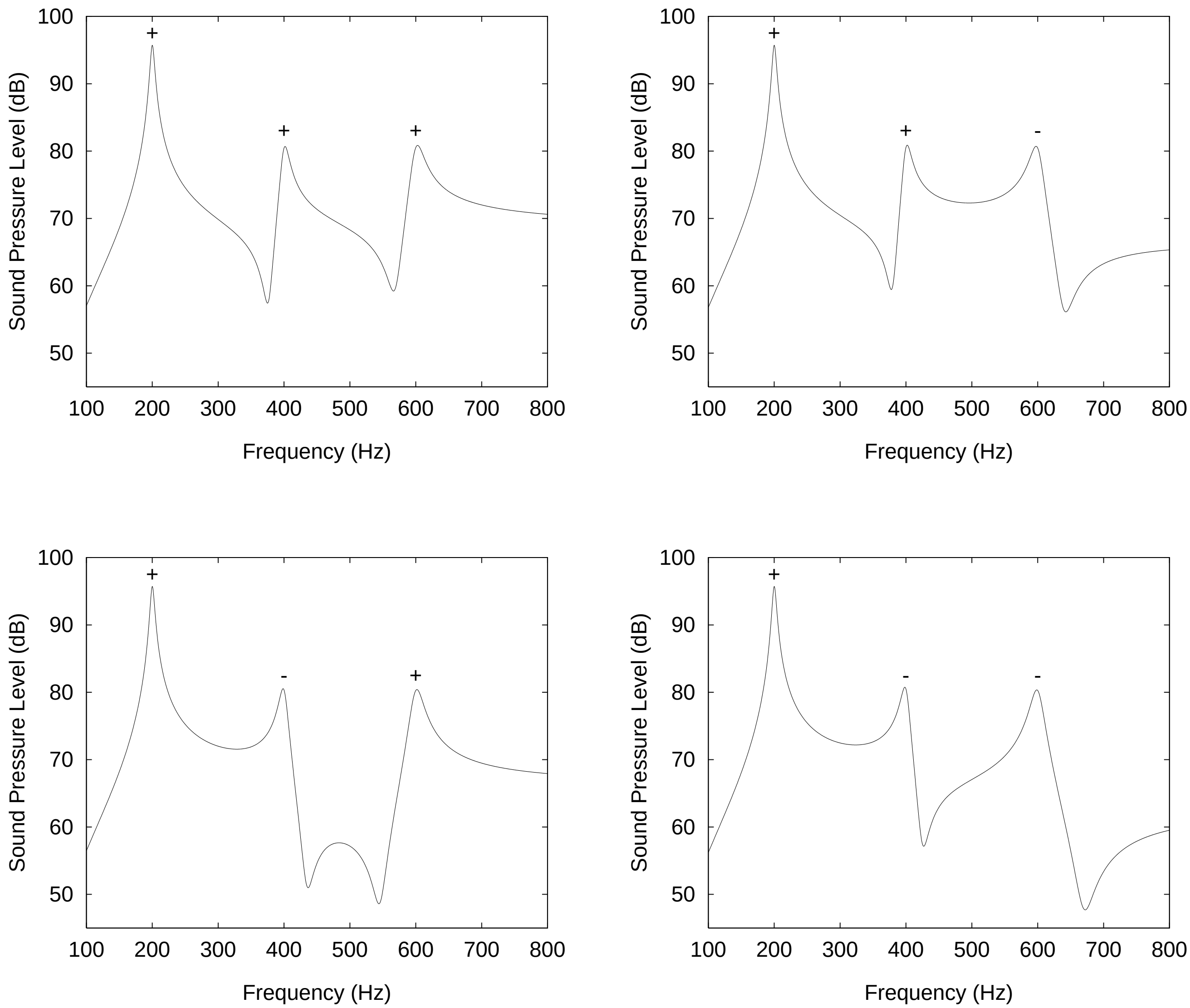


Figure 2.5: Different sound pressure responses achieved by varying the sign of the effective piston areas. Data for the frequency, Q-value, and A/m ratio is the same in all four responses (f=200, 400 and 600 Hz, A=0.06, 0.01 and 0.01 m^2; m=100 g and Q=30 for all oscillators). The sign of the effective area is indicated by a + or − above each peak. The sound pressure is calculated for a driving force of 1 N and a listening distance of 1 m. Figure adapted from Christensen (1984).

shows the five measured response curves and the best-fit lines obtained by modelling the instruments' sound pressure responses as a superposition of monopole sources. Curve-fitting was only attempted up to a frequency of around 600–800 Hz.

The mode parameters determined for the five guitars show that the ratio A/m for the fundamental top-plate mode is approximately one order of magnitude greater than the higher-frequency modes. The ratio A/m is proportional to the sound pressure radiated by a single mode and is a convenient measure of the relative radiative power of the different modes. Christensen shows that the contribution to sound pressure from one mode approaches a constant at high frequencies. Therefore, in the region where monopole radiation dominates (up to 1 kHz or more), the fundamental top-plate mode is the most important radiator of sound.

One other interesting feature that arises from the curve-fitting work is the differing contributions of the symmetrical top-plate modes to the sound pressure. As described in Section 3.7, modes such as the T(2,1), with a line of symmetry running from top to bottom through the centre of the top plate, produce no significant monopole radiation because the equal volume displacements created in the left and right halves of the top plate tend cancel each other out. Diagonal struts can be used on the top plate to introduce a degree of asymmetry into these modes, causing the left and right halves to produce different volume changes. The mode will then produce a significant net volume displacement so that its contribution to the sound pressure is increased. Christensen found that two of the five guitars examined (numbers 1 and 2 on Figure 2.6) had a diagonal strutting arrangement and the sound pressure responses of these two instruments showed a stronger contribution from the T(2,1) mode (at around 270 Hz) than the instruments with symmetrical strutting.

A study of the frequency distribution of the radiated sound energy from a guitar was made by Christensen (1983). Acoustic power density spectra were obtained from recordings of guitar music and many spectra were used to calculate a long time average. Each spectrum has a large number of peaks, some corresponding to body modes that radiate strongly, others corresponding to the discrete pitches of a certain musical key. The large number of peaks in the spectrum made interpretation difficult, so an integration was performed to calculate the percentage of the total radiated energy that was contained below a certain frequency. This distribution showed a number of steps that corresponded to modes of the guitar. The radiating power of the modes could be measured by the height of the steps. Christensen

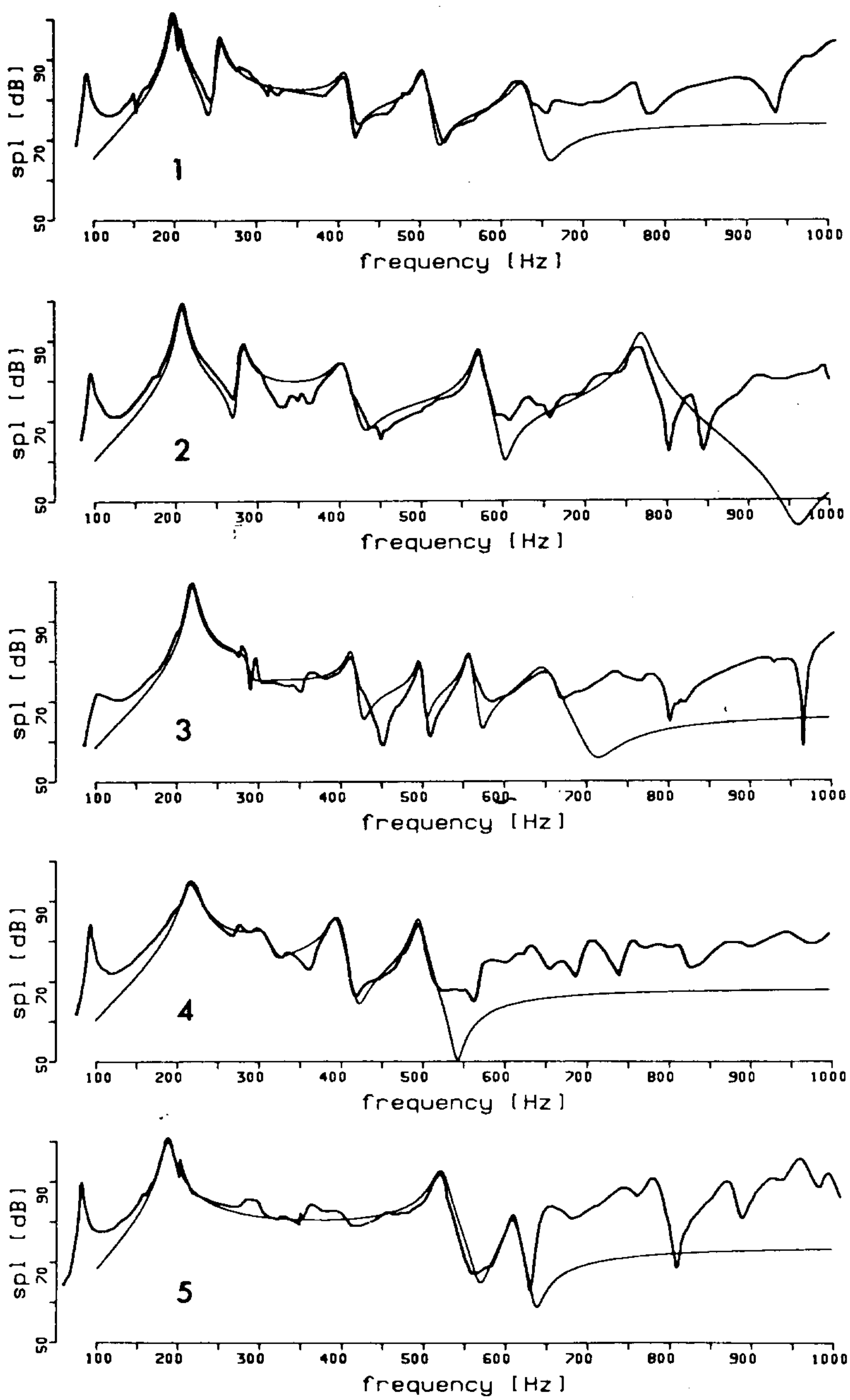


Figure 2.6: Comparison between measured sound pressure level responses (thin lines) and synthesised responses (heavy lines) for five guitars. Figure from Christensen (1984).

found that around 40% of the energy radiated below 4 kHz comes from the T(1,1) mode with a further 45% coming from the T(1,2) and T(3,1) modes.

Other work on the radiative power of the modes of the guitar was performed by Lai and Burgess (1990) who investigated the radiation efficiency of acoustic guitars by measuring the ratio of input power to radiated sound power. In the frequency range 50–550 Hz the radiation efficiency was measured to be about 0.13.

Some interesting work on radiativity of violins has been performed by Weinreich (1983, 1993). Measurements of violin radiativity were made using the principal of reciprocity, in which the violin body is excited with a signal from a loudspeaker (or combination of loudspeakers) and the resulting displacement of the bridge is measured. One of the important results stressed by Weinreich is the importance of the relative phases of sound pressures contributed by the different modes (also mentioned by Christensen (1984) above). Weinreich (1993) describes the sound pressure level between two modes as 'plateau-like' when the two modes have opposite phases for their radiativity, or 'notch-like' (containing an antiresonance) when the radiativity of the two modes has the same phase (see Figure 2.5). The antiresonance-type response can be inferred from local changes in the direction of change of the phase of the sound pressure.

Work by Rossing (1988) and Caldersmith (1985, 1986) discusses aspects of the guitar's radiation fields. The important points are that the geometry of the radiation fields is determined partly by the mode shape and partly by the frequency of the driving force. The triplet of modes formed by the coupling between fundamental top-plate, back-plate and air-cavity modes all involve motion of a single antinodal area in the top and back plates. The net result is an efficient pumping action that produces a large volume change. At frequencies close to their natural resonances (100–200 Hz), the wavelength of sound is of the order of one metre, rather larger than the dimensions of the guitar body. Such motion produces a spherical (monopole) radiation field. Other modes, such as the T(1,2) and T(3,1) are also relatively efficient volume pumps[3] although the net volume change results from motion of several unequal antinodal areas moving with opposite phases. At these low frequencies, such modes will also

[3]Caldersmith (1986) perhaps underestimates the radiation efficiency of the (1,2) mode calling it a 'poor radiator'. The net volume change produced by this mode is usually reasonably large, although he is correct to point out that the mode is often poorly driven by the string forces as its nodal line tends to lie very close to the bridge.

produce monopole-like radiation fields. At frequencies where the wavelength of sound in air is smaller than the dimensions of the guitar body, the radiation fields from these 'pumping' modes becomes less spherical and more directional, like a 'beam of sound'.

Multipole radiation from modes with high resonance frequencies becomes increasingly efficient at high frequencies. For modes with several antinodal areas, the radiation of sound may become concentrated into two narrow 'beams' above a so-called critical or coincidence frequency, at which the wavelength of the sound in air is equal to the wavelength of the bending in the top plate. Although there is no clear division between the frequency ranges in which monopole and multipole radiation dominates, it is convenient to create such ranges. At frequencies up to around 1 kHz the radiation from the instrument is dominated by monopole fields from modes that create a net volume change. Between 1 and 2 kHz the efficiency of this monopole radiation decreases and the efficiency of multipole radiation increases. Above 2 or 3 kHz the radiation from the instrument is likely to be dominated by multipole radiation.

2.6 Evaluation of an instrument's sound quality

Meyer (1983a) undertook a detailed investigation of the links between the subjective quality of a group of guitars, as judged by 40 people in a series of listening tests, and certain objective criteria determined by experiment. The listening tests involved the use of recorded extracts from six musical pieces performed on 15 different guitars. The extracts were presented in pairs and listeners were asked which guitar they preferred in terms of its tone quality.

The resonance characteristics of the guitars were measured by driving the guitar at the bridge and using the averaged signal from an array of six microphones to measure the sound pressure response. One potentially significant oversight of this work is the fact that only a single driving point is used. Some modes may be weakly driven at this driving point, and will not show up on the measured response curves. The same modes might be more strongly driven at other driving positions on the bridge.

Meyer then attempts to relate the subjective quality evaluations to the measured sound pressure responses by measuring the correlations between physical characteristics of the response (e.g. frequency, Q-value and peak amplitude of certain modes) and the subjective quality score. By comparing the measured values for each of these physical characteristics in turn with the subjective quality scores Meyer obtained 19 so-called quality criteria. Correlation values were obtained for 13 parameters, although it should be noted that only six of these correlation coefficients were greater than a half, the largest being 0.73. On reflection, it is not surprising that many characteristics achieved very low correlation scores or could not be correlated at all. The correlations measure the subjective importance of the parameter in isolation. Positive correlations of tone resulting from combinations of several quality criteria cannot be measured using this method. The use of only a single driving point may also have affected the number of criteria that could be correlated with sound quality.

The results, nevertheless, show that aspects of the guitar's sound pressure response can be linked to subjective assessments of its tone quality. Of the 13 quality criteria that could be correlated with subjective assessments of tone quality, 11 of these referred to the level of a resonance peak (or the average level in a certain frequency band) and the remaining two referred to the Q-values of the first and third resonance. Meyer emphasises that the contribution of the T(1,2) mode to the radiated sound pressure has a particularly strong effect on the perceived quality. Although the frequency position of the resonance peaks was tested, no correlation between mode frequency and subjective quality was found.

The amplitude of a resonance peak in the sound pressure response is determined by the ratio of effective area to effective mass (A/m) of each mode (Christensen, 1984). Meyer's work shows that the amplitudes of the peaks, and therefore the effective masses and areas of the modes, have the most significant effect on tone quality. This suggestion is confirmed by the results of the psychoacoustical listening tests presented in Chapter 7. Results from this chapter also indicate the relative unimportance of mode frequency in determining the tone quality of an instrument.

Further experimental work by Meyer (1983b) used a rigid, guitar-shaped frame onto which a variety of top plates were glued. Systematic changes were made to the top plates and the changes monitored in terms of the quality criteria described above. This detailed work highlights ways in which the tone quality of the guitar can be altered by suitable constructional changes. One of the most interesting experiments performed by Meyer was a study of the effects of using a variety of different bridges. Addition of a standard bridge adds both mass and stiffness to the top plate, resulting in a lowering of the output from many of the low-frequency modes. The response of the T(1,2) mode is seen to be be particularly strongly quelled when the bridge is added to the top plate. The bridge that had the most beneficial effect, as measured by nine of the quality criteria, was a bridge whose width (and therefore mass and stiffness) was considerably reduced.

2.7 Previous work at Cardiff

Much of the research in musical acoustics, carried out at the University of Wales in Cardiff over the past 15 years forms an important foundation for the work presented in this thesis. Some of the assumptions made in the modelling scheme outlined in Chapter 4 rely on results obtained by previous numerical modelling work by Walker (1991) and Brooke (1992). In this section I will briefly cover the historical background to the current research being undertaken at Cardiff.

Research into the modal properties of the guitar was started by Richardson (1982). The modes of the instrument were studied during different stages of the instrument's construction (e.g. before and after gluing the top plate to the rest of the body, and before and after addition of the bridge). Admittance and sound pressure response curves and holographic interferometry were used to study the body modes during the different construction stages. The measurements showed that the most severe change in the body's behaviour occurred after the bridge was glued to the top plate. Mode shapes obtained using holographic interferometry for a guitar complete with struts and bridge are given in Figure 2.7.

Work was also carried out on the factors which affect the radiated sound of the guitar. This highlighted the important differences between motion of the string in the 'vertical' (perpendicular to the top plate) and 'horizontal' (parallel to the top plate) planes. Figure 2.8 shows the measured input admittance of the guitar, measured at the bridge in three orthogonal directions. The input admittance in the 'vertical' direction is greatest throughout much of the frequency range. The strength of coupling between the string and body was shown to have a strong influence on the amplitude and decay of the radiated sound.

Finite element analysis

Finite element analysis (FEA) is a numerical technique that is used to calculate the vibrational response of an object to a sinusoidal driving force applied to a point on its surface. The object

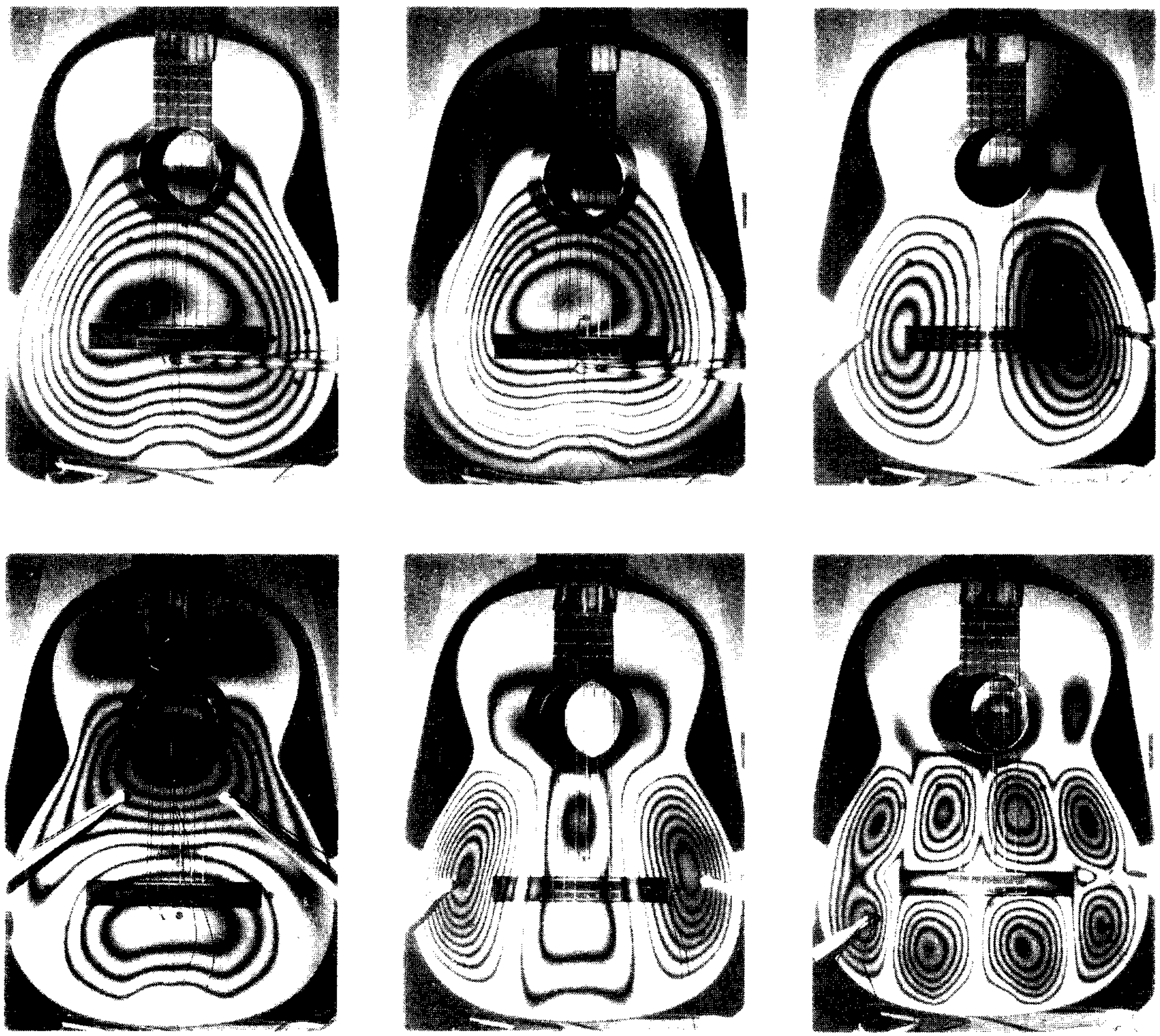


Figure 2.7: Modes of the guitar visualised using holographic interferometry, from Richardson (1990). Mode frequencies are (left to right, top to bottom) 106 Hz, 216 Hz, 268 Hz, 431 Hz, 553 Hz, 1010 Hz.

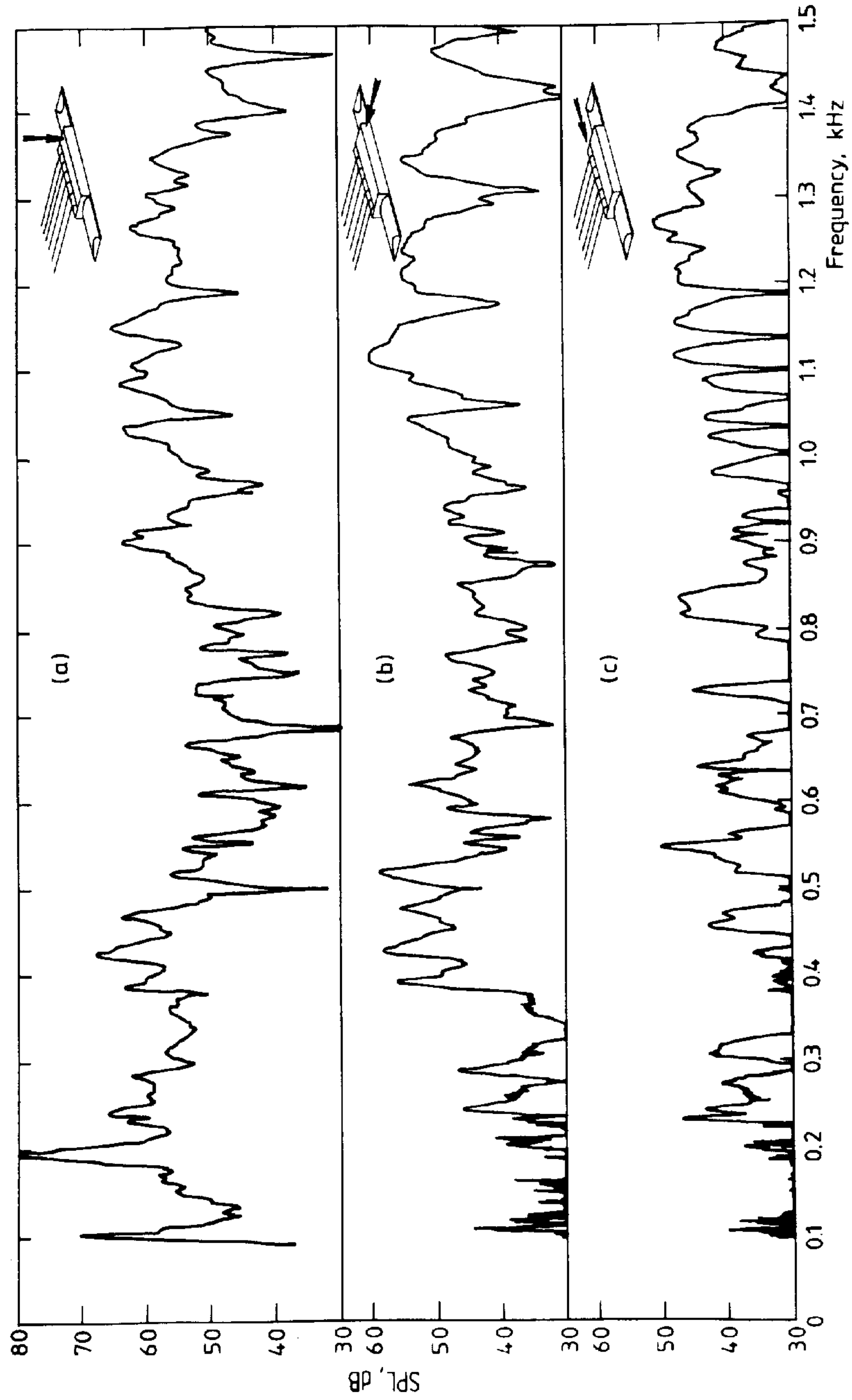


Figure 2.8: Sound pressure responses of guitar BR2 obtained by driving the bridge in three orthogonal directions, as indicated. The same driving force and microphone position were used for each measurement; the microphone was placed in front of the top plate at a distance of 1 m. Figure from Richardson (1982).

is broken up into a number of small elements, material properties and dimensions are input into the computer and the equations of motion for the elements are solved yielding the modes of vibration of the object. The technique allows the normal modes of complicated three-dimensional structures to be calculated. Without the implementation of FEA on powerful computers, the normal modes of complex structures would have to be approximated to those of objects with much simpler shapes and geometries. For the musical acoustician, FEA can yield very valuable information on the vibrational behaviour of certain instruments.

Early work using FEA to study the vibrations of a guitar top-plate was performed by Schwab and Chen (1976). Mode shapes and frequencies thus obtained compared favourably with measurements on real instruments, although the frequencies of modes which involved mostly across-grain bending were overestimated by the FEA. At Cardiff, FEA was first used by Roberts (1986) to model vibrations of conical bells, wooden plates and shells. Mode shapes and frequencies were obtained for a square, strutted spruce plate as well as a strutted, guitar top-plate. Results obtained from the FEA of the guitar top plate were in good agreement with data from holographic interferograms of real instruments. Roberts then used FEA to model the modes of violin plates as well as those of a violin body, complete with soundpost. Results obtained for the violin plates were again in good agreement with experimental data.

The finite element work was continued by Walker (1991) who modelled the modes of a guitar top-plate. Some results of this work at Cardiff are shown in Figure 2.9. The figure shows some of the normal modes of vibration calculated for a 2.6 mm thick spruce top plate complete with bridge and struts. Mode shapes given in Figures 2.9a, b, c and d are very similar to those obtained experimentally in Figure 2.7. The mode frequencies are also similar; the difference in the T(1,1) mode frequency is due to the absence of coupling to the air cavity in the FEA work. Walker used the data provided by the FEA in a numerical model of the guitar. Coupling of the top-plate modes to the fundamental modes of the air cavity and back plate was included, and admittance and sound pressure response curves for the complete guitar were calculated from the model. An experimental guitar rig was set up, similar to the one used by Meyer (1982 - see Section 2.2), so that the behaviour of the plate with and without coupling to the air cavity and back plate could be investigated experimentally. Holographic interferograms and frequency response measurements of the guitar rig were taken and compared with the results from the FEA and the numerical model.

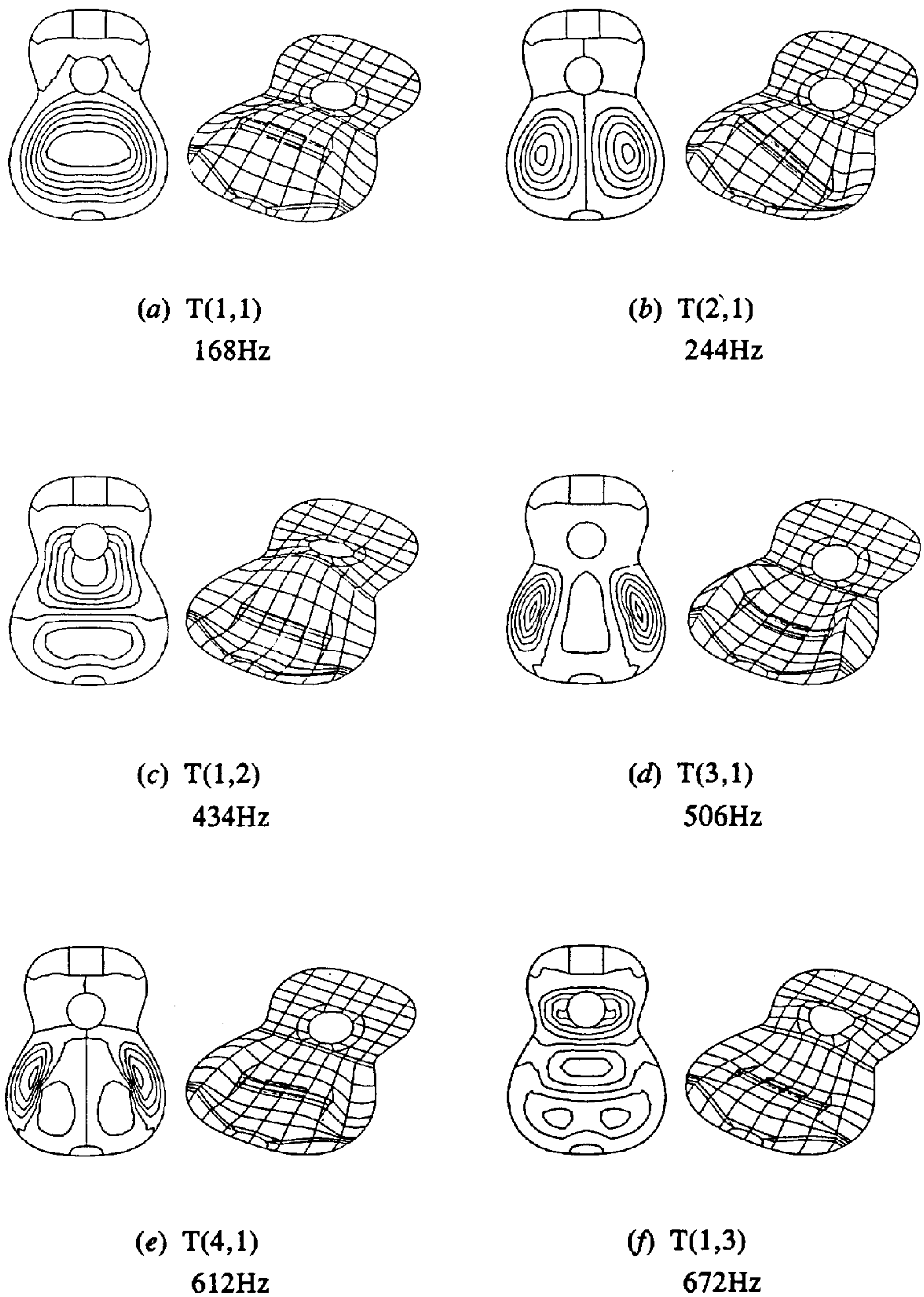


Figure 2.9: Finite element predictions for the normal modes of vibration of a guitar top plate. Contour plots of vibration amplitude and three-dimensional representations of the shapes are given for each mode. Figure from Brooke (1992), data from Walker (1991).

The boundary element method

Further numerical modelling of the classical guitar was performed at Cardiff by Brooke (1992). Mode data for the top plate was taken from the finite element work by Walker (1991), and the model calculated the coupling between the top plate and the fundamental air-cavity and back-plate modes. After coupling a string to the top plate, the radiation from the instrument was calculated using the boundary element method (BEM), a powerful numerical technique which calculates the three-dimensional sound radiation fields produced by bodies with a complex shape. The radiation fields for six top-plate modes radiating at 990 Hz are shown in Figure 2.10. The BEM predictions show that all of the radiation fields produced by the instrument can be well approximated by a combination of monopole and dipole sources. One further interesting feature of the BEM results is that the T(1,1) triplet of modes radiate more strongly than all of the other modes, despite being driven at a frequency around 800 Hz above their natural resonance. The T(4,2) mode (not pictured in Figure 2.10) has a resonance frequency of 970 Hz; despite being driven almost at resonance, it radiates more weakly than the T(1,1) modes.

2.8 Summary

The results of previous work which are of particular importance for this work can be summarised as follows. The fundamental modes of the top plate and air cavity are coupled together via common pressure changes in the cavity. A variety of models that describe this coupling has been proposed (Firth, 1977; Caldersmith, 1978; Christensen and Vistisen, 1980). The basis of the theoretical model outlined in Chapter 4 is a combination of two papers by Christensen; the first (1982) is the three-oscillator model which accounts for coupling between fundamental modes of the air cavity, top plate and back plate. The second (1984) is a model which predicts the far-field pressure response of a combination of top-plate modes. The coupling between string and top plate, as described by Gough (1981), is used to complement Christensen's oscillator model of the body, allowing the sound pressure response of the coupled string-body system to be evaluated.

The far-field sound radiation from the low-frequency body modes is assumed to be monopole in character, following Caldersmith (1978) and Christensen and Vistisen (1980). The

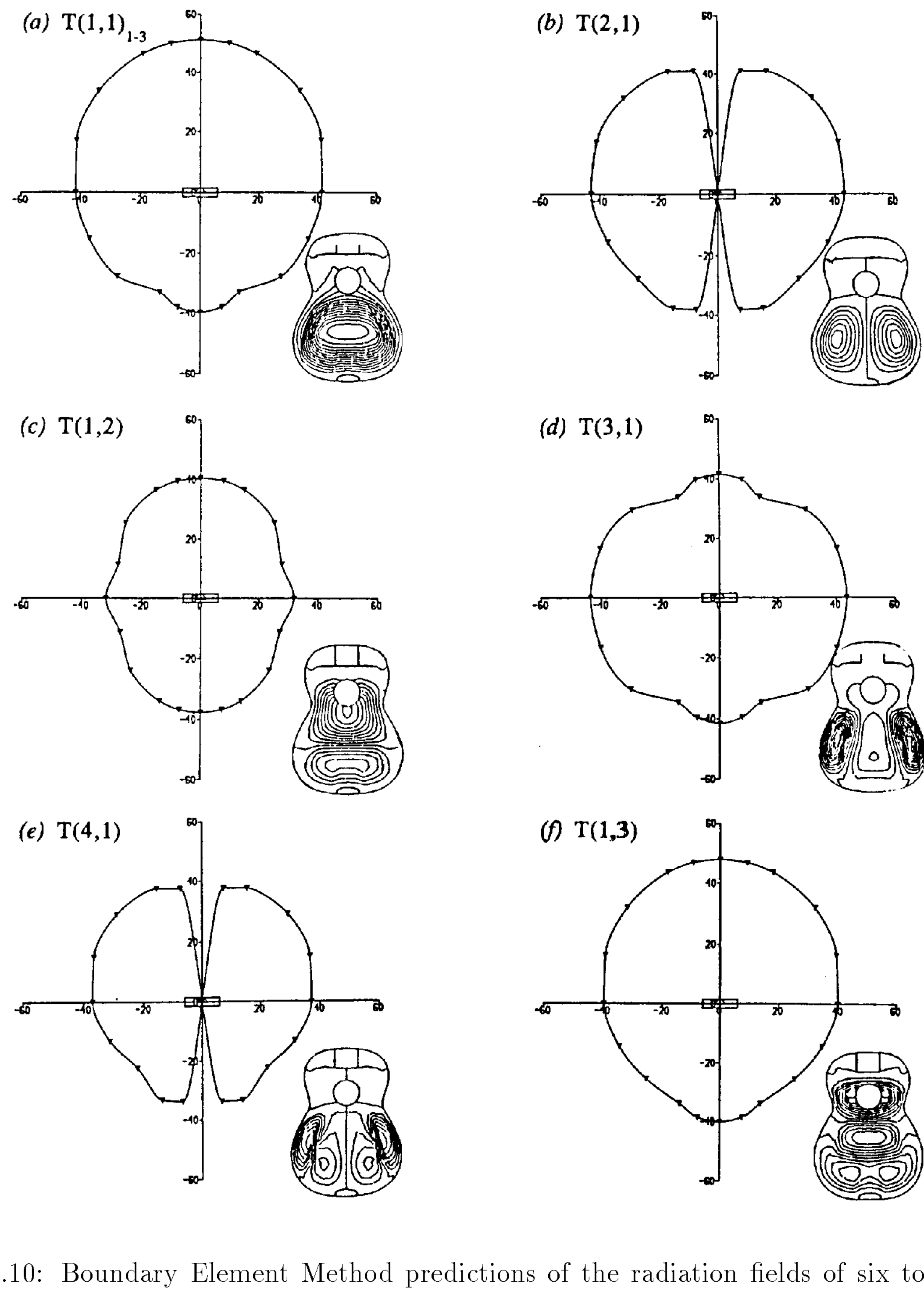


Figure 2.10: Boundary Element Method predictions of the radiation fields of six top-plate modes driven at 990 Hz. Sound pressure levels are calculated around a circle of radius 2.5 m in a plane perpendicular to the axis of the guitar. Mode frequencies are: (a) 98, 182 and 218 Hz (T(1,1) triplet), (b) 244 Hz, (c) 434 Hz, (d) 506 Hz, (e) 612 Hz, (f) 672 Hz. Figure from Brooke (1992).

BEM calculations by Brooke (1992) show that the radiation fields produced by many of the top-plate modes are, to a good approximation, monopole functions. The exceptions to this are the symmetric top plate modes which produce dipole radiation fields.

The numerical model of the guitar, presented in Chapter 4, replaces the BEM calculations of the guitar's sound radiation fields with the very much simpler method of using monopole sound sources. This saves a great deal of processing time; typical run-times on a mainframe computer for a single plucked note using the BEM were of the order of 10–20 hours. By avoiding the numerically intensive BEM calculations, the time needed to synthesise the sound pressure response of a single note could be reduced to around 5 or 10 minutes. This allowed a large number of sounds to be synthesised in a reasonable amount of time, so that more extensive psychoacoustical evaluation of the sounds could be achieved. One of the main objectives of this work was to examine the methods and results of previous modelling techniques used at Cardiff to find ways in which the essential sound-producing mechanisms of the guitar could be retained in the modelling scheme, while significantly reducing the amount of processing time needed by the model. The time-saving aspect allowed more emphasis to be placed on the psychoacoustical evaluation of the sounds synthesised from the model, so that the links between physical parameters used by the model and subjective assessments of tone quality could be examined.

In Chapter 4 I will outline the theoretical framework of the numerical model. Before this, in Chapter 3, I will present an overview of the sound-producing mechanisms in the guitar.

Chapter 3

The functioning of the guitar: an overview

In this chapter I will give a description of the functioning of the guitar as a musical instrument. The important processes that occur during sound production on the guitar have already been discussed in the literature review presented in Chapter 2. This chapter attempts to present the main ideas in a complete description of the sound production process of the guitar.

3.1 Plucking of the string

The process of sound production starts with the interaction of the finger and the string. The plucking interaction is often described in simplified form as a process in which a point on the string is pulled away from its equilibrium position, held for some time and then released with all points on the string having zero initial velocity. In reality the plucking process is more complicated, resulting in a complex distribution of velocities and displacements along the string at the time of release. The hardness or 'fleshiness' of the finger, the shape and stiffness of the nail, the movement of the finger and the frictional characteristics of the string and finger will all influence the way the string is released.

Rather little work has been undertaken on the details of this complex interaction. Current research at UWCC (Pavlidou, unpublished) involves numerical modelling of the string-finger interaction and is aimed at an understanding of how the properties of the string and finger influence the release conditions of the string and hence the final sound quality.

At the beginning of the pluck, the finger is moved by the player and the string moves with it due to frictional forces. When the frictional force reaches its maximum value, the string will begin to move relative to the finger. This motion is likely to have a 'sliding' component as well as a torsional or 'rolling' component. During this time travelling waves will move from the point of interaction towards the ends of the string. These waves will be reflected and return to the point of contact between finger and string influencing the local conditions. The string will continue to move over the fingertip and the nail to be released after an interaction time of between 10 and 50 ms.

The player can use a variety of techniques to control the release of the string. These include the speed with which the finger and nail are drawn over the string, the angle at which the string is released and the position of the finger along the string's length. The player can control these factors to vary the tone quality produced. Other subtleties of the contact between finger, fingernail and string are no doubt important to the player as they provide useful psychoacoustical information about the 'feel' of an instrument, but these are difficult to measure and quantify.

The terms string mode and string partial will be used throughout this thesis in preference to the terms harmonic and overtone. The effects of string stiffness and string-body coupling cause the frequencies of the string modes to deviate significantly from true harmonicity, so the term harmonic is rather misleading.

Plucking position

The influence of the plucking position on the string vibrations is well understood. When the string is plucked at a point corresponding to an antinode of a particular partial, this partial will be strongly excited in the initial vibration of the string. Similarly, plucking positions corresponding to a node of a particular partial will excite it only weakly. Simple theoretical treatments predict that such a partial, plucked at its node, will have zero amplitude. In reality, non-linear coupling between longitudinal and transverse string modes causes regeneration of suppressed partials. The position of the string while in contact with the finger or fingernail represents only its initial conditions and other processes are involved that influence the time-varying harmonic content of the string after it is released.

The fundamental string mode will be most strongly excited when plucked at its antinode, at the centre of the string. As the plucking position moves towards the bridge, the amplitude of the fundamental mode will fall relative to the other string partials. In general terms, the sound will become richer in high-frequency partials as the plucking position moves closer to the bridge giving a 'brighter' tone quality. This effect is illustrated in Figure 3.1 which shows the Fourier spectra of the same note plucked at two different positions on the string. The open low E string on guitar BR2 was plucked firstly at a position over the soundhole, and then at a position near the bridge. The note plucked over the soundhole excites only a small number of low-frequency string modes. The note plucked near the bridge has many more high-frequency modes excited, but the fundamental string mode (at 82.4 Hz) is almost absent.

Width of the fingernail

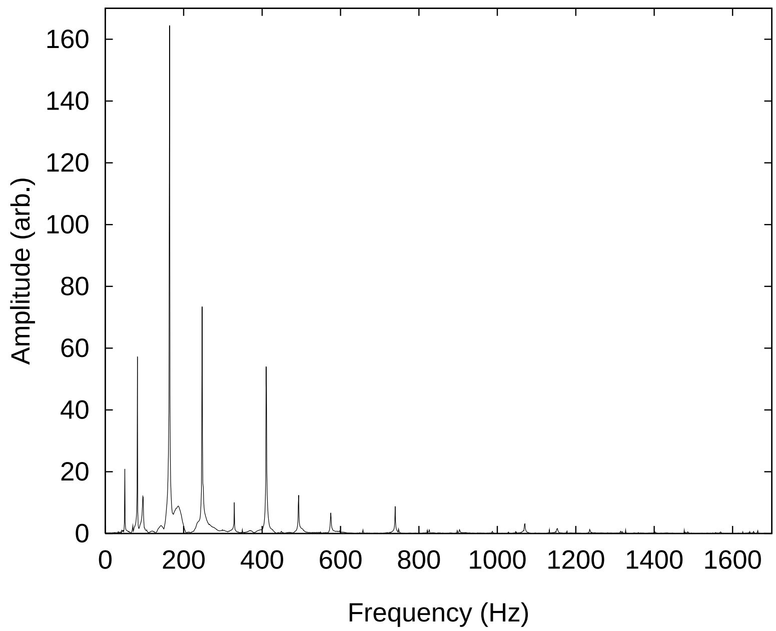
The shape, or more particularly, the width of the fingernail has an important effect on the excitation of the string modes. A fingernail (or plectrum) that is narrow or pointed gives the string a sharp corner at the point of contact, whereas a wider or blunter fingernail gives a more rounded corner. Fourier analysis of the shape of the string on release shows that a sharper corner will have considerably more high-frequency components present than a smoother, more rounded one. These high-frequency partials tend to decay relatively rapidly and are only significant in the early part of the string's motion.

3.2 Transverse string modes

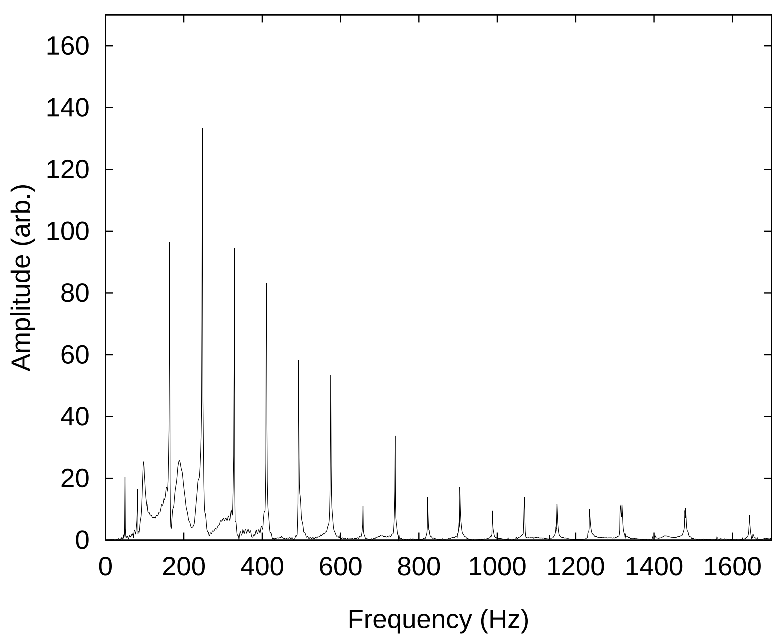
The generalised vibrations of a linear system are most easily understood as a superposition of independent normal modes. The frequencies of the normal modes of transverse vibration of a string are, to a first approximation, given by simple integer multiples of the fundamental mode, as seen in the simple relation:

$$F = \frac{n}{2l}\left(\frac{T}{\mu}\right)^{\frac{1}{2}} \tag{3.1}$$

where F is the frequency, n is the mode number, l is the length of the string, T is the string tension and μ is the mass per unit length of the string. This equation holds true only for an idealised, perfectly flexible string on rigid end supports. In realistic situations, as discussed below in Section 3.4, the movement of the end supports cause deviations from this harmonic



(a) Open low E string plucked over the soundhole



(b) Open low E string plucked near the bridge

Figure 3.1: Influence of the plucking position on the harmonic content of the string. In (b), a larger number of high-frequency string modes are excited and the fundamental mode at 82 Hz is very weakly excited. The body transient is more strongly excited in (b); the two broader peaks at around 100 and 200 Hz, corresponding to the two lowest body modes, are clearly visible. The small, sharp peak at 50 Hz, visible in both spectra, is due to mains hum.

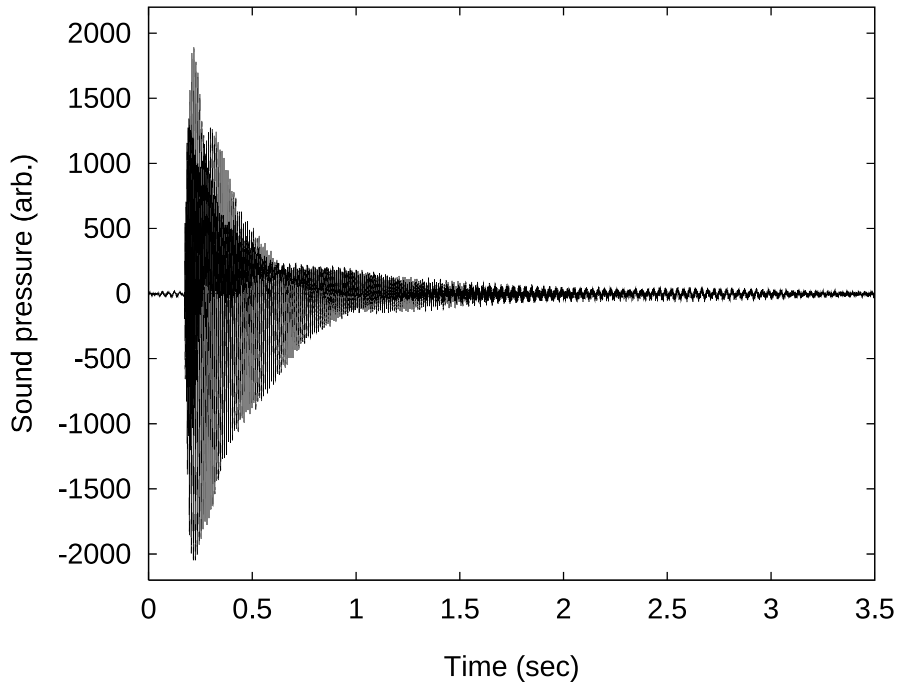
relation of the mode frequencies. However, we can still interpret the generalised, low-amplitude motion of a real string, to reasonable accuracy, as a superposition of harmonic normal modes.

The motion of the string can be described in terms of a component vibrating 'horizontally' (parallel to the top plate) and a component vibrating 'vertically' (perpendicular to the top plate). For an ideal string on a rigid mount, these two transverse modes are degenerate, but differing symmetries introduced by the bridge and differences in the coupling of the two transverse modes to body resonances lead to a breaking of this degeneracy. The most important consequence of this, for the player, is the different sound produced by the vertical and horizontal components of string motion. The player can release the string at different angles, and can hence control the initial proportion of vertical and horizontal motion. This has strong influences on the subsequent tone quality of the note. Vertical motion of the string couples readily to resonances of the body, as the top plate vibrates most easily in a direction perpendicular to its surface. String vibrations in this vertical plane drive the body well and result in a loud tone. Horizontal vibrations do not tend to couple so well to the body, as the bridge moves much less readily in a direction parallel to and across the top plate, and this results in a weaker sound.

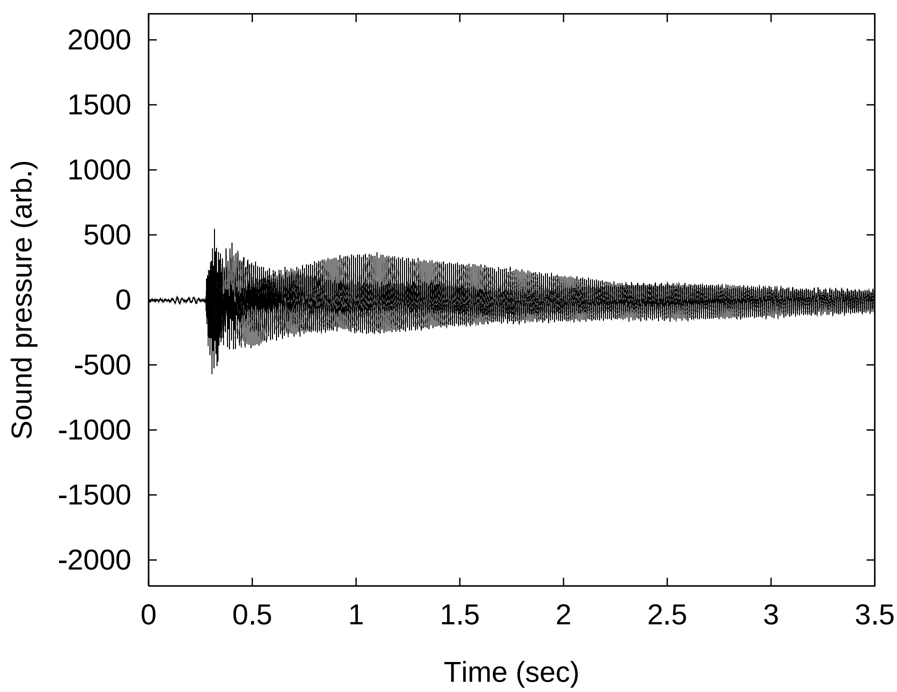
In addition, the stronger coupling between vertically polarised string motion and the body causes the string's energy to be radiated more quickly, resulting in a relatively rapid decay. Horizontally polarised string motion produces a longer decay. Classical guitar players exploit this difference when plucking the string. Usually the player pushes the string towards the top plate during the plucking process so that the string has a strong vertical component of motion on release, giving a relatively loud sound, but one which may be short-lived. If a quieter but more sustained sound is required, the guitarist imparts a stronger horizontal component of motion to the string.

The effect is illustrated in Figure 3.2. The open A string of a guitar was plucked both 'vertically' and 'horizontally' and the sound pressure at a position directly in front of the instrument was measured. The vertical pluck shows a large initial amplitude and relatively rapid decay; the horizontal pluck has lower initial amplitude but much slower decay.

The two transverse string modes will couple together via longitudinal vibrations of the string. The coupling causes the relative strength of the two transverse components to vary as energy is transferred back and forth between them. For a string which is given some angular



(a) Open A string plucked 'vertically'



(b) Open A string plucked 'horizontally'

Figure 3.2: Waveforms of the same note plucked in the 'vertical' and 'horizontal' directions; the plucking force was approximately the same in both. Note that the 'vertical' pluck (a) produces a sound which is initially loud but decays rapidly. The 'horizontal' pluck in (b) produces a quieter sound with a much longer decay. String fundamental frequency is 110 Hz.

momentum during the plucking process, its motion is described by a precessing, elliptical orbit (Gough, 1981). This cyclical transfer of energy between the vertical and horizontal string vibrations gives the sound a tone that evolves in time. For a string given zero angular momentum during the pluck, its motion remains confined to a plane and the produced sound is a simple decay of the initial tone.

The longitudinal vibrations responsible for the coupling between the two transverse modes are caused by the variations in string tension as the string vibrates about its equilibrium position. The tension reaches a maximum when the string's displacement is at either extreme, at which point the total length of the string is also a maximum. So, for one oscillation of the string, the tension passes through two maxima and so the longitudinal oscillations occur at twice the frequency of the transverse oscillations. These vibrations couple to some degree to the top plate, but the admittance of the bridge in this direction (parallel to the top plate and along the length of the string) is generally much lower than that for horizontal transverse motion (see Figure 2.8). It is unlikely that very much energy is transferred from longitudinal vibrations to the body modes, although very little work has been done to investigate the magnitude of such couplings. Torsional oscillations of the string are also possible due to frictional forces between the finger or fingernail and the string causing a twisting of the string before it is released.

3.3 Decay rates

The decay rates of the different string modes depend on a number of factors. The material properties of the strings themselves cause some damping and vibrational energy will be lost as heat as the string bends and twists. This internal damping tends to affect the high-frequency modes much more as these involve more bending of the string. The internal damping tends to be low for metal strings, but may be much more significant in gut and nylon strings (Fletcher and Rossing, 1991).

The string also has to work against the viscosity of the air, and this causes further damping. The damping imposed by the air is more significant at higher frequencies where the string vibrates more rapidly and has to work harder to overcome the friction due to the air. Both the internal damping and the fluid damping of the air cause the high-frequency energy of the string to be lost relatively rapidly.

The most important cause of energy loss from the string is through the bridge, where vibrational energy is taken from the string and radiated as sound energy from the body of the instrument. A small amount of energy may also be lost through the neck of the instrument but this probably contributes little to the radiated sound. The energy lost by the string to the body will be the dominant influence on the decay of the string vibrations for most frequencies. The rate at which energy is lost as radiated sound depends on the coupling of the string to the different body modes, and on the radiation efficiency of those modes and is likely to vary significantly between different instruments.

3.4 Coupling between string and body

The coupling between the string and the body of the instrument is essential for the production of sound in that it allows some of the string's energy to flow via the bridge into the body of the instrument. The body is driven into vibration by the string and the sound can be radiated from the body to the surrounding air mass. The large amplitude, low surface area vibrations of the string are converted to small amplitude, large surface area vibrations of the body. These body vibrations radiate sound much more effectively than the string, and so the body acts as a frequency-dependent amplifier of the string vibrations.

The simple, integer-multiple relationship for mode frequencies of the string (Equation 3.1) can only be used with caution on real instruments. The bridge of the guitar is by no means a rigid end support but moves with the top plate thus perturbing the normal modes of the string. The normal modes of the body are similarly affected by the coupling to the string and, when considering the string and body together, it is more accurate to talk of modes of the coupled system rather than 'string modes' and 'body modes'. In fact, it is correct to talk of normal modes of the body, or of the string, only when these elements are considered in total isolation. It is more useful to consider the instrument as a whole, and so the resonances of the coupled string-body system are the important features.

The coupling between the string and the body can be described as weak or strong. In the weak coupling limit the mode frequencies of the string and body before coupling are slightly modified, and the damping of the system is moderately increased. In the strong coupling limit the frequencies and damping of the uncoupled string and body modes are perturbed more severely. In extreme cases of strong coupling, where the frequency of a string fundamental coincides with a strong body resonance, the phenomenon of the guitar wolf-note can be observed. Here the two coincident modes couple to give rise to a split peak with two distinct, narrowly separated frequency components. This can often produce a dissonant, unpleasant sound. The damping of the two peaks of the split resonance is of the same order as the damping of the uncoupled body mode. In other words, the string changes from having a decay time of the order of a second or more in its uncoupled state, to having a decay time of the order of a tenth of a second when it is strongly coupled. This tends to increase the unpleasant sound associated with strongly coupled notes; not only do they sound 'out of tune' because of the two narrowly separated peaks, but they have a short decay time making them sound rather 'thumpy'.

This change in behaviour is illustrated in Figure 3.3 where the waveforms of three plucked notes are compared. All notes were played on the low E string of guitar BR2; the first note is the open E string, the second is stopped at the 3rd fret, the third at the 7th fret. The first and third notes have a relatively long sustain, but the second note decays much more rapidly because it has a string mode at 196 Hz that almost coincides with the $T(1,1)_2$ body mode and hence it couples much more strongly.

3.5 Normal modes of the body

The body of the guitar, like the string, has a set of vibrational modes. These modes are determined by the dimensions and construction of the instrument as well as the properties of the wood from which the guitar is made. We assume that the guitar body behaves as a linear system; the vibration amplitudes of the body are sufficiently small for this to be a reasonable assumption. Tests performed by Richardson (1982) provide experimental evidence that the vibrations of the body are indeed linear. The excellent fit obtained between velocity response measurements on real instruments and synthesised response curves in Chapter 5 provides further justification for the assumption that the guitar body is essentially linear. We can therefore accurately describe the vibrations of the guitar body as a superposition of these normal modes. The modes of the top plate, back plate and air cavity are all coupled together

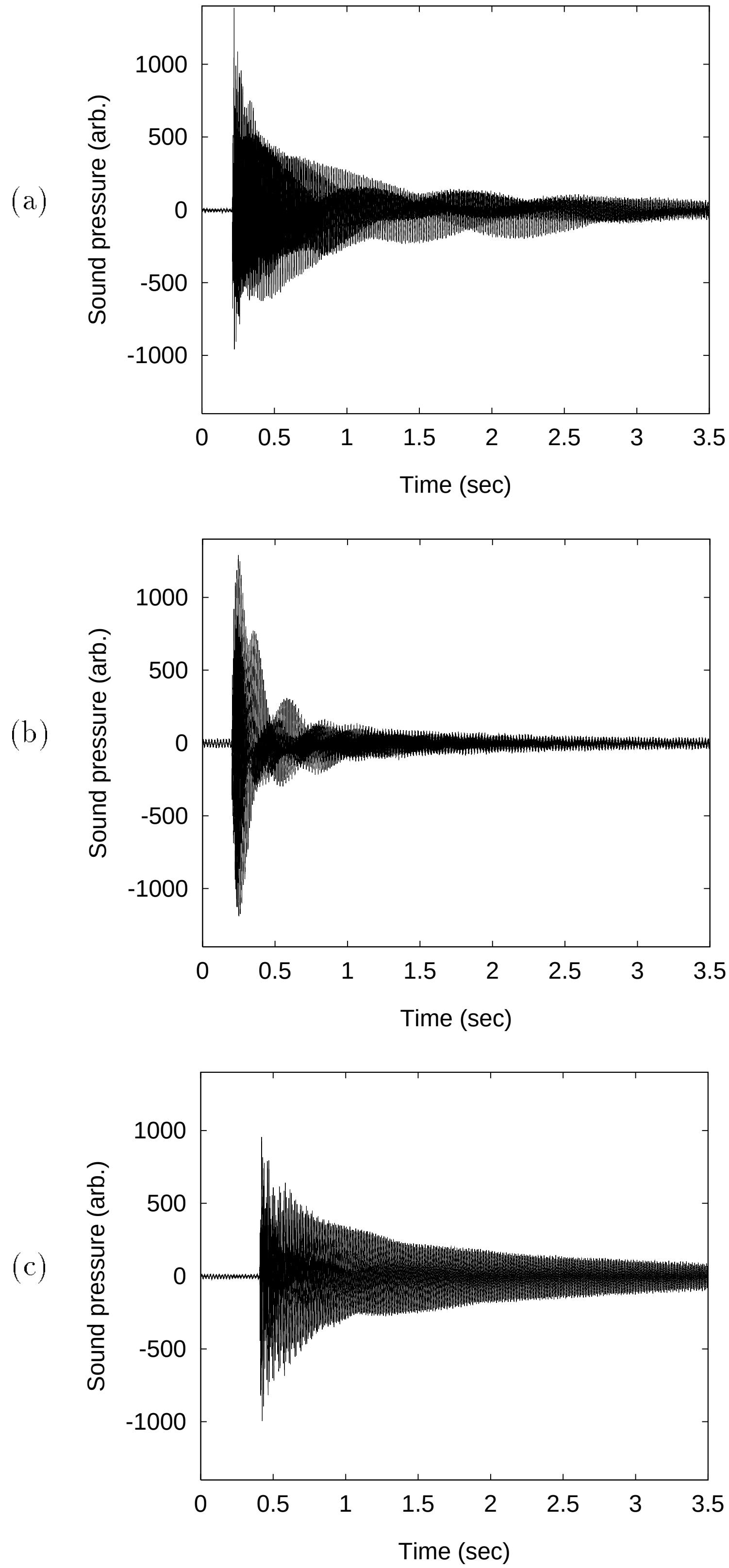


Figure 3.3: Waveforms of three plucked notes (fundamental frequencies: (a) 82 Hz, (b) 98 Hz, (c) 124 Hz). The note at (b) couples strongly with the $T(1,1)_2$ body mode resulting in a large initial amplitude and rapid decay.

via the common pressure changes in the air cavity. I will first discuss the uncoupled modes before commenting on the way the coupling influences the behaviour of the guitar body.

The top plate

Most of the movement occurs in the top plate of the instrument, and to a lesser extent in the back plate. The sides or ribs of the guitar act as fairly rigid supports for the top and back plates and do not vibrate a great deal so they contribute little to the radiated sound. The back has a large vibrating area, but vibrates rather less than the top plate which is responsible for most of the radiated sound energy. It should also be remembered that much of the guitar back and sides a will be in contact with the body of the player and so their contribution to the radiated sound is further reduced.

The vibrational modes of the isolated top plate can be characterised by their frequency, amplitude distribution (shape) and Q-value. Although the exact frequencies and Q-values will vary for different top plates, the shapes of the low-frequency modes for different guitars do not differ greatly and are seen to follow a similar hierarchy. Holographic interferometry is a technique commonly used to visualise the modes of a guitar. A laser beam is split into two, one part shining directly onto a photographic plate, the other part being reflected by the guitar which is driven with a sinusoidal force. Interference occurs between the two beams and a pattern of light and dark fringes is superimposed over the image of the guitar. The pattern of fringes can be read in a similar way to contour lines on a map and so the vibration amplitude over the surface of the instrument can be found. Examples of mode shapes visualised using this technique were given in Figure 2.7.

At high frequencies (above 1 kHz) the mode shapes become more complex, with a greater number of different areas vibrating with different phases. These high-frequency modes tend to occur closer and closer together and will overlap so that individual modes give way to what has been called a resonance continuum (Caldersmith, 1981). In this region the response of different top plates may differ more significantly, depending largely on the bending and twisting properties of the wood. The strutting arrangement of the top plate has an influence on the positions of the antinodal lines of these high-frequency modes. The top plate is much stiffer where a strut has been added, and so the antinodal lines of the modes tend to fall on top of the positions of the struts.

The back plate

The isolated back plate will have its own modes of vibration, similar in shape to those of the top plate. The amplitudes of the back vibrations are generally much lower than those of the top. Differences in the wood used for the top and back plates, and different arrangements of strengthening bars and struts, cause the resonance frequencies and damping of the back-plate modes to be different from those of the top plate.

The air cavity

Any enclosed volume of air with a small opening can be made to resonate at certain frequencies. The opening, which is the soundhole in the case of the guitar, can be visualised as containing a certain mass of air, or an air 'plug' which can oscillate against the stiffness of the rest of the air in the cavity. It does this at certain discrete resonant frequencies, the first of which is called the Helmholtz resonant frequency. In the guitar, however, the sides, top and back plates are not perfectly rigid, so the air plug is also working against their stiffness. The air cavity modes are thus coupled to the body and will influence the response of the whole instrument.

3.6 Coupling between air cavity, back plate and top plate

When the vibrational modes of the complete guitar body are studied, some similarities with the uncoupled top-plate modes are seen but the different boundary conditions imposed and the coupling between different elements of the body changes the modes. When considering the response of the guitar it is the modes of the whole body which govern its sound, so it is these modes which must be investigated.

Coupling between the top plate, back plate and air cavity arises from the common pressure changes in the cavity. As the top plate moves outward, the volume of the cavity increases and the pressure falls. The changes in pressure cause the 'plug' of air in the soundhole and the back plate to be forced into motion. In this way, the motion of the top plate, back plate and air cavity have a direct influence on each others.

The main result of this coupling is a change in the frequency and damping of the modes.

The strength of coupling is mainly determined by the net volume change produced by each of the plate modes. For example, the (1,1) modes of the top and back plate produce large volume changes, and so their resonance frequencies may be changed significantly by coupling to the air cavity. The (2,1) top-plate mode, which will produce a zero net volume change for a symmetrically strutted guitar, will not couple with the air cavity and will remain unaffected.

3.7 Radiation of sound

The guitar radiates sound from the whole of its surface, but the most important radiators are the soundhole and the top plate. The soundhole radiation is particularly important at the low-frequency end of the guitar's range where the air cavity acts as a reflex enclosure (see Section 2.2). At low frequencies, where the dimensions of the instrument are smaller than the wavelength of sound, the guitar radiates as a monopole producing spherical sound waves. At high frequencies, where the instrument dimensions are much greater than the wavelength of sound, the sound radiation becomes more directional.

The radiation from the top plate is most efficient when a large volume of air is displaced by the motion of the plate. In the T(1,1) mode, a large area of the plate moves in and out and will displace a large volume of air, so this mode radiates very strongly. The radiation field produced by the T(1,1) mode is essentially spherical (Figure 2.10). The T(2,1) mode, however, has two vibrating areas of equal area moving out of phase with each other. The out-of-phase motion of the air displaced by each tends to produce an acoustical short circuit resulting in poor radiation to the front of the instrument. It radiates more strongly to the sides of the instrument producing a dipole radiation field, as shown in Figure 2.10. Some guitars are built with a diagonal strut on the underside of the top plate to deliberately introduce more asymmetrical mode shapes. In this case the two vibrating areas of the T(2,1) mode will not produce the same volume change and hence an overall net volume displacement is achieved resulting in a combination of monopole and dipole fields. In a similar way, the monopole component of other guitar modes may be increased using asymmetrical struts on the top plate.

Many of the higher-frequency modes have several vibrating areas on the top plate. Sound radiation at high frequencies becomes increasingly directional and the high-frequency modes of the guitar tend to produce multipole radiation fields. In this frequency range the body modes occur very close together (see Section 3.5). Interference between the radiation patterns for these closely-spaced modes produces a sound radiation field that varies strongly with both frequency and listening position.

As mentioned in Section 1.6, when a note is plucked on the guitar, the top plate is deformed as the finger moves the string from its equilibrium position. As the string leaves the finger, an impulse-like force is transmitted to the body which excites its normal modes. The body emits a characteristic knocking noise, or body transient, which is made up of the combined impulse responses of the radiating body modes. This body transient lasts for just a fraction of a second and is excited every time a string is plucked. Note that in Figure 3.1, where the string is plucked near the bridge, the body transient is more pronounced, showing up in the spectrum in the 100–400 Hz region. Similar body transients occur in the violin for pizzicato notes, and in the piano where the supporting structure is similarly excited when the hammer hits the string.

A final consideration for the radiation process is the radiation damping. Modes which generate a large monopole component of radiation have to move a large volume, and hence a large mass, of air during each cycle. This can significantly affect the effective mass of these modes and can increase their damping considerably. Modes which tend to be the strongest radiators will suffer the greatest damping because of this fluid loading.

Chapter 4

An oscillator model of the classical guitar

4.1 Introduction

In this chapter I will describe the theoretical framework used for the numerical model of the guitar. The response of several top-plate modes is calculated, using Christensen's (1984) oscillator model. Coupling between the top plate and the fundamental air-cavity mode is first added, and coupling to a flexible back plate is then included, using ideas first presented by Christensen and Vistisen (1980) and Christensen (1982). The model of the guitar body thus includes all coupling between the top plate, back plate and fundamental air-cavity mode. A string is coupled to the body using the theory presented by Gough (1981). An expression for the response of the coupled string-body system to a driving force applied to a point on the string is then obtained.

4.2 Simple model of the top plate

To model the vibrational response of the guitar we associate one harmonic oscillator with each of the modes of the top plate, following the ideas of Christensen (1984). The response of the top plate is calculated by performing a summation of the frequency responses of each oscillator. This summation over the normal modes of the system assumes that the guitar body behaves as a linear system; the justification of this assumptions has already been discussed in

Section 3.5.

Each top-plate oscillator is characterised by a resonance frequency ω_0, an effective mass m and a damping factor γ. In addition, each oscillator drives a piston of effective area A. The effective monopole area is defined (Christensen, 1984) so that when the area A moves with the same displacement as the driving point, the volume of air displaced is equal to the volume displaced by the whole mode, creating the possibility of modes with negative effective areas. The volume displacement generated by the piston is then used as a simple volume source to calculate the monopole sound radiation from each mode. The equation describing the displacement of an oscillator for a sinusoidal applied force F of angular frequency ω is

$$m\frac{\partial^2 x}{\partial t^2} = F - kx - R\frac{\partial x}{\partial t} \ , \tag{4.1}$$

where x is its displacement from equilibrium, k is the stiffness constant and R the resistance to motion. The stiffness and resistance constants are directly related to the mass, damping factor and resonance frequency of the oscillator. Although the oscillator has its own natural resonance, this part of the motion is not sustained by the applied driving force F and the oscillator vibrates with frequency ω, the same frequency as the driving force. If the force is sinusoidal then the motion will also be sinusoidal so we can write

$$F = F_0 exp(i\omega t) \ , \tag{4.2}$$

$$x = x_0 exp(i\omega t) \ . \tag{4.3}$$

By differentiating this expression for x with respect to time we obtain

$$\frac{\partial x}{\partial t} = i\omega x \ , \tag{4.4}$$

$$\text{and} \quad \frac{\partial^2 x}{\partial t^2} = -\omega^2 x \ . \tag{4.5}$$

Substituting these into the original equation of motion and rearranging we find

$$x = \frac{F}{k - m\omega^2 + iR\omega} \ . \tag{4.6}$$

The resonance frequency of a lightly damped oscillator vibrating in free space is given by the ratio of its stiffness to mass:

$$\omega_0^2 = \frac{k}{m} \ . \tag{4.7}$$

Using Equation 4.7 and by defining the damping factor as $\gamma = \frac{R}{m}$ we can rewrite Equation 4.6 as

$$x = \frac{F}{m\left[\omega_0^2 - \omega^2 + i\gamma\omega\right]} \ . \tag{4.8}$$

This allows the displacement, due to one mode, of a point on the top plate to be calculated. By performing a summation of the displacement x over all modes of vibration, the total displacement at that point can be found. To obtain the velocity or acceleration at that point, the above expression is multiplied by $i\omega$ or $-\omega^2$ (see Equations 4.4 and 4.5). The motion of the top plate given by Equation 4.8 does not account for the influence of the air cavity or the vibrations of the back plate, which will directly affect the motion of the top plate. The air cavity and back plate have several modes of vibration, and these can be modelled as additional harmonic oscillators which are coupled to those of the top plate. I will first deal with the coupling of the top-plate and air-cavity modes.

4.3 Coupling between top plate and air cavity

When the top plate vibrates it alters the volume of the cavity and will force air in and out of the sound hole. The air in the sound hole can be visualised as being a cylindrical mass which vibrates up and down as the cavity volume is squashed and expanded by the motion of the top plate. Using the modelling scheme developed by Christensen and Vistisen (1980), the cylindrical mass of air is modelled as another oscillator with mass m_h, driving a piston with effective area A_h (the subscript h refers to the Helmholtz cavity mode). The restoring force for the air piston is provided by changes in pressure inside the cavity; as the piston moves outwards, the cavity pressure falls creating a force opposing the motion. The air piston and the top-plate pistons are thus coupled together by the common pressure changes in the cavity. The motion of one piston changes the cavity pressure and exerts a force on the other.

The equations of motion for the system of two coupled oscillators, driven by a sinusoidal force F applied to the top plate, are

$$m_t \frac{\partial^2 x_t}{\partial t^2} = F - k_t x_t - R_t \frac{\partial x_t}{\partial t} + A_t \Delta P \ , \tag{4.9}$$

and

$$m_h \frac{\partial^2 x_h}{\partial t^2} = A_h \Delta P - R_h \frac{\partial x_h}{\partial t} \ , \tag{4.10}$$

where ΔP is the change in cavity pressure, subscript t refers to the top plate and h to the air cavity. The solution of the equations of motion is given in detail in Appendix A, Section A.1. For now, only the final expressions giving the displacements of the top-plate and air-cavity pistons relative to their equilibrium position will be quoted. These are found to be

$$x_t = \frac{F D_h}{D_h D_t - {\alpha_{ht}}^2} \ , \tag{4.11}$$

and
$$x_h = \frac{-\alpha_{ht} F}{D_h D_t - {\alpha_{ht}}^2} \ , \tag{4.12}$$

where D_h, D_t and the coupling term α_{ht} are defined as follows:

$$\alpha_{ht} = \mu A_h A_t \ , \tag{4.13}$$

$$D_t = m_t(\omega_t^2 - \omega^2 + i\omega\gamma_t) \ , \tag{4.14}$$

$$D_h = m_h(\omega_h^2 - \omega^2 + i\omega\gamma_h) \ . \tag{4.15}$$

In reality, the displacements of the pistons are not steady but vary sinusoidally with time (for a sinusoidal driving force); the $exp(i\omega t)$ term has been omitted for clarity. A more useful expression is that giving the velocities of the two pistons. When measuring frequency responses of guitars, the admittance is a commonly-measured quantity. This is simply the velocity (usually measured at the bridge) divided by the driving force. The relation between displacement and velocity of the pistons is known from Equations 4.4 and 4.5 so the expression for the admittance at the bridge is written as

$$\chi_t = \frac{i\omega D_h}{D_h D_t - {\alpha_{ht}}^2} \ . \tag{4.16}$$

Another useful quantity to be able to model is the sound pressure response. This is the sound pressure radiated by the pistons to a point in space a distance r from the bridge of the guitar. It is, in effect, the pressure wave that would reach a listener's ears at that point in space. A detailed calculation of the sound radiation would have to take into account the three-dimensional shape of the guitar as well as the position and nature of reflective walls and surfaces of the room. If it is first assumed that the guitar is situated in an anechoic environment, then reflections from walls and other surfaces need no longer be considered. However the calculation is still complex and it is necessary to simplify it further.

At distances from the guitar which are comparable or greater than the longest wavelengths being considered, the radiated sound waves are spherical, to a reasonable approximation. It is then possible to treat the top-plate modes of the guitar as point sources of sound. The source strength of a point source is the net volume displaced. Since the effective areas and velocities of each piston are known, the volume displacement of the top plate can be found by summing contributions from each piston, and the radiated spherical pressure waves can be calculated.

The complex pressure due to a monopole source of sound is given by

$$\hat{p} = \frac{i\omega\rho}{4\pi r} U exp(-ikr) \; , \tag{4.17}$$

where ρ is the equilibrium density of air, r the distance from the source and U the volume velocity of the source. In the case of a piston with effective area A moving with velocity u the volume velocity is $U = uA$. So by assuming all modes of the guitar to be monopole sources of sound, the complex sound pressure at any point in space can be calculated.

Previous numerical modelling of radiation fields at Cardiff (Brooke, 1992) showed that many top-plate modes could be very well approximated by a monopole source, but other modes had a strong dipole character and radiate strongly to the sides of the instrument, but very poorly to the front. The radiation from these dipole modes was modelled by defining two monopole sources of equal source strength having a phase difference of π radians and having a separation distance d. By defining a monopole and dipole effective area for each mode the radiation fields could be made wholly monopole, wholly dipole or a mixture of the two.

When measurements are made on guitars by driving the bridge with a sinusoidal force and recording the output of a microphone over the frequency range of interest, it is a time average of the pressure which is measured. The sound pressure level (SPL) is defined as

$$SPL = 10 log_{10} \left[\frac{(p^2)_{av}}{(p_{ref})^2} \right] \; , \tag{4.18}$$

where $(p^2)_{av}$ is the average squared pressure and p_{ref} is a reference pressure that equals 2×10^{-5} Pa. The average squared pressure is obtained from the complex pressure by using

$$(p^2)_{av} = \frac{1}{2}[Re(\hat{p})^2 + Im(\hat{p})^2] \; . \tag{4.19}$$

By substituting for the volume velocity in Equation 4.17 and summing contributions from the soundhole and top-plate pistons, Equations 4.18 and 4.19 can be used to calculate the SPL response of the modelled instrument.

Some results from the numerical model are shown in Figures 4.1 and 4.2. Figure 4.1 compares the calculated admittance response of an uncoupled top plate with the response of the same top plate coupled to an air cavity. Figure 4.2 compares the pressure responses for the same two cases. The results are shown for a driving force of 1 N, and the pressure response is calculated at a distance of 2.5 m directly in front of the top plate. Unless otherwise stated, all subsequent velocity and pressure responses will be calculated using these values. The coupling to the air cavity gives rise to the extra peak at around 100 Hz and so improves the response of the instrument at these low frequencies. Without this coupling the lowest notes in the guitar's range would have a very weak fundamental, hence the air cavity helps to provide a stronger bass response.

So far I have considered the motion of the top plate and the air in the soundhole only, assuming the back plate to be rigid. This assumption now needs to be relaxed in order to see the influence of coupling a flexible back plate to the air-cavity and top-plate pistons.

4.4 Adding a coupled back-plate piston to the system

The influence of the back plate is modelled by introducing another piston with effective area A_b, mass m_b working against a spring of stiffness k_b, as described by Christensen (1982). A diagram representing the system of coupled top-plate/back-plate/air-cavity pistons is given in Figure 4.3.

The equations of motion for the three pistons in the coupled system are

$$m_t \frac{\partial^2 x_t}{\partial t^2} = F - k_t x_t - R_t \frac{\partial x_t}{\partial t} + \Delta P A_t \ , \tag{4.20}$$

$$m_h \frac{\partial^2 x_h}{\partial t^2} = \Delta P A_h - R_h \frac{\partial x_h}{\partial t} \ , \tag{4.21}$$

and
$$m_b \frac{\partial^2 x_b}{\partial t^2} = \Delta P A_b - k_b x_b - R_b \frac{\partial x_b}{\partial t} \ , \tag{4.22}$$

where R_b is the resistance to motion felt by the back-plate piston and ΔP, as before, is

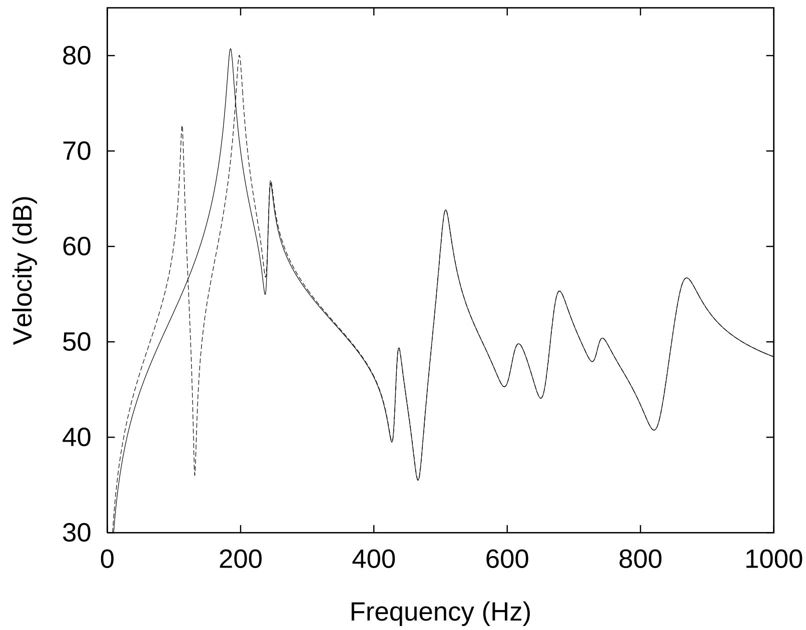


Figure 4.1: Calculated velocity response of uncoupled top plate (solid line) and top plate coupled to an air cavity (dotted line), both measured at the bridge. Data for top-plate modes from Walker (1991). Driving force was 1 N.

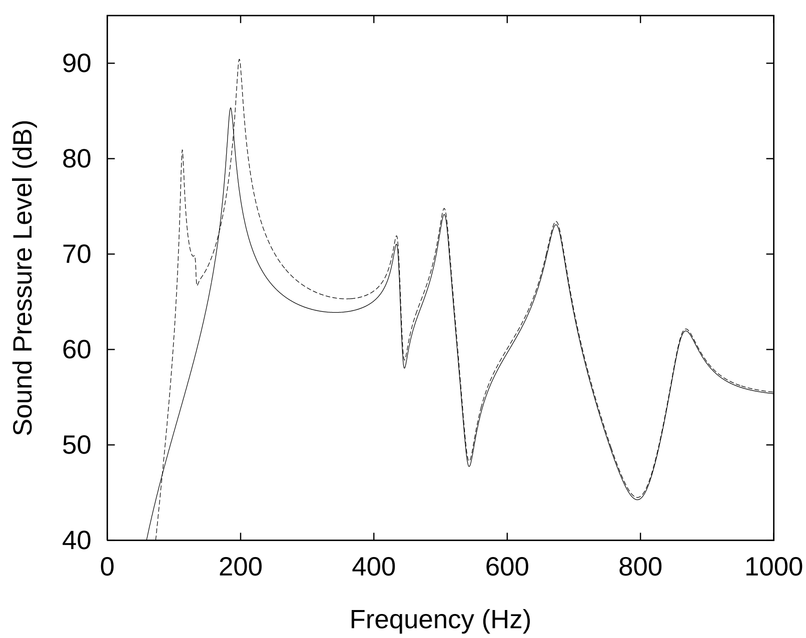


Figure 4.2: Calculated pressure response of uncoupled top plate (solid line) and top plate coupled to an air cavity (dotted line). Data for the top-plate modes is the same as that used in Figure 4.1 above. Not all peaks in the velocity response give rise to a peak in the sound pressure response because some top-plate modes have zero effective areas. The pressure is calculated at a distance of 2.5 m directly in front of the bridge, with a driving force of 1 N.

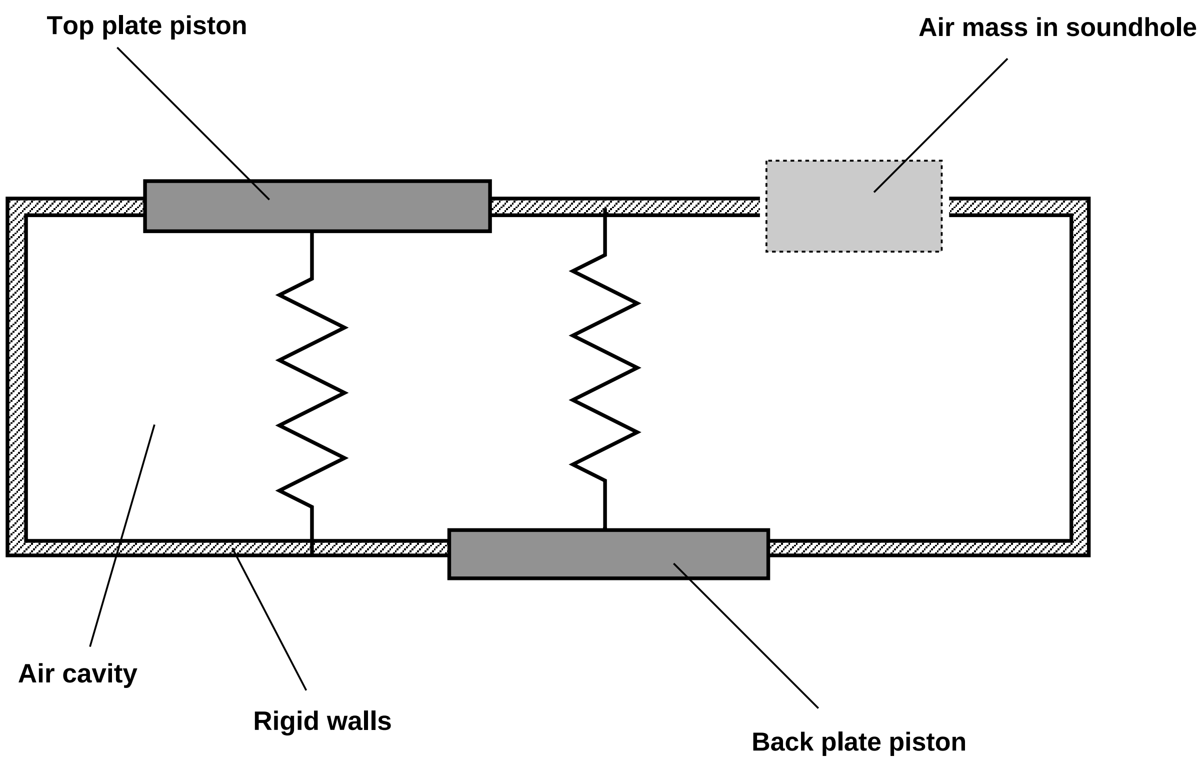


Figure 4.3: Schematic view of the guitar body as a system of coupled pistons. The motion of the top plate is modelled as that of a piston with mass m_t and effective area A_t, connected to a spring of stiffness k_t; the back plate is modelled similarly. The soundhole is modelled by considering the motion of an mass m_h of air, with effective area A_h, oscillating against the stiffness of the air in the cavity. Motion of any one piston causes a change in the pressure inside the cavity and will affect the motion of the other two pistons.

the pressure change in the cavity resulting from movement of the pistons. The procedure for obtaining solutions to these equations is given in Appendix A, Section A.2. The displacements of the three pistons are found to be

$$x_t = \frac{F(D_b D_h - {\alpha_{hb}}^2)}{D_t D_h D_b + 2\alpha_{ht}\alpha_{hb}\alpha_{bt} - [D_t(\alpha_{hb})^2 + D_h(\alpha_{bt})^2 + D_b(\alpha_{ht})^2]} \; , \tag{4.23}$$

$$x_h = \frac{-F(D_b \alpha_{ht} - \alpha_{hb}\alpha_{bt})}{D_t D_h D_b + 2\alpha_{ht}\alpha_{hb}\alpha_{bt} - [D_t(\alpha_{hb})^2 + D_h(\alpha_{bt})^2 + D_b(\alpha_{ht})^2]} \; , \tag{4.24}$$

and

$$x_b = \frac{F(\alpha_{ht}\alpha_{hb} - D_h \alpha_{bt})}{D_t D_h D_b + 2\alpha_{ht}\alpha_{hb}\alpha_{bt} - [D_t(\alpha_{hb})^2 + D_h(\alpha_{bt})^2 + D_b(\alpha_{ht})^2]} \; . \tag{4.25}$$

As already stated, the piston velocity is a more useful quantity to calculate than the displacement, and is obtained by multiplying the above expressions by $i\omega$ (see Equations 4.4 and 4.5). Expressions are obtained which give the piston velocities for the system of one air piston coupled to one back-plate piston and one top-plate piston, driven by a sinusoidal force with magnitude F and frequency ω. To obtain the solution for n top-plate pistons a summation is performed. To calculate the frequency response of, for example, the top-plate velocity, the expression must be calculated at a number of discrete points in the frequency range of interest. The admittance response can be calculated by dividing the expression for top-plate velocity by F, or, by making use of Equations 4.17–4.19, the SPL response of the instrument can be evaluated.

Figure 4.4 illustrates the effect of including a flexible back plate in the model. In this case the back-plate coupling introduces a third peak at around 220 Hz and also modifies the responses of the other two peaks. The frequency of the 200 Hz peak is lowered slightly, and its height reduced by around 3 dB. The lower peak suffers similar changes but to a lesser degree. The overall effect of introducing the flexible back to the system can be visualised as a slight increase in the instrument's response in the region 220–350 Hz at the expense of moderate decreases in response at lower frequencies.

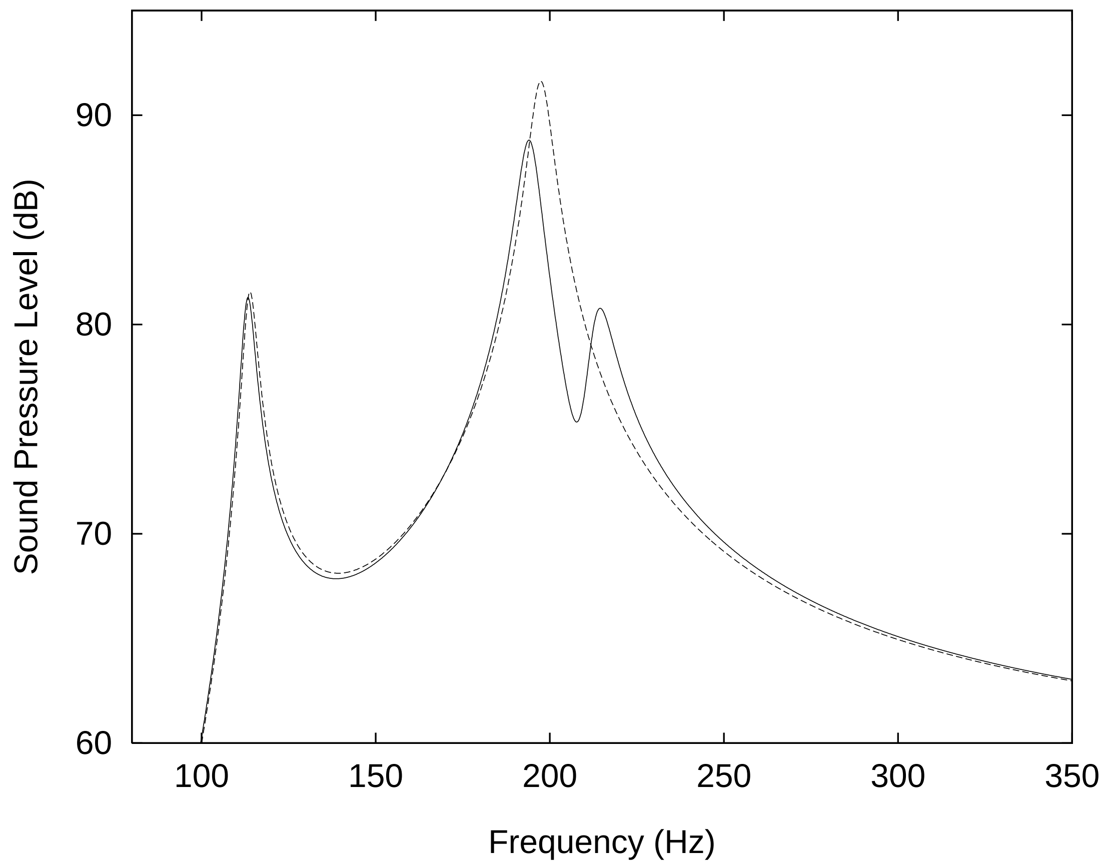


Figure 4.4: Comparison of guitar body response with (solid line) and without (dotted line) coupling to the fundamental back-plate mode. The introduction of coupling to the back plate introduces a third resonance, the T(1,1)$_3$, at around 220 Hz, and modifies the amplitudes and frequencies of the two lower resonances. Data for fundamental top-plate mode (uncoupled values): f_t = 185 Hz, m_t = 60 g, A_t = 0.038 m^2, Q_t = 25. Data for back-plate mode (uncoupled values): f_b = 210 Hz, m_b = 180 g, A_b = 0.02 m^2, Q_b = 25. Both plate modes were coupled to the fundamental air-cavity mode whose uncoupled resonance frequency was 133 Hz.

4.5 Fluid loading

One effect that has so far not been included in the modelling scheme is fluid loading of the pistons. When the top plate vibrates, the air it has to displace exerts a force on the top plate opposing the motion. This fluid loading has a significant effect on the motion of the guitar body, mainly causing increased damping of the vibrations. To account for this fluid loading an extra term must be included in the equations of motion for the system.

An exact model of the fluid loading of the guitar would be difficult to achieve; it would have to take into account the three-dimensional shape of the guitar as well as the two-dimensional pattern of vibration amplitude for each of the guitar's normal modes. A simpler approach can be used to investigate the qualitative effect of fluid loading on the vibrational response of the guitar. Solutions for the motion of a fluid-loaded circular piston mounted in an infinite coplanar baffle are available. Clearly the body of the guitar surrounding the vibrating area does not constitute an infinite baffle, but this simple model can be useful as a first approximation to the real processes occurring on the guitar.

Caution must be used when relating the physical area of the piston in the fluid loading model to the effective piston area in the oscillator model of the guitar. For the lowest (1,1) triplet of modes, the motion of the top and back plates can be reasonably approximated to that of a circular piston since the entire vibrating area of the plate moves with the same phase. For this reason, the effective area parameter used by the model is similar in magnitude to the actual physical area of the plate that vibrates. Values for the effective area of the T(1,1) mode, calculated from response curves of real instruments using the curve-fitting routines presented in Chapter 5, are of the order of 0.05 m^2, equivalent to a circular piston of radius 13 cm. If the lower bout of the guitar were approximated to a circular area, its radius would be around 17 or 18 cm, so the piston radius calculated from the curve-fitting routine corresponds well with the actual physical area of the plate that vibrates.

For most of the other top-plate modes, this is not the case. The effective piston area may be considerably larger than the area of the lower bout of the instrument. The differences arise because the volume displacement of the mode is the quantity that determines the radiated sound pressure, but it is the area of vibration that determines the amount of fluid damping. The effective area of the mode (defined in Section 4.2) is calculated by dividing the net volume displacement of the mode by the displacement at the driving point, hence when the driving point displacement is small, the effective area may become larger than the actual area of the plate. When driven at the bridge, modes such as the T(1,2) tend to produce only small vibration amplitudes because they have a high effective mass. They can nevertheless produce a significant net volume displacement, resulting in very large values of effective area for driving points at the bridge. If the simple model for fluid loading were applied to such top-plate modes, the effects would be greatly exaggerated since the effective area of the mode would be considerably larger than the physical area of the plate actually vibrating. For this reason, fluid loading was not included when calculating sound pressure responses from the model, but is included here to show the effects on the (1,1) triplet of body modes.

Pierce (1981) gives the fluid reaction force for a piston of radius a mounted in an infinite coplanar baffle as:

$$\hat{F}_{fluid} = \rho c \hat{v} \pi a^2 \left[\frac{(ka)^2}{2} + i\frac{8ka}{3\pi} \right] , \tag{4.26}$$

where $\hat{v}$ is the normal velocity of the piston, k is the wavenumber, c is the speed of sound and ρ is the density of air. When a fluid force term for each piston ($\Delta_t, \Delta_h, \Delta_b$) is introduced, the equations of motion for the system become

$$m_t \frac{\partial^2 x_t}{\partial t^2} = F - k_t x_t - R_t \frac{\partial x_t}{\partial t} + \Delta P A_t - \Delta_t \frac{\partial x_t}{\partial t} , \tag{4.27}$$

$$m_h \frac{\partial^2 x_h}{\partial t^2} = \Delta P A_h - R_h \frac{\partial x_h}{\partial t} - \Delta_h \frac{\partial x_h}{\partial t} , \tag{4.28}$$

and $$m_b \frac{\partial^2 x_b}{\partial t^2} = \Delta P A_b - k_b x_b - R_b \frac{\partial x_b}{\partial t} - \Delta_b \frac{\partial x_b}{\partial t} . \tag{4.29}$$

The definition of the Δ_x terms is

$$\Delta_t = \rho c \pi {a_t}^2 \left[\frac{(ka_t)^2}{2} + i\frac{8ka_t}{3\pi} \right] , \tag{4.30}$$

$$\Delta_h = \rho c \pi {a_h}^2 \left[\frac{(ka_h)^2}{2} + i\frac{8ka_h}{3\pi} \right] , \tag{4.31}$$

and $$\Delta_b = \rho c \pi {a_b}^2 \left[\frac{(ka_b)^2}{2} + i\frac{8ka_b}{3\pi} \right] , \tag{4.32}$$

where a_t, a_h and a_b are the radii of the pistons. Proceeding exactly as before, we substitute expressions for the resonance frequency of the pistons, and by assuming sinusoidal displacements for a sinusoidal driving force we can derive expressions for the derivatives of the displacements. Three expressions involving the displacements of the three pistons are obtained:

$$x_t = \frac{F - \alpha_{ht}x_h - \alpha_{bt}x_b}{m_t({\omega_t}^2 - \omega^2 + i\omega\gamma_t) + i\omega\Delta_t}\ , \tag{4.33}$$

$$x_h = \frac{-\alpha_{ht}x_t - \alpha_{hb}x_b}{m_h({\omega_h}^2 - \omega^2 + i\omega\gamma_h) + i\omega\Delta_h}\ , \tag{4.34}$$

and

$$x_b = \frac{-\alpha_{bt}x_t - \alpha_{hb}x_h}{m_b({\omega_b}^2 - \omega^2 + i\omega\gamma_b) + i\omega\Delta_b}\ . \tag{4.35}$$

Substituting again for D_h, D_t and D_b and rearranging we obtain

$$x_t(D_t + i\omega\Delta_t) + \alpha_{ht}x_h + \alpha_{bt}x_b = F\ , \tag{4.36}$$

$$x_h(D_h + i\omega\Delta_h) + \alpha_{ht}x_t + \alpha_{hb}x_b = 0\ , \tag{4.37}$$

and

$$x_b(D_b + i\omega\Delta_b) + \alpha_{bt}x_t + \alpha_{hb}x_h = 0\ . \tag{4.38}$$

As before, these equations are arranged in matrix form and Cramer's rule is used to find the solutions for the three piston displacements. To simplify matters, I have used the definition $X_t = D_t + i\omega\Delta_t$ and similar definitions for the air cavity and back plate. The solutions for the displacements of the three coupled pistons, when driven by a sinusoidal force F and subject to fluid loading, are given below:

$$x_t = \frac{F(X_bX_h - {\alpha_{hb}}^2)}{X_bX_tX_h + 2\alpha_{ht}\alpha_{hb}\alpha_{bt} - [X_t(\alpha_{bh})^2 + X_h(\alpha_{bt})^2 + X_b(\alpha_{ht})^2]}\ , \tag{4.39}$$

$$x_h = \frac{-F(X_b\alpha_{ht} - \alpha_{hb}\alpha_{bt})}{X_bX_tX_h + 2\alpha_{ht}\alpha_{hb}\alpha_{bt} - [X_t(\alpha_{bh})^2 + X_h(\alpha_{bt})^2 + X_b(\alpha_{ht})^2]}\ , \tag{4.40}$$

and

$$x_b = \frac{F(\alpha_{ht}\alpha_{hb} - \alpha_{bt}X_h)}{X_bX_tX_h + 2\alpha_{ht}\alpha_{hb}\alpha_{bt} - [X_t(\alpha_{bh})^2 + X_h(\alpha_{bt})^2 + X_b(\alpha_{ht})^2]}\ . \tag{4.41}$$

The results of using this simple modelling scheme for the fluid loading are shown in Figure 4.5. The solid line gives the calculated sound pressure response of a guitar (including flexible back plate) with the fluid loading scheme outlined above incorporated into the model. The dotted line gives the response of the same guitar without fluid loading. The mode frequencies of the two peaks at around 200 Hz are lowered by 5–10 Hz, the amplitudes of the peaks are lowered by 3–5 dB and the Q-values are decreased (increased damping of the modes). The peak at 110 Hz suffers a larger decrease in its amplitude and resonance frequency, and a moderate increase in its damping. Results from the listening tests in Chapter 7 suggest that changes of this magnitude would produce only small differences in sound. Three-fold changes in Q-value are shown (Chapter 7) to produce only very small changes in tone. The changes in damping indicated by Figure 4.5 are significantly less than this, although work by Gough (1983) suggests that, for strongly radiating modes, fluid loading may indeed cause three-fold changes in Q-value. The reductions in the mode frequencies appear reasonably large, but results presented later suggest that the differences in tone produced by such changes in mode frequency are fairly small. The small decrease in the amplitudes of the peaks may also cause slight changes in tone.

The modelling scheme for the fluid loading outlined here is, however, only a simple approximation of the actual processes occurring on real instruments. The simple model allows a qualitative investigation of the nature of the changes imposed by fluid loading of the guitar body, but more detailed models are required to make accurate quantitative predictions.

Until now only the response of the body, comprising back plate, top plate and air cavity, has been calculated by assuming a sinusoidal driving force is applied to the bridge. The actual driving force is supplied by the vibrating string. The next section deals with the the theory of the string vibrations and their coupling to the top plate of the guitar.

4.6 The vibrating string

When a guitar string is plucked and set into motion it is the components of the string tension acting at the bridge which act on the top plate and force it into oscillation. In general there will be three forces acting at the bridge acting in directions perpendicular to the top plate, parallel and along the length of the instrument and parallel and across the instrument. The top plate is most easily driven when the driving force acts perpendicularly to the top-plate surface, so only these perpendicular forces will be considered in the model. For a string at

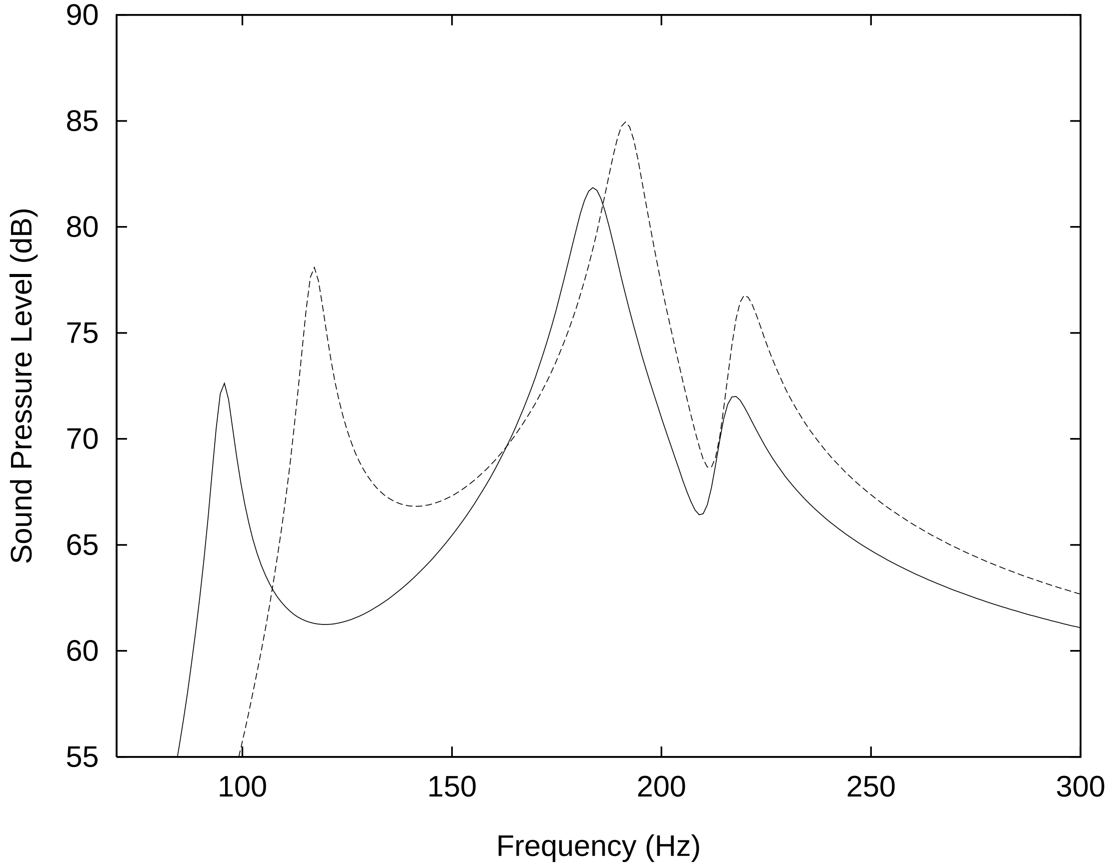


Figure 4.5: Comparison of guitar body response calculated with (solid line) and without (dotted line) the fluid loading scheme outlined in Section 4.5. The nature of the changes imposed by fluid loading of the guitar body (increase in the damping of the modes, reduction in the frequencies and amplitudes of the peaks) is clearly shown. The magnitude of the changes in mode frequency and amplitude predicted by this simple model of fluid loading may be overestimated.

tension T, making an angle θ with the top plate, the force acting in a direction perpendicular to the top plate is $T sin\theta$, as illustrated below in Figure 4.6.

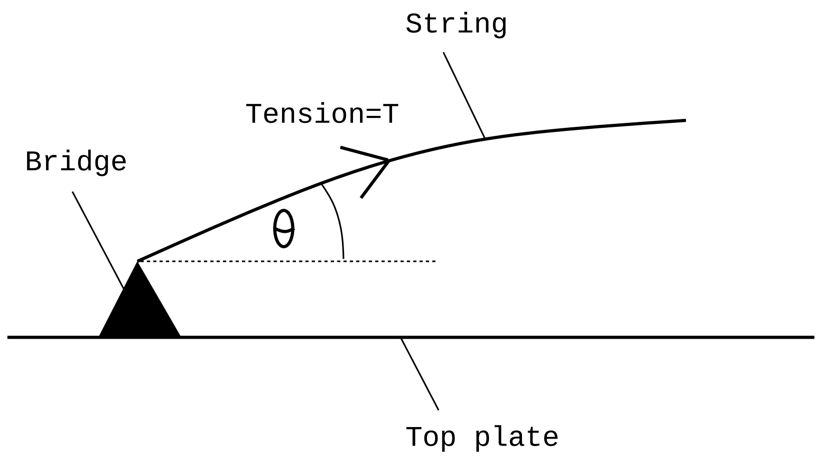


Figure 4.6: Forces imposed on the top plate by the vibrating string. The string tension T acts in a direction along the string, as indicated. The force acting at the bridge in a direction perpendicular to the top plate is $T sin\theta$.

If θ is small, $sin\theta$ and $tan\theta$ are approximately equal. If the string's displacement at distance x from the bridge is denoted as Ψ then

$$sin\theta \approx \frac{\partial \Psi}{\partial x} \ . \tag{4.42}$$

The force acting at the bridge in a direction perpendicular to the top-plate surface can therefore be written as

$$F \approx T\frac{\partial \Psi}{\partial x} \ . \tag{4.43}$$

The displacement of the string at a given distance x from the bridge can be expressed as a summation of Fourier components, each with amplitude a_n:

$$\Psi = \sum \Psi_n \ , \tag{4.44}$$

$$\text{and} \qquad \Psi_n = a_n sin(k_n x) exp(i\omega t) \ , \tag{4.45}$$

where k_n is the wavenumber of the nth mode. The derivative $\frac{\partial \Psi_n}{\partial x}$ due to one string mode is

$$\frac{\partial \Psi_n}{\partial x} = a_n k_n cos(k_n x) exp(i\omega t) \ . \tag{4.46}$$

Evaluated at the bridge ($x = 0$) this gives

$$\left(\frac{\partial \Psi_n}{\partial x}\right)_{x=0} = a_n k_n exp(i\omega t) \ . \tag{4.47}$$

So the perpendicular force F_n due to one string mode is found to be

$$F_n = T a_n k_n exp(i\omega t) \ . \tag{4.48}$$

The total force at the bridge is found by performing a summation over the n string modes:

$$F = \sum F_n \ . \tag{4.49}$$

However, we cannot simply substitute into Equation 4.48 the wavenumbers and Fourier coefficients of the modes of a string stretched between rigid supports. The motion of the top plate alters the both the body modes and the string modes and this coupling must be accounted for in the model.

The displacement x_t of the top plate is given in Equation 4.39. Substituting the expression for the string force from Equation 4.48 we find

$$x_t = \frac{T a_n k_n (X_b X_h - \alpha_{hb}{}^2)}{X_b X_t X_h + 2\alpha_{ht}\alpha_{hb}\alpha_{bt} - [X_t(\alpha_{bh})^2 + X_h(\alpha_{bt})^2 + X_b(\alpha_{ht})^2]} \ . \tag{4.50}$$

Following the ideas of Gough (1981) we write down expressions for the different energy flows into the system. The power supplied by the driving force, P_d, is equal to the driving force multiplied by the velocity of the string at that point:

$$P_d = F_n exp(i\omega t) \left(\frac{\partial \Psi}{\partial t}\right)_{x=x_0} , \tag{4.51}$$

for a force F_n applied at $x = x_0$. Differentiating equation 4.45 with respect to time we find

$$P_d = F_n a_n sin(k_n x_0) i\omega exp(2i\omega t) \ . \tag{4.52}$$

Note that this is the power supplied to one string mode only. The total energy in a rigidly supported, lossless string of mass m and length l under tension T is given by:

$$E_s = \int_0^l \frac{m}{2l} \left[\left(\frac{\partial \Psi}{\partial t}\right)^2 + c^2 \left(\frac{\partial \Psi}{\partial x}\right)^2 \right] \partial x \ , \tag{4.53}$$

where $c^2 = (Tl/m)^{\frac{1}{2}}$. When losses in the string are represented by a damping factor γ, the total rate of change of energy in the string is given by:

$$\frac{\partial}{\partial t} E_s = \int_0^l \frac{m}{2l} \frac{\partial}{\partial t} \left[\left(\frac{\partial \Psi}{\partial t}\right)^2 + c^2 \left(\frac{\partial \Psi}{\partial x}\right)^2 \right] \partial x + \int_0^l \frac{m\gamma}{l} \left(\frac{\partial y}{\partial t}\right)^2 \partial x \ . \tag{4.54}$$

After substituting for Ψ_n and its derivatives, the following expression for the rate of change of energy (power loss) in the nth string mode is obtained:

$$\frac{\partial}{\partial t}E_s = \frac{i\omega m {a_n}^2}{l}({\omega_n}^2 - \omega^2 + i\omega\gamma_n)exp(2i\omega t)\int_0^l sin^2(k_n x)\partial x \; . \tag{4.55}$$

The integral is found to be $\frac{l}{2}$ and so the power loss P_s in one string mode is

$$P_s = \frac{i\omega {a_n}^2}{2}D_s exp(2i\omega t) \; , \tag{4.56}$$

where $D_s = m({\omega_n}^2 - \omega^2 + i\omega\gamma_n)$. At the bridge, the tension of the string results in a vertical component of force equal to $Ta_nk_nexp(i\omega t)$. To balance this there must be a force of $-Ta_nk_nexp(i\omega t)$ at the bridge. If the velocity of the bridge is written as $\frac{\partial}{\partial t}(x_t exp(i\omega t))$ then the power supplied to the bridge can be written as

$$P_{br} = -i\omega T k_n a_n x_t exp(2i\omega t) \; . \tag{4.57}$$

We now have three expressions for the power. Equating the powers as follows

$$P_d = P_s + P_{br} \; , \tag{4.58}$$

the following expression for a_n, the Fourier coefficient of the nth string mode, is obtained:

$$a_n = \frac{2}{D_s}(F_n sin(k_n x_0) + T k_n x_t) \; . \tag{4.59}$$

Combining equations 4.50 and 4.59 to eliminate a_n we obtain an expression for the top-plate displacement, x_t , which includes coupling between one string mode, one top-plate mode, one back-plate mode and the the fundamental air-cavity mode:

$$x_t = \frac{2Tk_nF_nsin(k_nx_0)(X_bX_h - {\alpha_{hb}}^2)}{D_s\left[X_tX_hX_b + 2\alpha_{hb}\alpha_{ht}\alpha_{bt} - (X_t(\alpha_{hb})^2 + X_h(\alpha_{bt})^2 + X_b(\alpha_{ht})^2)\right] - 2T^2{k_n}^2(X_bX_h - {\alpha_{hb}}^2)} \; . \tag{4.60}$$

To obtain the total displacement of the top plate at the driving point, a summation over all string and body modes of interest is performed.

Chapter 5

Experimental work: the guitar body

This chapter will examine the limitations of the model by comparing the synthesised guitar-body response curves with measurements made on real instruments. Curve-fitting methods were used on the measured response curves to extract the values of the four basic resonance parameters for 10 to 15 different body modes on two different instruments. The variability of some of these parameters is emphasised by the differences in response curves observed when the guitar body is driven at different points along the bridge. Mode data was obtained for each guitar at three different driving positions. This chapter deals exclusively with the response of the guitar body in the absence of any coupling to the strings. At all stages, agreement between modelled and observed behaviour is examined and some conclusions are drawn pointing out those aspects of real instruments which are not adequately dealt with in the model. Measurements of the coupling between string and body are presented in the next chapter.

5.1 The guitar body

To be able to perform valid and useful psychoacoustical listening tests using synthesised sounds, we must be sure that the input data for the model accurately represents the response of real instruments. Since we are primarily interested in the influence of the body of the instrument on the sound quality, the input values describing the body modes must be carefully chosen.

Previous work by Brooke (1992) made use of data taken from finite element analysis (FEA) of a spruce top-plate for synthesis of guitar tones. The modelled spruce plate was thinned in steps of 0.1 mm (Walker, 1991). FEA yielded the mode shapes as well as values of resonance frequency, effective monopole area and effective mass for each of the vibrational modes at each stage of the thinning process. These values could then be used to synthesise guitar sounds and investigate the perceived differences in timbre as the plate was thinned. Not all important physical properties of the spruce plate could be modelled using FEA. In particular the damping properties of the wood could not be modelled so that data for the Q-values of the body modes had to be chosen to represent 'typical' values. Furthermore, the plate was assumed to be isotropic, so that potentially important features of the wood grain were overlooked.

FEA is a complex and time-consuming procedure which requires sophisticated computer processing. It is therefore desirable to have a relatively simple and fast method of extracting physical data for real guitars, so that a computer model of one particular guitar can be built. The model can then be used to investigate the changes in sound that would occur when one or more of its physical parameters are varied. The body parameters required by the model are the frequencies, Q-values, effective masses and effective areas of the body modes. Admittance measurements of the instrument will easily yield the resonance frequencies and Q-values of the most prominent peaks, but data for the masses and areas is much harder to obtain.

Schelleng (1963) describes a method for obtaining the value of the effective mass of a mode, by measuring the change in its resonance frequency as small masses are attached to the driving point. This works reasonably well provided the mode under investigation is well separated from all others. If this is not the case, when the masses are added and the resonance frequency is lowered, the mode may overlap with another and it may be impossible to be sure which peak belongs to which mode. The first two or three modes of a guitar are usually well separated, and Schelleng's method can be used here to measure effective masses. However, many of the other modes will overlap each other, and other methods must be used to measure their effective masses.

To enable data for mass, area, frequency and Q-value for all body modes to be obtained, curve-fitting techniques were developed to find the best-fit line for a given response curve. The method used was the Marquardt non-linear least-squares fit and the code was taken from Numerical Recipes (Press et. al, 1989). The method requires the definition of a function

$y = f(x)$ which is to be used as the model for the curve-fitting. Derivatives of y must be given with respect to all parameters which can be varied in the curve fitting.

5.2 Velocity response measurements and curve-fitting

I first took measurements of the velocity response of an instrument and tried to fit single peaks of the response using the function below, which describes the time-averaged velocity of a single oscillator of effective mass m, Q-value Q and resonance frequency ω_0, driven by a force of magnitude F and angular frequency ω:

$$y = 10log_{10}\frac{\omega^2 F^2}{2m^2 ref\left[(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2\right]} \ , \tag{5.1}$$

where ref is an arbitrary constant which defines the zero level on the decibel scale. The experimental set up for the response measurements was as follows. The guitar was supported at the neck in an upright position leaving the top, sides and back free to vibrate. All strings were slackened and damped to avoid coupling between top-plate modes and string modes. The guitar was driven at the bridge saddle using a small wooden mounting device (of mass 0.45 g) which fitted over the saddle and allowed the guitar to be driven at the exact points at which the strings pass over the bridge saddle and drive the top plate. An accelerometer (Brüel & Kjær type 4374) was attached on top of the mounting device and its output fed to a charge amplifier (Brüel & Kjær type 2635). A small magnet was placed on top of the accelerometer and a coil placed over the magnet. A variable frequency, constant amplitude sinusoidal current was passed through the coil to drive the system. The masses of the accelerometer and magnet were 0.65 and 0.69 g respectively. The total mass loading of the top plate was thus less than 2 g. The magnet was placed on top of the accelerometer for all velocity response measurements; the point of detection is therefore the same as the driving point for all velocity response curves shown.

The driving signal was generated using a Brüel & Kjær heterodyne analyser (type 2010) and amplified by a Quad 303 power amplifier. The signal from the accelerometer was connected to the charge amplifier and then fed back into the heterodyne analyser to be bandpass filtered using a bandwidth of 3 Hz. A dc logarithmic output voltage was fed to a PC which sampled and stored the signal. A Brüel & Kjær X-Y recorder (type 2308) was used to control the frequency sweeping of the driving signal from the heterodyne analyser.

The fitting routine was applied to all of the peaks in turn, and values of frequency, Q and effective mass were easily extracted. However, to obtain properly callibrated values for the effective mass, the driving force must be known. Furthermore, if curve-fitting is applied to each peak in isolation, it does not account for the velocity contributions of the other modes.

For this reason, a second curve-fitting program was developed which modelled the response curve data as that of a sum of harmonic oscillators. Although the initial guesses for the frequencies, Q-values and masses needed to be chosen more carefully in order for the program solutions to converge, good fits to many peaks were obtained by first fitting the data to a single peak, then adding data for the other peaks one by one. The program could be used to adjust the parameters for selected peaks, or for all of the peaks, or for a selected region of the response curve, or for the whole frequency range.

Some problems were encountered when trying to obtain the best-fit curve. If two or more peaks lay very close together, there was a tendency for the curve-fitting routines to 'merge' them, as a small change in the resonance frequency of either peak caused them to overlap. It also became increasingly hard to add more peaks once best-fit data for the most prominent peaks had been found. At higher frequencies there was a tendency for several peaks to overlap; often a sharp peak would 'emerge' from a broader peak. Obtaining good fits for these situations was more difficult. Additions were made to the curve-fitting programs which allowed the frequency of a mode to be held fixed, while the effective mass, Q-value and effective area were varied.

In order to obtain callibrated values for the effective masses, Schelleng's method was applied to the fundamental top-plate mode. Measurements were made at several string positions of the change in resonance frequency as 5 g and 10 g masses were attached to the driving point. The values of the T(1,1) effective mass obtained were used to callibrate the values obtained from the curve-fitting. By fixing the input value of the fundamental mode's effective mass, all other masses could be scaled accordingly.

This method was tried on several velocity response curves, but one problem remained. With the soundhole still open, all top-plate modes coupled to the air cavity, with the result that the peaks seen on the response curve are at different frequencies than the natural resonance frequencies of the uncoupled top plate. The coupling to the air cavity also affects the effective masses and Q-values of the top-plate modes. Since we require the values of the uncoupled top-plate modes to use as inputs for the numerical model (the coupling effects being calculated by the model) it was necessary to seal the soundhole with a polystyrene plug, so that coupling between the top plate and the fundamental air-cavity mode was removed. This sealing of the soundhole meant that the air-cavity stiffness felt by the top plate was increased, resulting in a small increase in resonance frequencies, but the Q-values, effective masses and areas were those of the uncoupled top plate.

Figures 5.1–5.3 show the velocity responses measured on guitar BR2 at three different driving positions. The soundhole was sealed for all measurements, and the driving points were all situated on the bridge. The best-fit lines obtained with the curve-fitting programs are indicated as dotted lines. The best-fit values for effective mass, frequency and Q for the three driving positions are given in Tables 5.1–5.3.

Errors

The curve-fitting routine adjusts the parameters so that the errors over the whole response curve are minimised. When inspecting the goodness of fit by eye, greater importance tends to be associated with the positions of the peaks. Better agreement between the frequencies of the peaks in the measured and best-fit responses could be obtained, but only at the expense of a greater error over the whole frequency range. The resonance frequencies of the modes can be more accurately estimated for the sharper peaks (i.e. those with high Q-values). Figure 5.1, for example, indicates that the resonance frequencies of the two lowest modes are accurate to around 1 Hz. It is clear that the errors in the estimates of the frequencies of the higher-frequency peaks are larger than this, perhaps as great as 10 Hz. All best-fit values for the modes' resonance frequencies are quoted to the nearest Hertz, but typical errors are likely to be $\pm$ 5 Hz.

The nature of the curve-fitting makes it difficult to assess the errors for individual values of effective mass, effective area and Q-value. The amplitude of any one peak in the sound pressure response is determined by the values of all three parameters. A best-fit Q-value which is lower than the actual value will cause the errors in the estimation of the effective mass and area for that peak to be greater. In addition, errors in the effective masses and areas of the low-frequency modes make it more difficult to achieve a good fit at high frequencies. Visual inspection of Figures 5.1–5.3 confirms that the errors for the high-frequency peaks are considerably greater than those for the low-frequency peaks. Best-fit values for effective mass and area are quoted to three decimal places; typical errors are likely to be between 3 and 4%. Q-values have been rounded to the nearest whole number; errors are of the order of 5%.

5.2.1 Mode identification

By looking at the response curves it is usually not possible to identify more than two or three modes using information about the frequencies of the peaks. For responses without coupling to the air cavity, the first peak must be the T(1,1) mode, and the second the T(2,1). At higher frequencies, extra evidence is needed to obtain a positive identification of the modes. Holographic data provides the best method for clearly identifying the different modes. Interferograms for guitar BR2 taken by Richardson (1992) were used for this purpose. These holographic interferograms were performed on the fully coupled guitar (i.e. the soundhole was open). In my experiments the soundhole was plugged, so some allowances had to be made for small changes in the frequencies of the modes.

Another method for mode identification is the use of Chladni patterns. This technique is in effect a simple predecessor of holographic techniques. A fine powder is sprinkled onto the top plate of the guitar which is supported by clamps in a horizontal position. The top plate is driven with a sinusoidal force using a coil and magnet. At frequencies corresponding to a particular body mode, the powder lying in the antinodal regions is vibrated and tends to be moved by the vibrations towards the nodal lines. After some time the powder collects along the nodal lines providing a means of identifying the mode. A number of Chladni experiments were tried using dried tea as the 'powder' but the top plate of the guitar was found not to be sufficiently smooth and flat for the tea to collect along the nodal lines. However, the antinodal regions of the top plate were easily identified by the motion of the grains of tea. By distributing the tea evenly over the top plate, modes with up to nine or ten different areas of vibration could be identified with relative ease. The frequencies of each mode were noted and the the pattern of vibrating areas was sketched. A number of different driving points were used to make sure that as many as possible of the modes under 1 kHz were identified.

Eleven top-plate modes were identified for guitar BR2 using this Chladni technique. Additional modes were identified by examining the holographic data by Richardson and allowing for small differences in mode frequency that result from the plugging of the soundhole. By

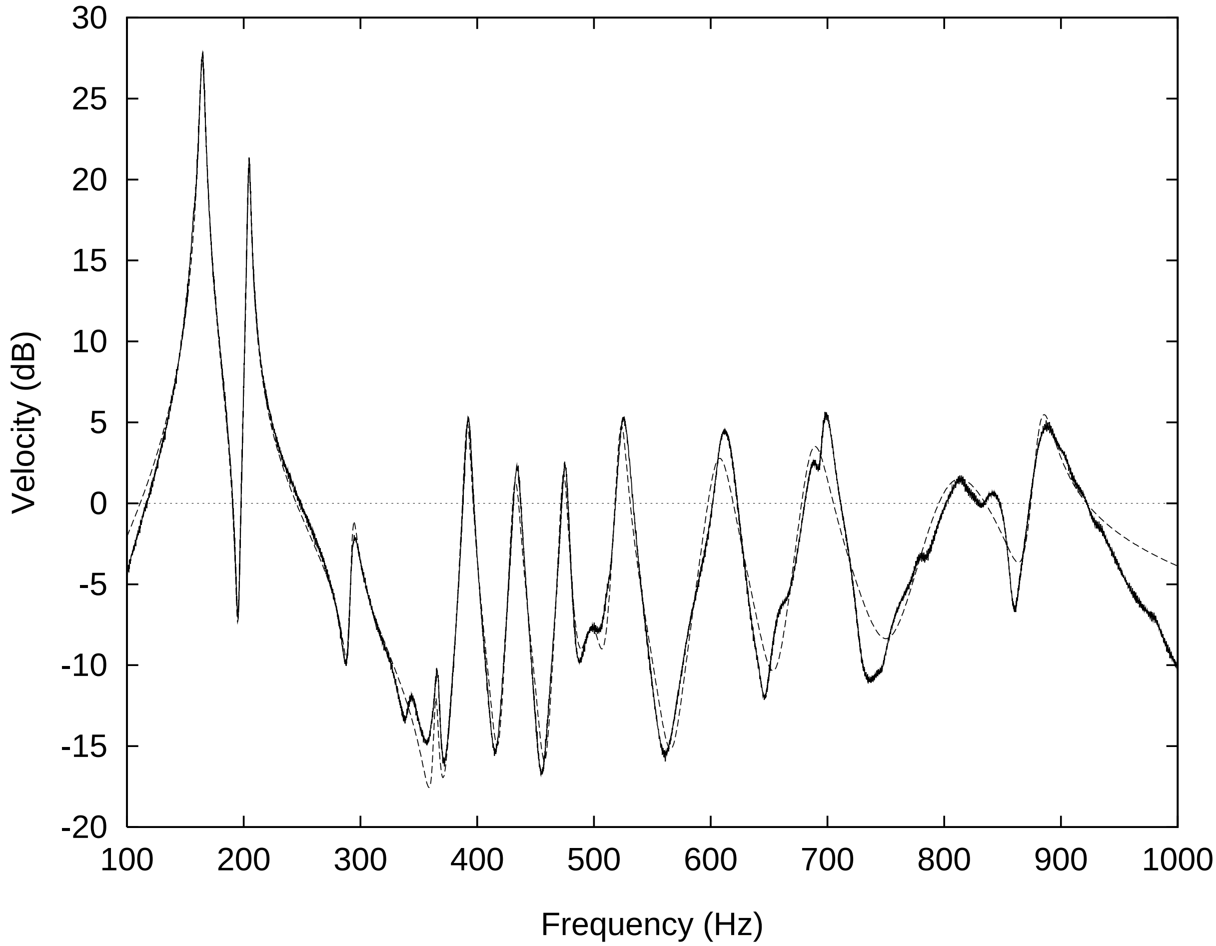


Figure 5.1: Measured velocity response of guitar BR2 (solid line) and best-fit curve (dotted line). The guitar was driven on the bridge at the low E string position with the soundhole sealed. Mode data taken from the best-fit curve is given in Table 5.1

Mode	Resonance frequency (Hz)	Effective mass (kg)	Q-value
T(1,1)	165	0.062	36
T(2,1)	204	0.201	66
B(1,2)	293	1.920	50
$T(1,2)_1$	364	11.374	84
$T(1,2)_2$?	391	0.612	60
T(3,1)	432	0.726	51
T(4,1) ?	474	0.830	66
T(4,1) ?	497	1.001	21
$T(2,2)_1$	523	0.451	57
$T(2,2)_2$	607	0.209	25
T(5,1)	688	0.197	28
???	806	0.096	12
T(4,2) ?	882	0.263	49

Table 5.1: Mode data extracted from velocity response of guitar BR2. Driving point was on the bridge at the low E string position.

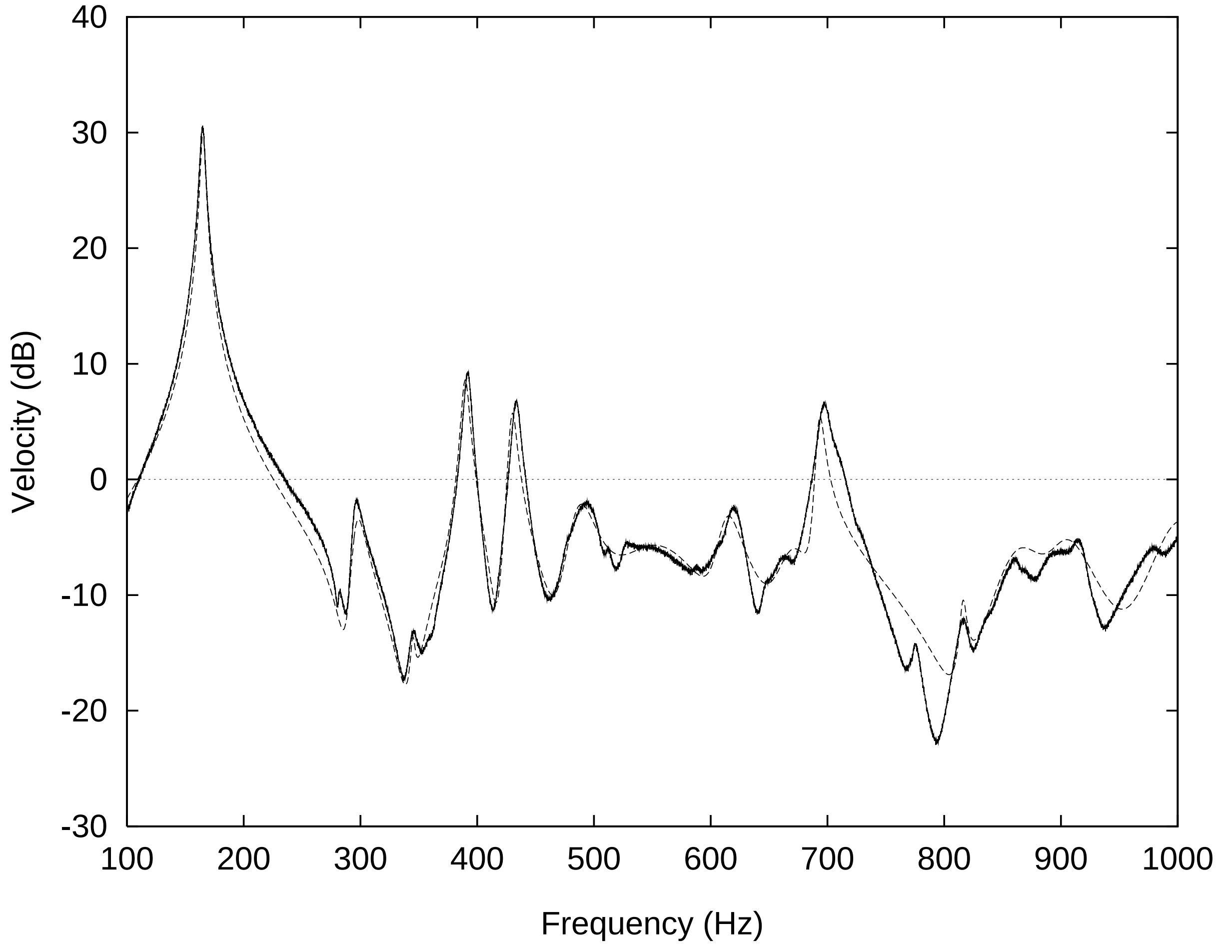


Figure 5.2: Measured velocity response of guitar BR2 (solid line) and best-fit curve (dotted line). The guitar was driven on the bridge at the G string position with the soundhole sealed. Mode data taken from the best-fit curve is given in Table 5.2

Mode	Resonance frequency (Hz)	Effective mass (kg)	Q-value
T(1,1)	165	0.062	41
B(1,2)	296	1.254	24
$T(1,2)_1$	345	23.153	74
$T(1,2)_2$	390	0.457	55
T(3,1)	430	0.623	58
T(4,1)	487	0.571	19
T(2,2) ?	547	0.240	6
T(2,2) ?	613	0.857	28
???	670	0.567	16
T(5,1)	693	0.721	96
???	816	8.383	127
T(4,2)	864	0.439	17
???	904	0.614	22
T(5,2)	1000	0.443	21
???	1085	0.075	13

Table 5.2: Values extracted from velocity response of guitar BR2. Driving point was on the bridge at the G string position.

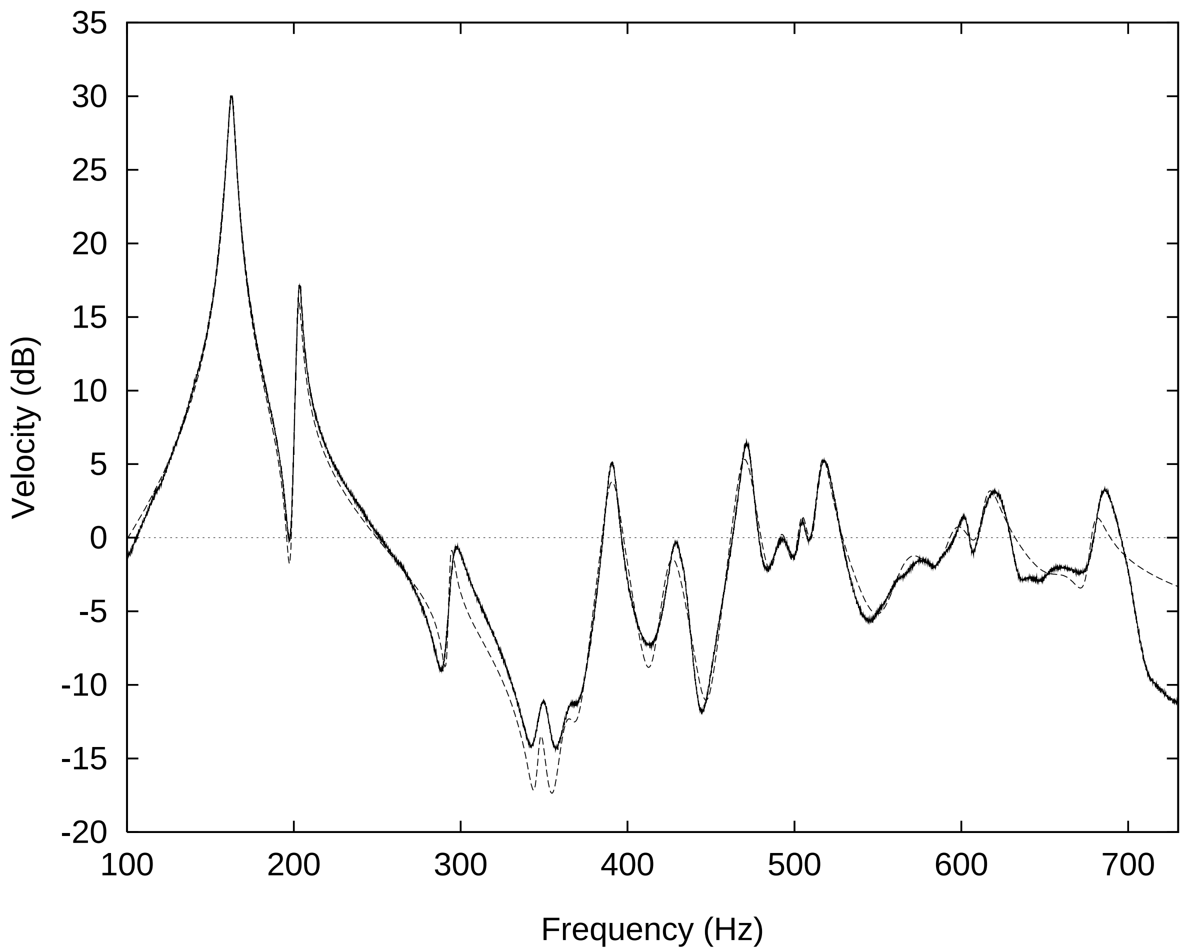


Figure 5.3: Measured velocity response of guitar BR2 (solid line) and best-fit curve (dotted line). The guitar was driven on the bridge at the top E string position with the soundhole sealed. Mode data taken from the best-fit curve is given in Table 5.3

Mode	Resonance frequency (Hz)	Effective mass (kg)	Q-value
T(1,1)	163	0.054	37
T(2,1)	203	0.394	59
B(1,2)	294	3.664	80
$T(1,2)_1$	348	10.875	62
???	364	3.888	25
$T(1,2)_2$	390	0.446	33
T(3,1)	426	0.678	26
T(4,1) ?	470	0.339	35
T(4,1) ?	492	1.006	40
T(2,2) ?	504	1.343	58
T(2,2) ?	516	0.636	57
T(2,2) ?	567	0.395	17
T(2,2) ?	595	0.582	27
T(2,2) ?	615	0.849	49
???	659	1.334	20
???	679	1.351	67

Table 5.3: Values extracted from velocity response of guitar BR2. Driving point was on the bridge at the top E string position.

comparing the frequencies of the peaks in the response curves with the frequencies of the modes visualised using the Chladni technique, most of the modes could be identified. Day-to-day variations in the mode frequencies meant that not all modes could be identified with absolute confidence. In particular, at high frequencies, the modes occur with smaller frequency separations and it becomes increasingly difficult to be sure exactly which peak on a response curve corresponds to which mode.

There are further subtleties to be taken into account when identifying the different modes. Some of the internal air-cavity modes will couple with the plate modes to produce 'double modes'. This is a relatively common occurrence with the T(1,2) mode. In an isolated plate, the mode will appear only once, but in the complete instrument two (1,2)-type modes are often seen. The holographic data by Richardson showed that several symmetrical top-plate modes (e.g. the T(4,1) and T(2,2)) occur at two different frequencies, suggesting some coupling effects with the back or sides of the instrument. Subscripts are used to distinguish between such modes (e.g. $T(1,2)_1$, $T(1,2)_2$).

5.2.2 Coupling between the top and back plates

The top-plate velocity responses show very little influence from the fundamental back-plate mode. Normally the B(1,1) mode couples to the T(1,1) mode to produce two peaks at around 200 Hz in the response. Although the soundhole was sealed, it was expected that coupling between top and back plate would continue in a similar way due to the pressure changes in the cavity. Only one peak corresponding to the fundamental top-plate modes was observed in the measured response.

Further experiments showed that the manner in which the soundhole was sealed had a strong influence on the degree of coupling between fundamental top- and back-plate modes. When the soundhole was plugged with a polystyrene disc, the back plate coupled very weakly to the top plate. When the soundhole was sealed by using masking tape, the back seemed to couple more strongly, and the two peaks were seen as expected. A comparison of the two responses is shown in Figure 5.4. In the response with the soundhole sealed with tape, the results of the stronger coupling between back plate and top plate are clear. The first peak is at a much lower frequency, the height of the peak is lower due to the increase in effective mass resulting from the coupling, and the Q-value of the peak is noticeably lower due to increased damping.

Other peaks in the response remain unaffected apart from the peak at around 310 Hz whose effective mass and damping both increase when the back couples more strongly to the top plate. This was also observed when comparing responses with a sealed soundhole, and those with an open soundhole; the height and damping of the peak were significantly increased in the fully coupled system. This suggested that the mode at 310 Hz was a back-plate mode rather than a top-plate mode. This was confirmed by the holographic data which shows the B(1,2) mode occurring at a frequency of around 300 Hz.

The fact that the coupling of the fundamental back-plate mode could be largely excluded by using a polystyrene plug was advantageous since the response curves were to be used to estimate values of the uncoupled top-plate mode parameters. The absence of the back plate coupling was a welcome finding, although it indicated that the interaction of the top, back and air cavity was more complicated than the model suggested.

5.3 Sound pressure responses

Using the single and multi-peak fitting programs, values of ω_0, m and Q were obtained from the velocity responses of the instrument, as described in Section 5.2. Sound pressure responses were needed to obtain values of A, the effective monopole area. Velocity and pressure responses were measured in pairs, leaving the magnet and coil arrangement the same for both so that the same driving force was used for both responses.

The pressure responses were measured in an anechoic chamber with a Brüel & Kjær condenser microphone (type 4144) connected to a power supply (type 2807). The microphone was placed directly in front of the guitar, level with the bridge at a distance of 0.9 m. The microphone signal was fed into the heterodyne analyser to be filtered and rectified as before. The function used for the curve-fitting routines modelled the sound pressure radiation from the guitar as that of a sum of monopole sources, the source strength for each mode being given by the product of its velocity and effective area. In the far-field, where the distance of the microphone from the top plate is greater than the wavelength of sound, the low-frequency radiation fields can be reasonably well approximated by a sum of monopole sources. The distance of the microphone from the guitar used in these measurements was limited by the size of

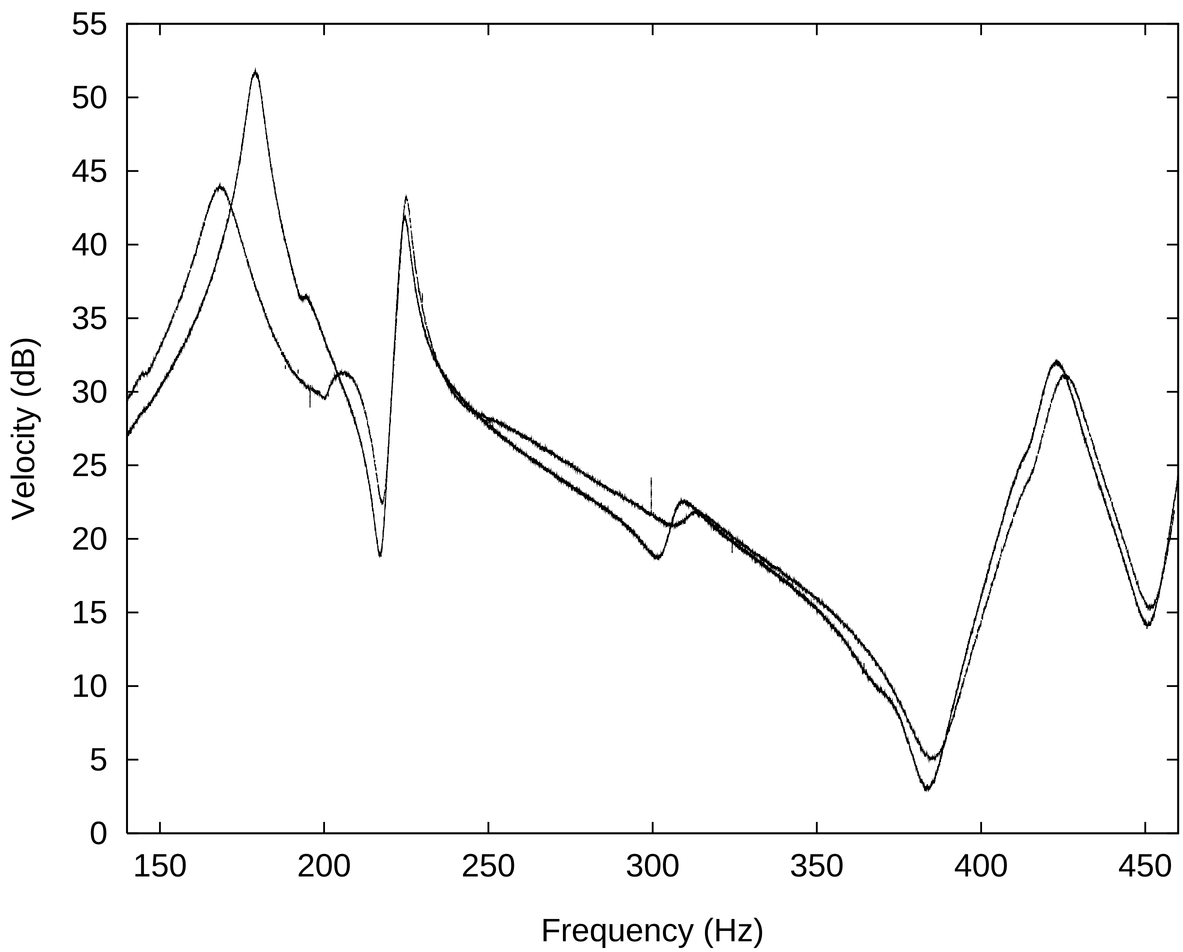


Figure 5.4: Comparison of response curves for guitar BR2 obtained by sealing soundhole with polystyrene plug and masking tape. The response using the polystyrene plug gives a sharper and greater-amplitude peak at around 175 Hz. The guitar was driven on the bridge at the top E position.

the anechoic chamber, and at around 1 m only satisfies the far-field condition for frequencies of around 300 Hz or more.

Once data for the Q-value, frequency and effective mass had been found from the velocity responses, these values could be held constant as the effective area was adjusted to obtain the best fit. Occasionally, small adjustments in the frequency or Q-value of one of the modes were required to improve the pressure fit.

Measurements were made on two guitars using three different driving positions on each instrument. The driving points were on the bridge saddle at the low E, G and top E string positions. For most guitars, the responses at the different driving positions will be very different. Although the same body modes are being driven, the positions of the nodal lines relative to the driving point cause the values of effective mass and effective area to vary considerably between different string positions.

The curve-fitting techniques were applied to the three velocity and three pressure responses of each guitar. This eventually yielded sets of data for each driving position giving values of resonance frequency, effective mass, effective area and Q-value. The comparison of measured and best-fit pressure responses for guitar BR2 are shown in Figures 5.5–5.7. The values obtained from the best-fit curves are given in Tables 5.4–5.6.

5.4 Back plate response curves

Once values of the top-plate mode parameters had been determined, data relating to the back-plate modes and the fundamental air-cavity mode was also required in order to be able to investigate the model's ability to accurately predict the coupling between top plate, back plate and air cavity. The area of the soundhole is easily measured; the other parameters required for the air cavity were the frequency and Q-value of the uncoupled Helmholtz mode. It is possible to measure these, but there is some difficulty in removing the coupling of the top and back plate so that values for the uncoupled air cavity are obtained. Christensen (1984) measured values of f_h for five guitars and found the frequencies to be in the range 122–135 Hz. Since the most important factors influencing the frequency and Q-value of the Helmholtz mode are the volume of the cavity, the shape of the cavity and the size of the soundhole, and since different guitar makers tend only to experiment with small variations on a fairly standard construction, it is not surprising to find little variation in these values. Huber (1991) gives data on the dimensions of a variety of handmade instruments. The diameters of the soundholes were found to be in the range 83–89 mm and the depth of the guitar body (although not constant along its length) was found to be between 90 and 107 mm. Such variations are likely to lead to only very slight differences in the Helmholtz mode parameters from instrument to instrument. For this reason it was decided not to measure the frequency and Q-value of the Helmholtz modes of the two guitars but simply to assume 'standard' values and make small adjustments as necessary. The values that produced the most satisfactory results from the model were $f_h = 122$ Hz, $Q_h = 40$ for guitar BR1 and $f_h = 122$ Hz, $Q_h = 45$ for guitar BR2.

To obtain estimates of the back-plate mode parameters pairs of velocity and sound pressure response curves were measured. The experimental arrangement was largely the same as before. The sound hole was sealed, and the back plate was driven in the centre at a position roughly equivalent to the bridge position on the top plate. The fundamental back-plate mode is usually the most important as it produces the largest volume change and couples most strongly with the air cavity and top plate. Values for the fundamental mode and one or two higher modes were obtained for both instruments as shown in Tables 5.7 and 5.8.

5.5 Comparisons between measured and synthesised response curves

The previous sections have shown how measurements on the guitar body, together with curve-fitting techniques, allow good estimates of the four mode parameters to be obtained for several modes of a real instrument. These estimates can then be used as input data for the model, and the results of the model compared with other measurements on the instruments. This is an important step as it highlights parts of the model which accurately reflect the behaviour of real instruments, as well as drawing attention to aspects of real instruments which are not adequately dealt with in the modelling scheme.

We have so far modelled the response of a combination of 10–15 top-plate modes by using values extracted from real instruments. Similarly, values for the first few back-plate modes were obtained. The best-fit curves obtained for the velocity response of a simple summation

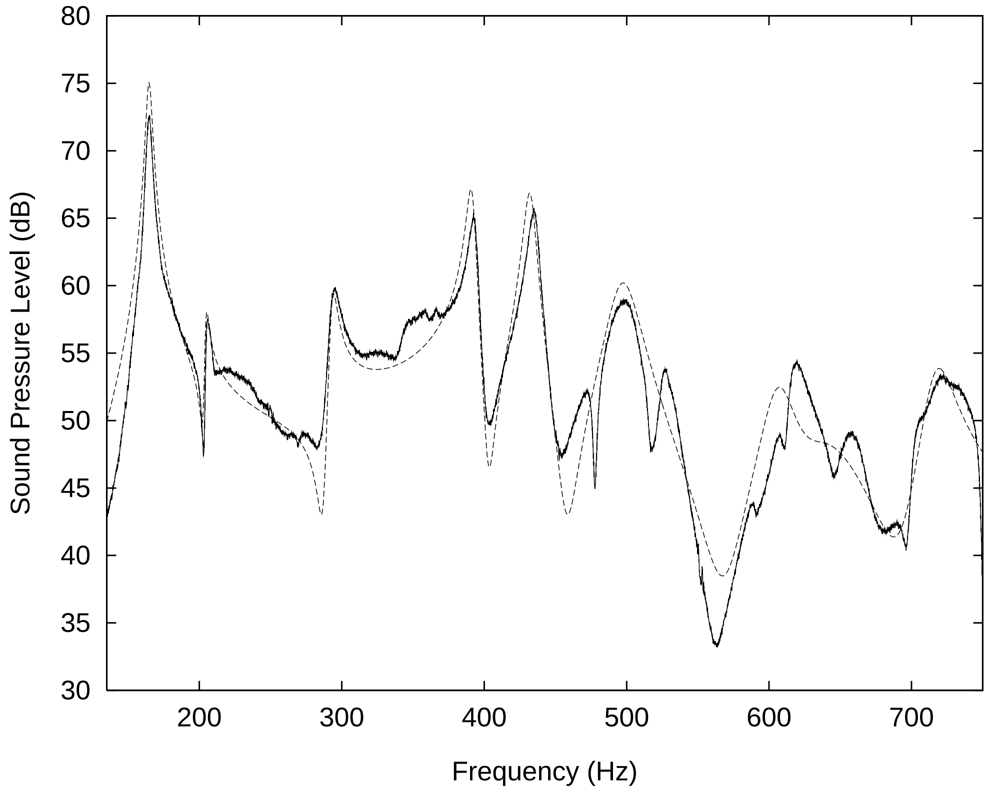


Figure 5.5: Measured sound pressure response for guitar BR2 (solid line) and best-fit curve (dotted line). The guitar was driven on the bridge at the low E string position with the soundhole sealed. The microphone was placed directly in front of the bridge at a distance of 0.9 m.

Mode	Resonance frequency (Hz)	Effective mass (kg)	Q-value	Effective area (m^2)
T(1,1)	165	0.062	36	0.059
T(2,1)	204	0.201	66	0.010
B(1,2)	293	1.920	50	0.186
T(1,2)	391	0.612	60	−0.133
T(3,1)	432	0.726	51	−0.189
T(4,1) ?	497	1.001	21	−0.290
T(2,2)	607	0.209	25	−0.017
???	639	0.500	11	−0.044
T(3,2)	717	0.300	33	−0.024

Table 5.4: Values extracted from sound pressure response of guitar BR2. Driving point was on the bridge at the low E string position.

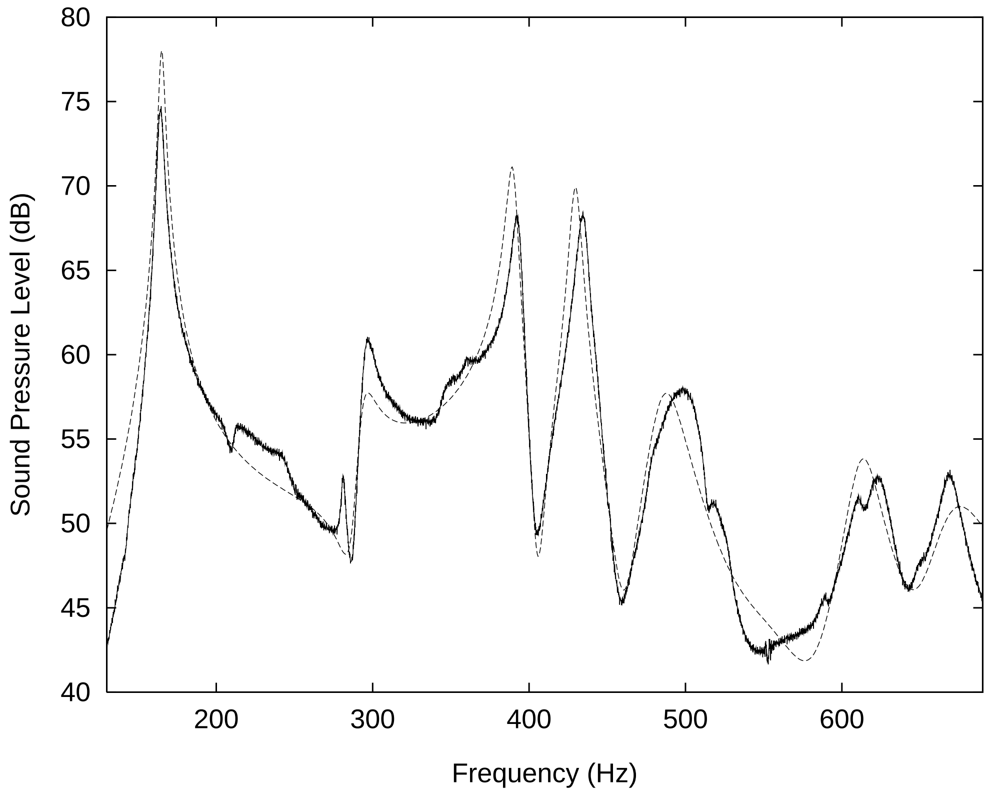


Figure 5.6: Measured sound pressure response of guitar BR2 (solid line) and best-fit curve (dotted line). The guitar was driven on the bridge at the G string position with the soundhole sealed. The microphone was placed directly in front of the bridge at a distance of 0.9 m.

Mode	Resonance frequency (Hz)	Effective mass (kg)	Q-value	Effective area (m^2)
T(1,1)	165	0.062	41	0.061
B(1,2)	293	1.254	24	0.132
$T(1,2)_1$	390	0.457	55	−0.145
T(3,1)	430	0.623	58	−0.165
T(4,1) ?	487	0.571	19	−0.105
???	547	0.240	6	−0.016
T(2,2)	613	0.857	28	−0.064
???	670	0.567	16	−0.046

Table 5.5: Values extracted from sound pressure response of guitar BR2. Driving point was on the bridge at the G string position.

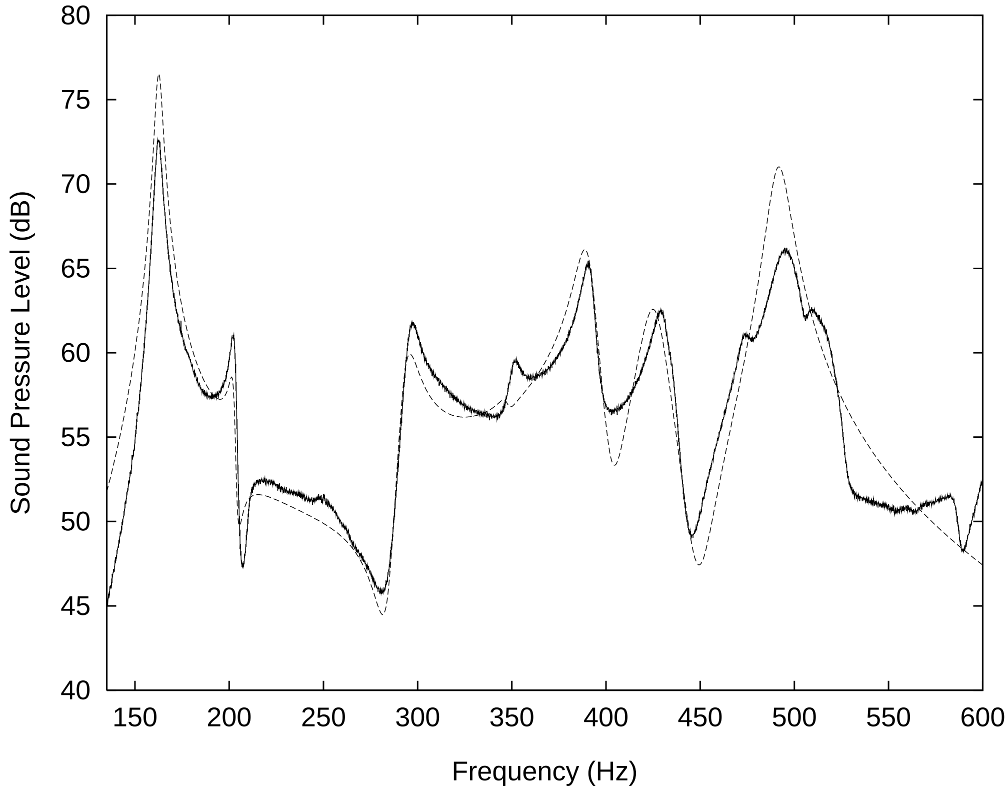


Figure 5.7: Measured sound pressure response of guitar BR2 (solid line) and best-fit curve (dotted line). The guitar was driven on the bridge at the top E string position with the soundhole sealed. The microphone was placed directly in front of the bridge at a distance of 0.9 m.

Mode	Resonance frequency (Hz)	Effective mass (kg)	Q-value	Effective area (m^2)
T(1,1)	163	0.054	37	0.053
T(2,1)	203	0.394	59	−0.018
B(1,2)	294	3.664	28	0.556
$T(1,2)_1$	348	12.875	62	−0.070
$T(1,2)_2$	390	0.446	33	−0.126
T(3,1)	426	0.678	26	−0.170
???	492	1.006	40	−0.474

Table 5.6: Values extracted from sound pressure response of guitar BR2. Driving point was on the bridge at the top E string position.

Mode	Resonance frequency (Hz)	Effective mass (kg)	Q-value	Effective area (m^2)
B(1,1)	221	0.071	31	0.065
B(1,2)	310	0.370	18	0.073
B(?,?)	412	0.310	42	−0.071

Table 5.7: Mode data for the back plate taken from the measured pressure response of guitar BR1 driven at the centre of the back plate with the soundhole sealed. The microphone was directly in front of the bridge at a distance of 0.9 m.

Mode	Resonance frequency (Hz)	Effective mass (kg)	Q-value	Effective area (m^2)
B(1,1)	207	0.051	40	0.060
B(1,2)	277	0.446	13	0.214

Table 5.8: Mode data for the back plate taken from the measured pressure response of guitar BR2 driven at the centre of the back plate with the soundhole sealed. The microphone was directly in front of the bridge at a distance of 0.9 m.

of modes are extremely close to the measured data, indicating that just three parameters (frequency, Q-value and effective mass) for each of the body modes can very adequately describe the velocity response of the guitar. Best-fit curves for the radiated sound pressure are still good, but the agreement between measured and synthesised responses is not as good as that for the velocity responses, indicating that the fourth parameter, the effective piston area, does not properly describe the sound radiation properties of the modes.

Calculating a response curve for a summation of uncoupled oscillators does not properly test all predictive powers of the model. Many of the most important features of an instrument occur as a result of coupling between its different parts. For this reason a second series of measurements were made on the two guitars. This time the soundhole was left open to allow the normal coupling between top plate, back plate and air cavity. The strings remained slackened and damped, so that the response of the body alone was measured. Velocity and sound pressure measurements were made up to a frequency of around 1 kHz, the experimental arrangement being the same as before.

The values estimated from the curve-fitting routines were then used as input for the model. This time the full coupling between the top-plate modes and the fundamental modes of the air cavity and back plate was calculated in the model. The synthesised response curves again show some shortfalls of the model. The frequencies of some of the coupled modes are not correct, pointing to errors in the estimation of the effective area. Although the frequencies and Q-values of body modes will show some variation from day to day due to different room temperatures and humidity levels, the mismatch between synthesised response and measured response was up to 50 Hz which is far larger than can be accounted for by changes in atmospheric conditions. This disagreement must be due to the calculated coupling being too large. The model uses the effective area parameter to calculate the volume velocity of a mode which determines the degree of coupling to the back plate and air cavity, as well as the level of sound radiation. The values of the effective areas are estimated from measurements of the sound pressure response of the guitar. It seems likely that the simplified model of the sound pressure radiation leads to an incorrect estimate for the effective area. This leads to errors when calculating the degree of coupling between top plate, back plate and air cavity.

The most important coupling occurs between the fundamental resonances of the top plate, back plate and air cavity. The interaction produces a triplet of resonance peaks, the lowest of which is usually found at around 100 Hz, the other two at around 200 Hz. When the data from the best-fit curves is used to calculate this resonance triplet, the lowest peak is around 30 Hz too low, the middle peak around 50 Hz too low, and the highest peak around 25 Hz too high, indicating that the coupling between the three modes is too strong. Adjustments were made to the values of effective area for the fundamental top and back-plate modes until the correct degree of coupling was achieved, and the results from the model showed good agreement with experimentally measured response curves. Since not all measurements were made on the same day, there are some small variations in the frequencies of some of the modes due to atmospheric effects. The coupled mode frequencies predicted by the model are therefore not always exactly the same as the measured frequencies, but were usually within around 5 Hz of the measured values.

To achieve a good match between synthesised and measured responses, the values of the effective areas had to be reduced by around 20% in the case of the fundamental top-plate mode, and by around 60% in the case of the back plate. Some adjustments to the estimated values of the effective area for some of the higher modes were also made to improve the match between measured and synthesised response. This inability to model both the sound pressure radiation and the mode coupling with a single parameter, the effective piston area, is the first major shortcoming of the model. It is perhaps not surprising that the pressure radiated from each of the modes cannot be accurately modelled by assuming that the volume displaced by the mode can be equated with the source strength of a monopole radiator. The modelling scheme involves the movement of a piston with an associated area. In reality, the amplitude distribution of a mode over the surface of the top plate may be rather complex. The details of this distribution, as well as the three-dimensional shape of the guitar itself will have important consequences for the sound radiation.

As well as highlighting the inadequacy of the effective area parameter to accurately model the sound pressure radiation, the comparison between synthesised and measured response curves also showed other aspects of real instruments not dealt with in the modelling scheme. In the frequency range 350–550 Hz some modes present in the responses measured with the soundhole sealed are absent in the equivalent response measured with the soundhole opened. In addition, some new modes appear when the soundhole is open. This is most likely due to the coupling of internal air-cavity modes to top-plate modes. In most guitars, the first internal air-cavity mode occurs at a frequency of around 450 Hz. The frequency of the T(1,2) mode is also found to be fairly close to 450 Hz in many guitars, being primarily determined by the geometry of the guitar rather than the properties of the materials used. These two modes, having similar amplitude distributions, will tend to interact to produce two coupled modes. This coupling is at present not included in the modelling scheme. The details of the interaction are more complicated than those which describe the coupling to the fundamental air-cavity mode. The internal air modes may have several areas vibrating in anti-phase; this is also true for top-plate modes above the fundamental resonance. One or more of these areas may interact producing the coupling effect. To be able to predict the change in frequency, damping and effective mass of the original modes is beyond the scope of the present model. Instead, simple adjustments to the acoustical parameters of some of the top-plate modes were made to obtain a better match with the measured response. In many cases a shift in frequency was all that was required, but small adjustments to the Q-values, effective masses and areas were sometimes necessary. The coupling interaction meant that 'new' peaks appeared on the response curves, but attempts were not made to obtain best-fit data for these peaks.

As already stated, matching the precise frequency of every mode was not a high priority since changes atmospheric conditions between measurements makes this almost impossible to achieve. Atmospheric changes also account for the different Q-values measured for the same mode on different responses. However, an agreement in frequency to within 5 or 10 Hz, and more importantly a good match in the general shape of the curve and the relative heights of the different peaks was obtained. Some examples are shown in Figures 5.8–5.11. The poor match between modelled and measured response above 500 Hz is clear and indicates features of real instruments that are not adequately modelled in the current scheme. Such features include the coupling between internal air-cavity modes and top-plate modes, mechanical coupling between the top and back plate via the sides of the instrument, and the frequency dependence of the values of effective area. In addition, there is a small peak at around 120 Hz on both synthesised sound pressure responses that is absent on the measured data. This is a result of the coupling calculated between the T(1,2) mode, which has a fairly large effective area, and the Helmholtz cavity mode. The coupling on the real instrument is clearly smaller, again indicating the problems of using a single parameter to model the coupling between plate modes and the air cavity as well as the sound radiation from the instrument.

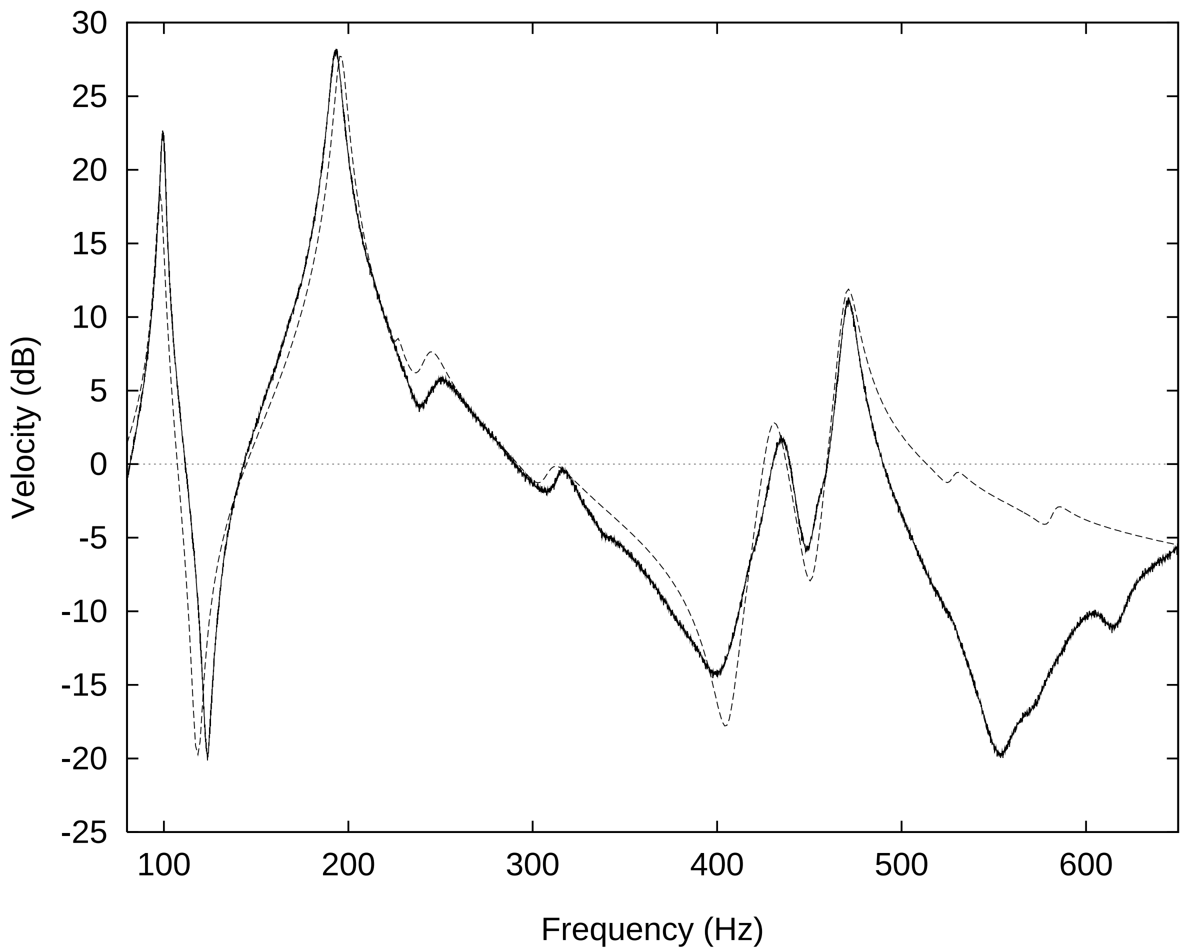


Figure 5.8: Measured velocity response of guitar BR1 (solid line) and modelled velocity response (dotted line). The guitar was driven on the bridge at the G string position with the soundhole open.

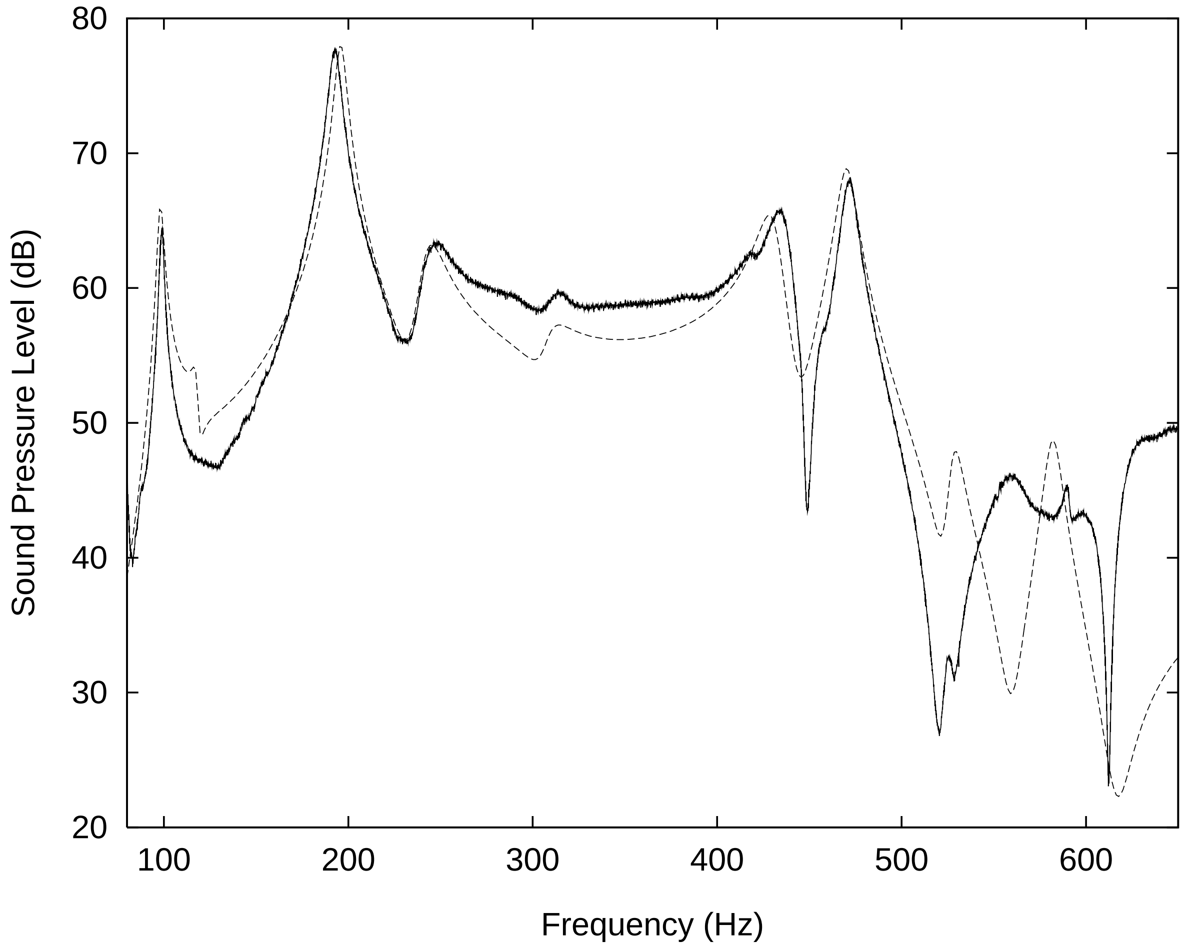


Figure 5.9: Measured sound pressure response of guitar BR1 (solid line) and modelled pressure response (dotted line). The guitar was driven on the bridge at the G string position with the soundhole open.

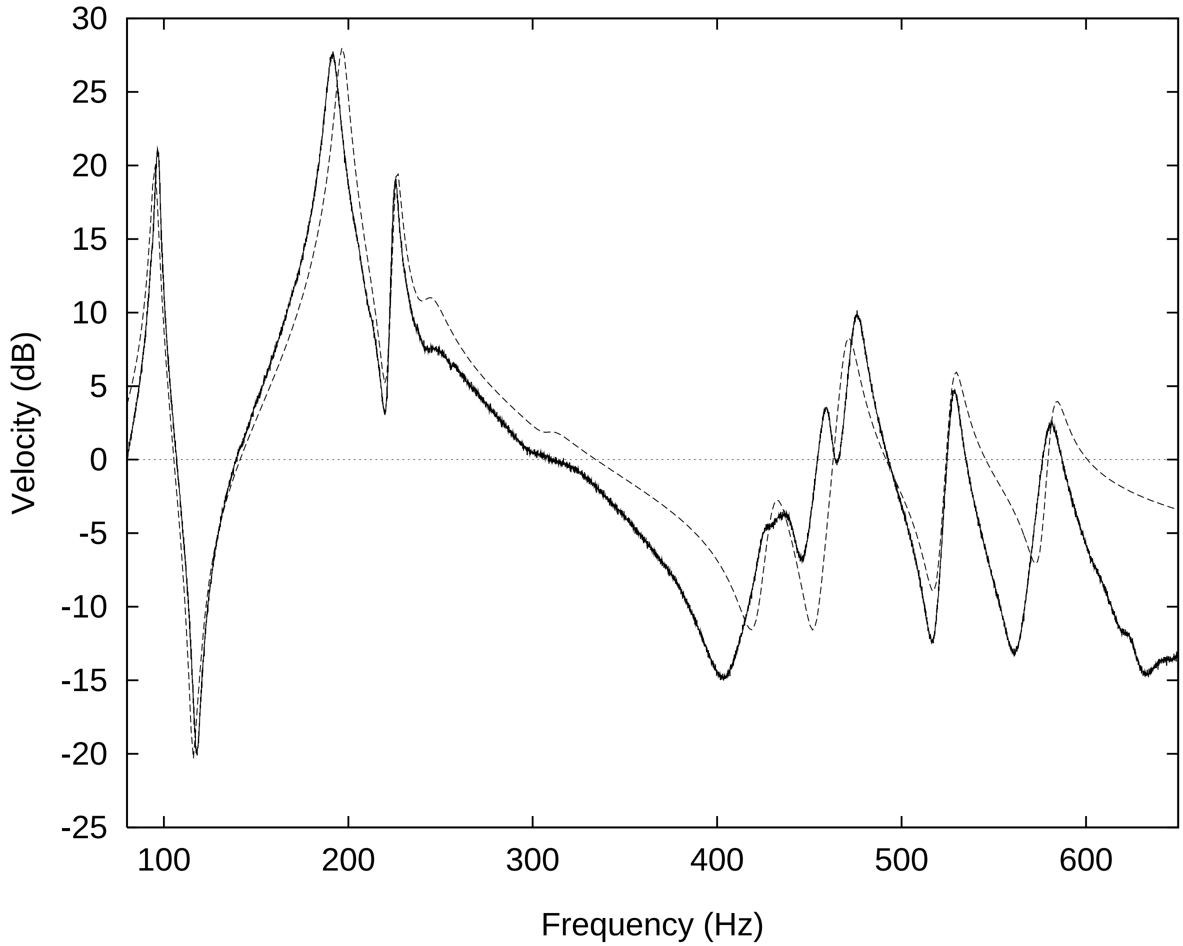


Figure 5.10: Measured velocity response of guitar BR1 driven at the top E string position (solid line) and modelled velocity response (dotted line). The guitar was driven on the bridge at the top E string position with the soundhole open.

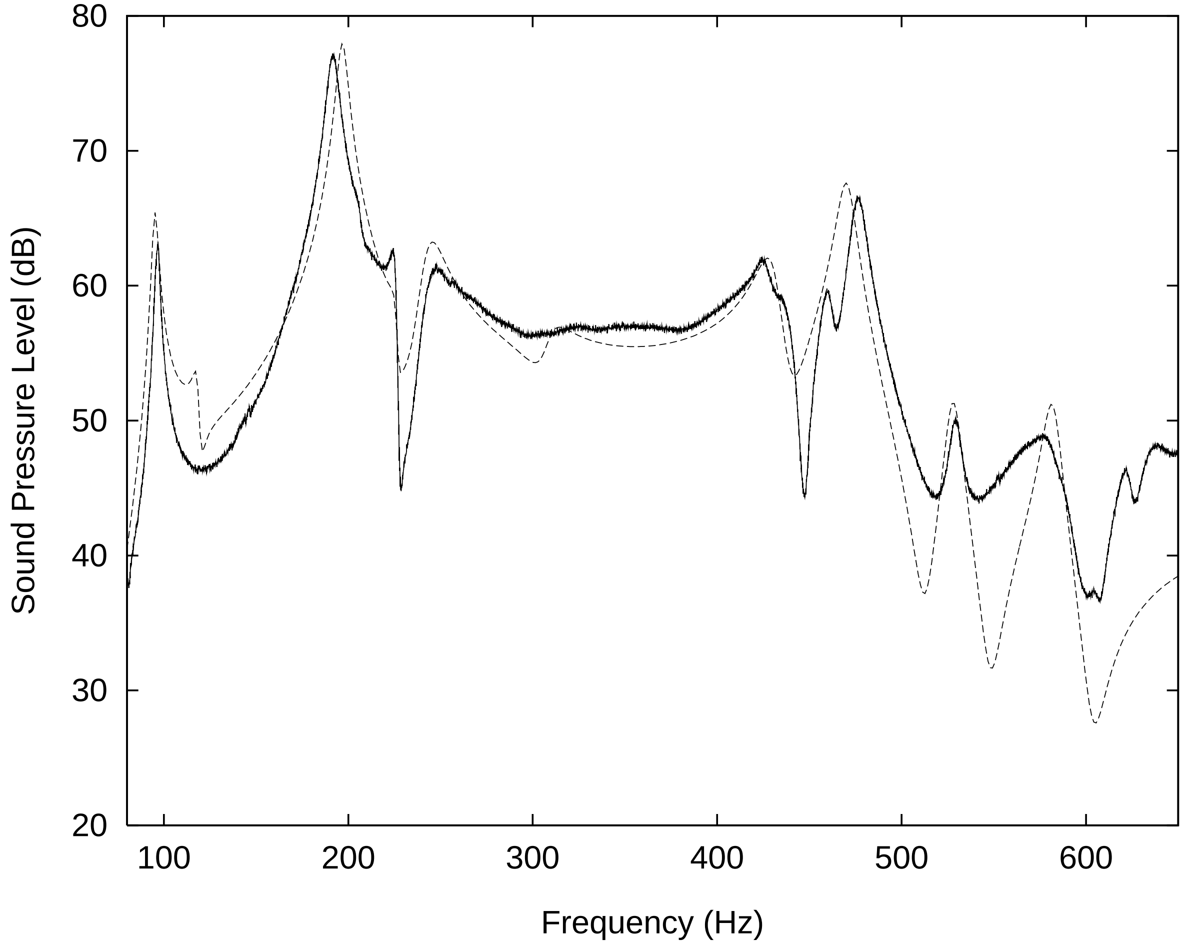


Figure 5.11: Measured sound pressure response of guitar BR1 (with soundhole open) driven at the top E string position (solid line) and modelled pressure response (dotted line). The guitar was driven on the bridge at the top E string position with the soundhole open.

5.6 Summary

In this chapter I have investigated the validity of the modelling assumptions for the guitar body by comparing measured responses on real instruments with responses synthesised from the model. Velocity and sound pressure responses were measured on two guitars (with the soundhole sealed) using three different driving positions. Using a curve-fitting function which modelled the response curves as a summation of contributions from several (uncoupled) oscillators, excellent fits were obtained for the velocity responses. Good fits were also obtained for the sound pressure responses at frequencies below 700 Hz. Estimates of the four acoustical parameters associated with each body mode were taken from the best-fit curves. Values of these parameters for 10 to 15 uncoupled top-plate modes were found for each of the three driving positions. The response of the back plate of each instrument was also measured, and best-fit data for the first few back-plate modes was similarly obtained.

Using the estimated values for the uncoupled plate modes, responses were calculated from the model which included full coupling between the top plate, back plate and fundamental air-cavity mode. Comparisons between synthesised responses and the measured responses of the instrument (with the soundhole open) showed a number of differences. The model calculates an excessively large coupling between the fundamental modes of the top plate, back plate and air cavity resulting in some coupled modes being 40–50 Hz away from the measured resonance frequencies. This indicates that the effective area parameter could not be used to calculate both the volume displacement and sound radiation properties of a mode. A reduction in the estimated value of the effective area of the fundamental top and back-plate modes led to a significantly better match between the synthesised response and the measured data in the low-frequency region. At frequencies above 500 Hz, other features in the measured response were not reflected in the model's predictions. Some peaks appeared as doublets, and the frequencies and amplitudes of others were significantly changed, largely as a result of coupling between internal air-cavity modes and plate modes. Mechanical coupling between the top and back may also alter the response of the real instrument. The model, in its current form, makes no attempt to account for these interactions.

Despite the shortcomings of the model, a set of body-mode parameters for the two instruments was obtained. These parameters described, to reasonable accuracy, the velocity and sound pressure response of the instrument up to a frequency of 600–700 Hz in response to a force applied to one of three driving positions on the bridge. It was now possible to use the body-mode parameters to calculate the modelled response of the guitar body when coupled to a string. The next chapter deals with this and compares the predicted behaviour of the string-body system with measurements made on the two instruments.

Chapter 6

Experimental work: the coupled string-body system

The calculation of the coupling between string and body by the model must be validated by comparisons with measurements on real instruments. The modelling scheme assumes the string is driven at a point and vibrates in a plane perpendicular to the surface of the top plate. The model then calculates the velocity response at a point on the bridge. Two approaches were used to try and obtain measurements of the coupled string-body system that matched the assumptions used in the modelling scheme. One was to pluck the string and measure the velocity response at the bridge with an accelerometer. The time-domain velocity signal could then be Fourier transformed to produce the frequency response of the coupled string-body system. The second approach was to pass a sinusoidal current along the string and pass the string between two poles of a strong magnet to produce a sinusoidal driving force along a short length of the string in a direction normal to the surface of the top plate. The frequency of the the driving force could then be swept through the region of interest and the velocity at the bridge measured with an accelerometer.

6.1 The free string

Before the model can synthesise the response of the guitar body to a force applied to one of its strings, it must have the relevant input data to describe the string. The vibrating length of the string, its mass and tension, the frequencies and Q-values of the string modes and the position of the driving force along the string must all be measured or assigned an appropriate value. The lengths of the string when stopped at the different frets are found in the literature on guitar construction (e.g. Sloane, 1976). Values for the mass and tension of the string will vary somewhat between different makes of string; for our purposes 'typical' values will suffice, and values were taken from measurements by Brooke (1992) and Richardson (1982).

The frequencies of the string modes can, as a first approximation, be assumed to be integer multiples of the fundamental frequency. To achieve a synthesised sound that is somewhat closer to the real instrument, the inharmonicity of string modes should be considered. The small shifts in frequency of the string modes from harmonic behaviour may be due to a number of causes. Strong coupling between a string and body mode will cause the frequencies of the string modes to be shifted away from their harmonic values. This will be most obvious when string modes couple to the fundamental top-plate mode. The deviation from harmonicity is part of the reason for the unpleasant character imparted to these strongly coupled notes. This coupling is already included in the model. Inharmonicity of the string modes can also result from the effects of string stiffness, which is not accounted for in the modelling scheme. The higher-frequency string modes must fit a larger number of half waves along their length and so the string suffers greater bending. The reluctance of the string to bend, due to its stiffness, causes these higher modes to be slightly higher in frequency than the true harmonic relations of the ideal string.

Fletcher (1964) measured the deviation from harmonicity of stiff piano strings and derived the expression given in Equation 2.1 (page 29) for the frequencies of the string overtones. The partial frequencies described by the equation were found to match the experimental data measured by Fletcher very well, and Equation 2.1 was therefore used to calculate the frequencies of the string modes so that the effects of inharmonicity due to string stiffness were included in the model.

6.1.1 Decay rates of the string modes

The relative decay rates of the string partials have an important effect on the tone quality of a guitar note. Synthesis of realistic sounds from the model required that all input data, including the Q-values of the string modes, was chosen to reflect the behaviour of real instruments. The Q-values of fundamental modes of different guitar strings have been measured previously

(Richardson, 1982; Brooke, 1992), but there was little available data on the Q-values of higher string modes.

An experiment was set up to measure the Q-values for several string modes. The guitar string was mounted on a heavy metal bar. The bar was filled with sand to further increase its mass and hence reduce coupling between the string and the bar as far as possible. The string was plucked using a length of fine copper wire. The wire has a low breaking strain and so when wrapped around the guitar string and pulled it will snap allowing a plucking force of consistent strength to be applied to the string. The plucking point was chosen to be approximately one thirteenth of the way along the string so that the first 12 partials would all be fairly strongly excited by the pluck. The vertical component of the force was measured using a Brüel & Kjær force transducer (type 8200), fed into a conditioning amplifier (Brüel & Kjær type 2626) and then to a heterodyne analyser. The signal was filtered using a bandwidth of 30 Hz and then converted to a linear dc voltage that was sampled on a PC. The decay envelope of the string was displayed on screen and a point soon after the initial maximum force value was chosen. The time taken for the force to drop to $1/e$ of this value was measured.

The first experiment used a number of different B strings and these were all tuned to 247 Hz. Some time was allowed for the string to 'settle in' and a small amount of retuning was usually necessary after having left the string for a few minutes. The decay rates of the first and fifth string modes were measured on a variety of string types including nylon, gut and steel. The decay times for each string were measured three times and an average taken. The results are shown in Table 6.1.

A second experiment was performed using one of the nylon B strings and a metal-wound A string to determine the decay rates of all of the first ten modes as well as the 15th and 20th modes. Results are given in Table 6.2. It is clear that there is a relatively complex variation in decay times for the different string modes although there seems to be a moderate rise in Q-value as the frequencies of the modes increase.

By plucking the string in the vertical direction, the subsequent motion of the string was largely confined to this plane. However, the shape of some of the decay envelopes showed deviations from the expected exponential behaviour showing that energy was being exchanged between the vertical and horizontal planes of vibration. On some of the decay envelopes clear dips and peaks were superimposed on this exponential decay corresponding to the losses and

String type	Decay times (sec)	
	1st mode	5th mode
Nylon A	0.830 ± 0.01	0.392 ± 0.005
Nylon B	0.776 ± 0.04	0.368 ± 0.01
Gut	0.887 ± 0.01	0.400 ± 0.002
Varnished Gut	0.903 ± 0.02	0.419 ± 0.004
Polyester	0.889 ± 0.01	0.322 ± 0.008
Steel	3.118 ± 0.02	1.333 ± 0.01

Table 6.1: Measured decay rates for a number of guitar strings, mounted on a rigid, massive bar. The force at the string termination was measured and band-pass filtered to isolate individual string modes. Decay times given are the times taken for the force amplitude to drop to $1/e$ of its initial value.

Mode No.	B string		A string	
	Decay time (sec)	Q-value	Decay time (sec)	Q-value
1	0.94 ± 0.03	730 ± 20	6.89 ± 0.45	2390 ± 160
2	0.37 ± 0.004	580 ± 10	4.75 ± 0.05	3300 ± 30
3	0.52 ± 0.008	1220 ± 20	3.42 ± 0.04	3560 ± 40
4	0.48 ± 0.005	1500 ± 20	2.39 ± 0.10	3310 ± 140
5	0.43 ± 0.01	1680 ± 20	2.26 ± 0.11	3920 ± 190
6	0.37 ± 0.006	1730 ± 30	1.59 ± 0.04	3310 ± 80
7	0.33 ± 0.003	1810 ± 20	1.37 ± 0.02	3330 ± 50
8	0.19 ± 0.002	1190 ± 10	1.79 ± 0.06	4970 ± 170
9	0.20 ± 0.005	1410 ± 40	1.39 ± 0.19	4350 ± 600
10	0.24 ± 0.02	1880 ± 160	1.55 ± 0.04	5380 ± 140
15	0.15 ± 0.001	1780 ± 20	0.63 ± 0.04	3290 ± 210
20	0.12 ± 0.002	1920 ± 30	0.43 ± 0.03	2990 ± 210

Table 6.2: Measured decay rates for different modes of a nylon B string ($f_0 = 247$ Hz) and a metal-wound A string ($f_0 = 110$ Hz) mounted on a rigid, massive bar.

gains of energy to and from the horizontal modes. Gough (1981) has described how longitudinal vibrations of the string cause coupling between the horizontally and vertically polarised motion, resulting in exchanges in energy between the two transverse modes. Other important effects may arise from small differences in the string terminations, which lead to the two transverse modes experiencing different boundary conditions and will break the degeneracy of the transverse modes. In assuming an exponential decay of the vertical component of force at the bridge we are overlooking all these effects, and this will contribute to errors in the measurement of the decay times. The relatively large errors in the decay times of 8th and higher modes of the A string are due to these effects.

6.2 Response of the bridge to an applied plucking force

To achieve an plucking force of consistent strength, lengths of fine copper wire were used, as before. Using the copper wire to pluck the string also satisfies the model's conditions that the string should be driven at a point. The wire was pulled so that the guitar string moved initially in the plane perpendicular to the surface of the top plate. On release the string continued to vibrate predominantly in this plane, but coupling to the body modes or to the longitudinal vibrations of the string produced some motion in the plane parallel to the top plate as the string vibrations decayed. When the time-domain velocity signal measured at the bridge was Fourier transformed, only the initial portion of the signal was used where it could more reasonably be assumed that the string motion was predominantly in the vertical plane.

An accelerometer was used to measure the velocity response of the bridge to the plucking force applied to the string. The signal from the accelerometer was fed to a charge amplifier and then low-pass filtered before being sampled by a computer. The signal was displayed on screen so that the correct portion to be Fourier transformed could be chosen.

The measurements made previously on the instrument had used a small wooden mounting device placed over the saddle of the bridge at the appropriate string position, and so it was decided to try the same approach here. However, the mounting device damped the string's motion quite considerably, and subsequent measurements were made with the accelerometer attached to the flat portion of the bridge just behind the saddle. Comparison of the Fourier transforms of the signals measured at the two different positions on the bridge showed that there were no significant differences between the two, and in particular, the heights of the major peaks of the response were within one or two decibels of each other. The Fourier transforms of the measured signals and the comparisons with modelled response curves are shown in Figures 6.1–6.3.

The essential features of the responses are well predicted by the model. The taller, sharper peaks correspond to modes of the string, other peaks correspond to modes of the body. The coupling between string and body modes matches the real data well, although at higher frequencies not all body modes have been included in the modelled response. The real data shows a larger number of smaller peaks, corresponding to body modes above 400 Hz. In Figure 6.1, the first string mode is below the Helmholtz resonance and the second string mode is below the fundamental top-plate resonance. When the open string is stopped at the second fret, as in Figure 6.2, both the first and second string partials roughly coincide with the body modes. The second partial at 180 Hz couples strongly with the $T(1,1)_2$ mode to produce a split peak.

The major point of disagreement between modelled and measured data is in the height of the string peaks. The modelled response typically shows a moderate increase in the amplitude of the string peaks at higher frequencies. The measured data shows that the higher string modes tend to have a lower peak height. This is largely due to the nature of the force applied in the experiment using the copper wire. When the wire breaks, the force applied to the string will be similar to a step function in time. The Fourier transform of a step function varies as the inverse of the frequency; the energy imparted by the wire-pluck will therefore be more strongly concentrated in the low-frequency string partials, as observed in the experimental results. The modelled responses use an impulse force to excite the system, so that energy is distributed equally in the frequency domain. Differences between the actual Q-values of the string modes and the values used in the model may also contribute to the differences in the peak amplitudes. A change in the Q-value of a string mode by a factor of two will cause a change in peak amplitude of 6 dB.

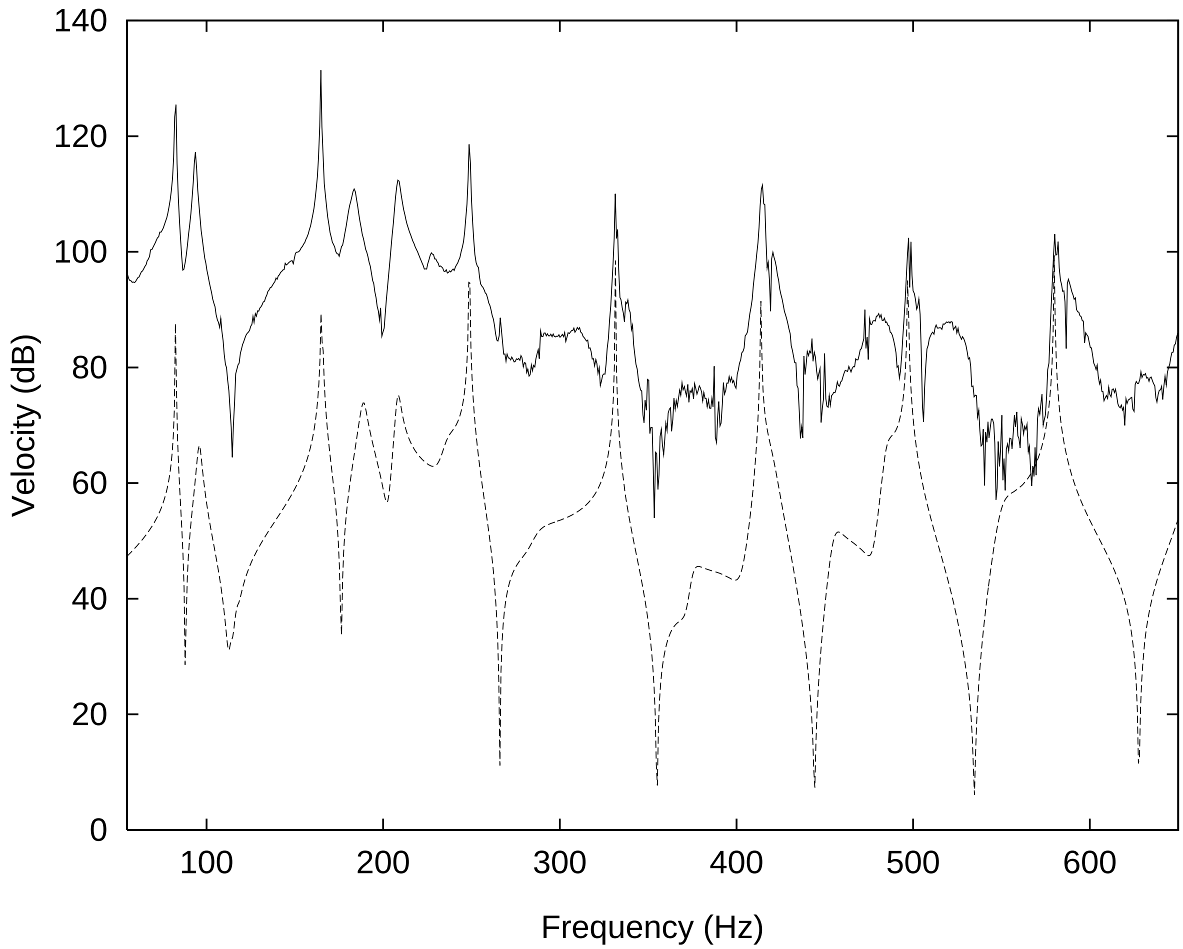


Figure 6.1: Response of guitar, measured at the bridge, to an plucking force applied to the open low E string and comparison with modelled response (dotted line). The modelled response has been displaced for clarity. Tall, sharp peaks at near-harmonic frequencies correspond to the coupled string modes; broader peaks are associated with modes of the guitar body.

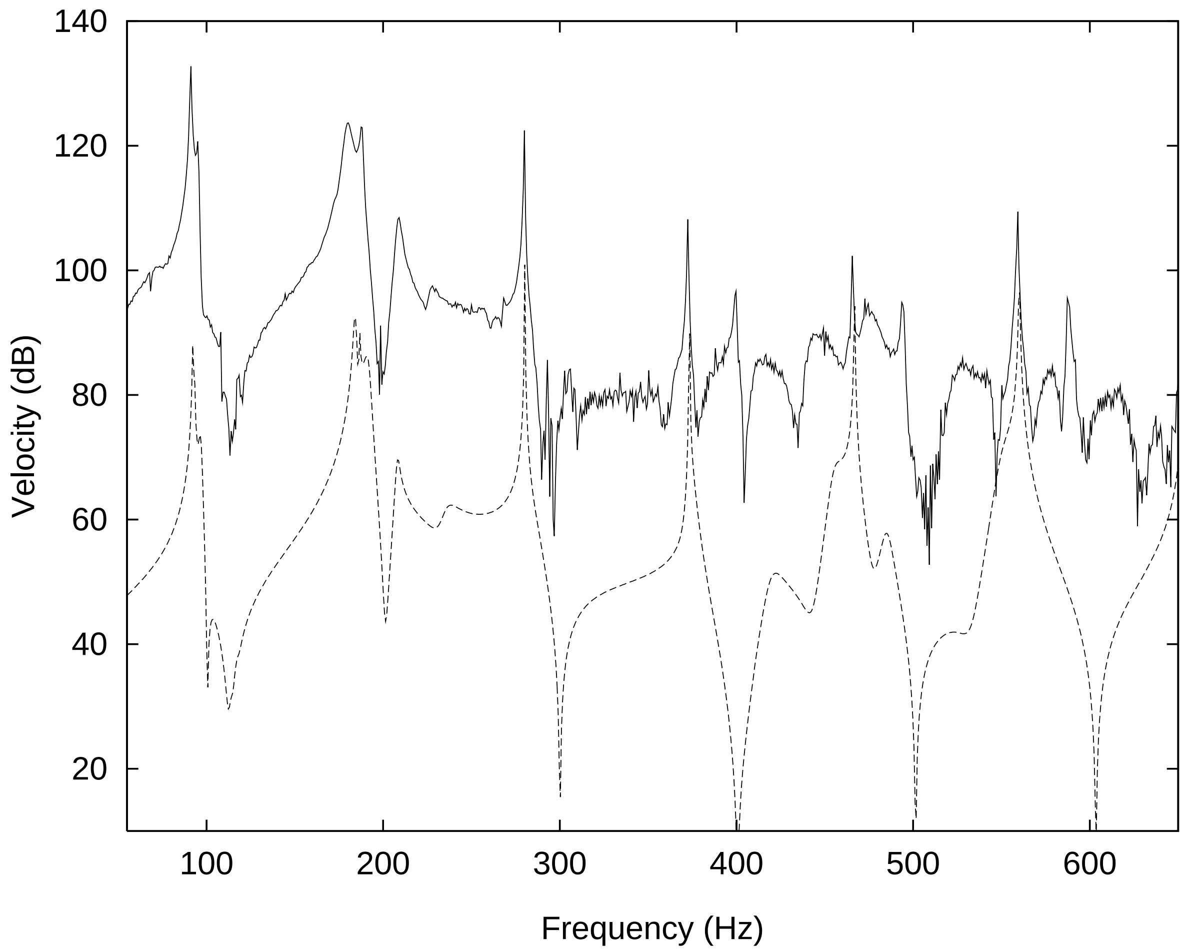


Figure 6.2: Response of the guitar, measured at the bridge, to an plucking force applied to the low E string, stopped at the 2nd fret, and comparison with modelled response (dotted line). The modelled response has been displaced for clarity. Note that the first and second string partials (at 92.5 and 185 Hz) coincide with the first two body modes, resulting in strong coupling that creates split resonances. The coupling between the second string partial and the $T(1,1)_2$ mode at 185 Hz is particularly strong, creating a highly damped split resonance whose two component frequencies lie either side of the true harmonic frequency of the string partial. This produces a rapidly decaying sound which may also sound somewhat 'out of tune', depending on the shift in frequencies associated with the split peak. This phenomenon is sometimes referred to as the guitar wolf-note.

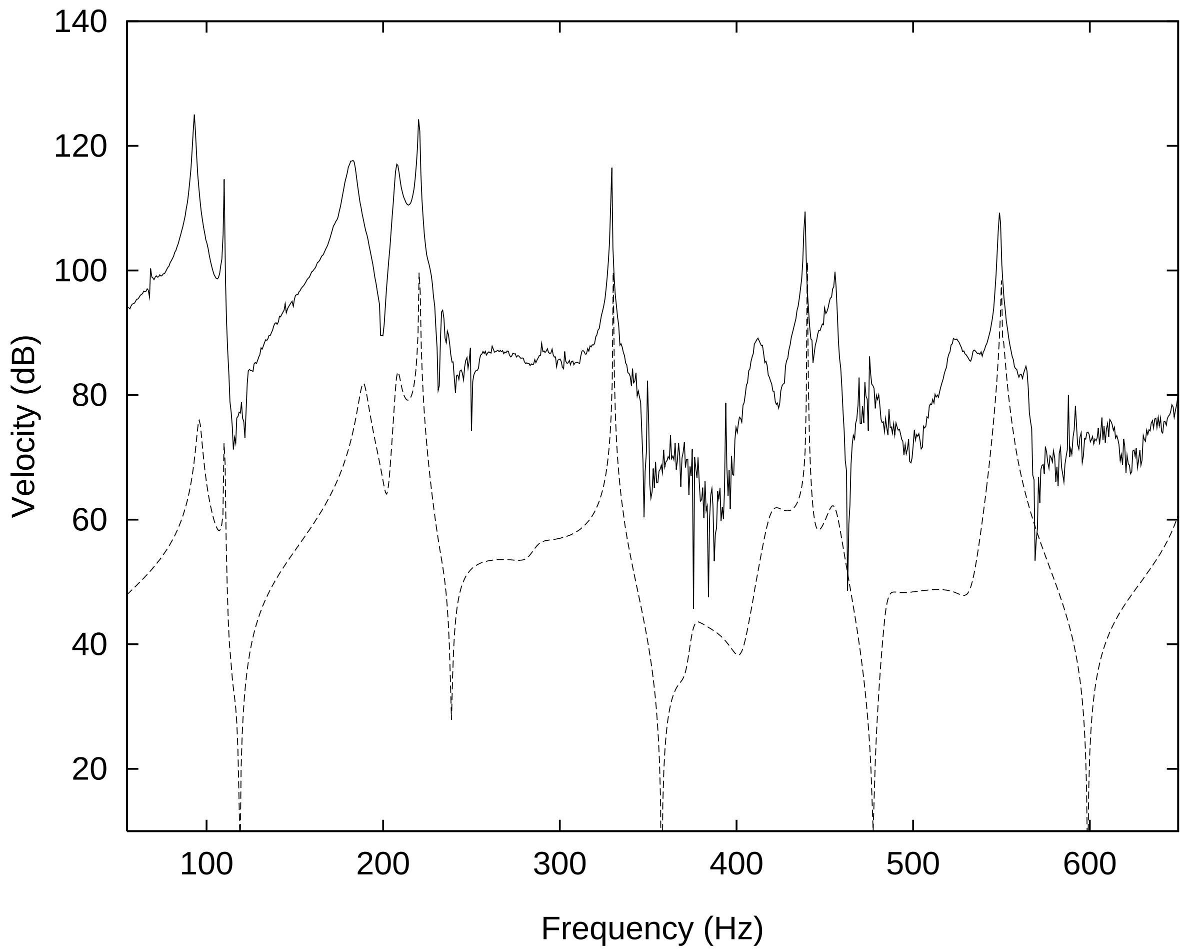


Figure 6.3: Response of the guitar, measured at the bridge, to an plucking force applied to the low E string, stopped at the 5th fret, and comparison with modelled response (dotted line). The modelled response has been displaced for clarity.

6.3 Response of the body to a sinusoidal force applied to the string

A second experiment was set up to try to drive the string in a plane perpendicular to the top plate and measure the response at the bridge. The driving force on the string was achieved by using a sinusoidal voltage from the heterodyne analyser, amplifying it using the Quad amplifier, then applying it to the two ends of the guitar's low E string. The lowest three strings are metal wound and so can conduct electricity along their length. Electrical connections were made by attaching crocodile clips to the string at the tie block at one end and to the part of the string leading to the tuning heads at the other. The vibrating length and mass of the string were hence unaffected by the connections. The string was passed between the poles of a powerful magnet. The interaction of the current through the string and the magnetic field provided a force perpendicular to the guitar's top plate. An accelerometer was used to measure the response of the body of the guitar to the driving force applied at the string. The signal from the accelerometer was fed to the heterodyne analyser where it was converted to a dc logarithmic output voltage. The output voltage was sampled and stored on a PC.

A measurement of the horizontal and vertical movement of the string was needed to show whether the string was vibrating predominantly in a vertical plane. The devices used had a small emitter of infra-red light with a photo-diode mounted opposite. Between them was a small slot through which the string could pass, as shown in Figure 6.4. The output voltage of the device is dependent on the amount of light reaching the photo-diode so as the string moves it casts a shadow on the photo-diode and the output voltage falls. By using two such devices a measurement of the horizontal and vertical displacement of the string was obtained. A small circuit (see Figure 6.5) was built for each switch to provide the necessary supply voltage and current.

6.3.1 Opto-switch callibration

In order to callibrate the opto-switches, a small screw was mounted on the end of a micrometer. The opto-switch and micrometer were mounted so that when the micrometer was turned the screw was slowly pushed into the slot of the opto-switch casting a shadow on the light-sensitive surface. The displacement of the screw could be measured to an accuracy of around one

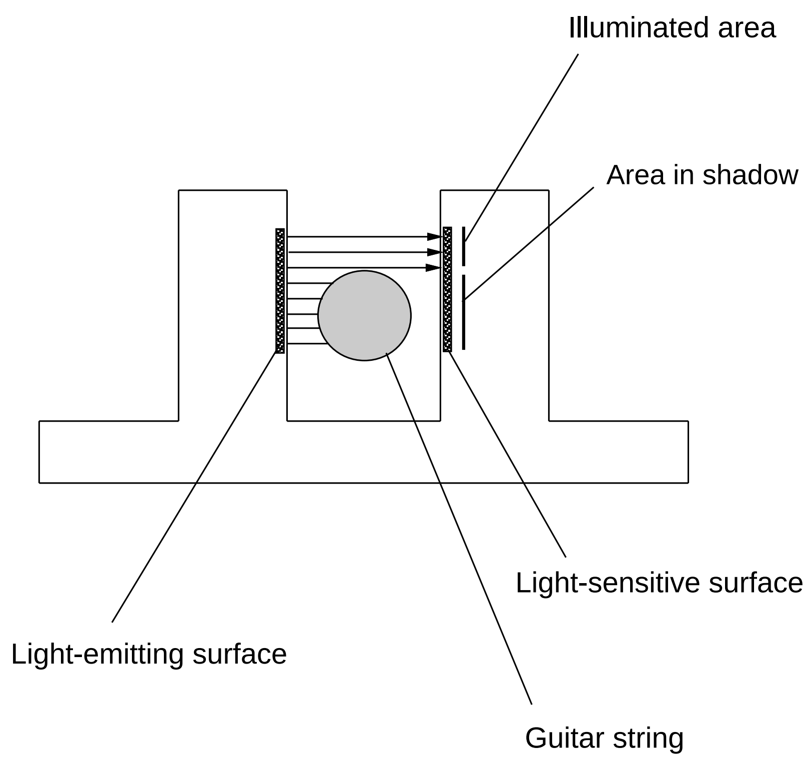


Figure 6.4: Opto-switch in cross-section, mounted on the guitar so that the string passes in front of the light-sensitive surface.

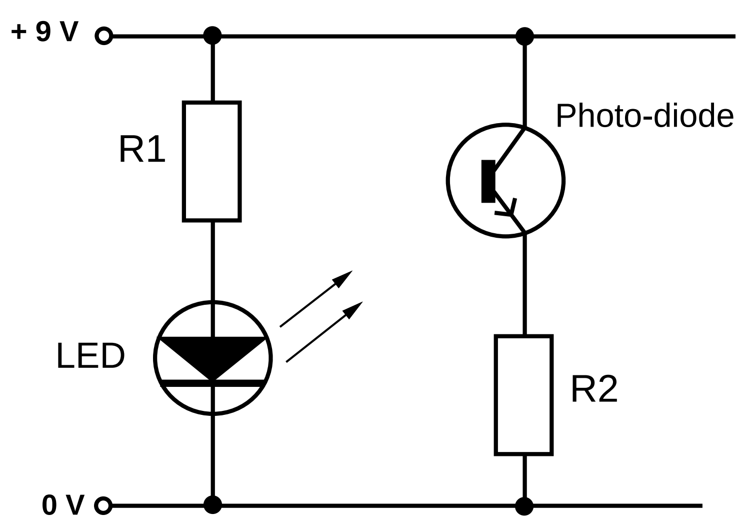


Figure 6.5: Circuit used with the opto-switches. Resistor R1 is used to limit the current through the light-emitting diode (LED). The output voltage was measured across resistor R2. A value of 1 kΩ was used for both resistors. The LED and the photo-diode are both contained inside the opto-switch.

hundredth of a millimetre, and the output voltage of the device was measured using an AVO meter. The light-sensitive area of the device was found to be situated 2.15 mm from the top edge, and extended a further 1.10 mm down. The width of the sensitive area was 0.95 mm. The voltage-displacement characteristics of one of the devices are shown in Figure 6.6 along with the best-fit straight line for the middle (linear) portion.

The voltage-displacement characteristics of the two devices were measured three times. For one measurement the circuits were left switched on for several hours to ensure that the circuit and opto-switches were not significantly affected by heating effects. The voltage-displacement characteristics for each device were plotted and a best-fit straight line found for the linear region of each curve. The three values found for the gradient of the best-fit line for each device were in good agreement, and an average value was calculated. Data is given in Table 6.3. By measuring the output voltages of the opto-switch circuit during the experiment, the displacement of the string could then be calculated using the gradient of the best-fit lines.

During the experiment the output voltage of the opto-switch was sampled and stored on a PC for later scaling and conversion. The callibration of the computer's analogue to digital (A to D) converter needed to be obtained so that the integer values stored by the computer could be converted back into the actual output voltages of the devices.

6.3.2 Callibration of the PC's analogue to digital converter

The callibration of the computer's A to D converter was measured by applying a d.c voltage to the analogue input, sampling this, and using an AVO meter to measure the voltage. The sampling time was set to several seconds and the average integer value stored by the PC over this period was obtained. Each individual sample was represented by an integer value, but the average of several of these values tended to be non-integer. The results and the best-fit line obtained are shown in Figure 6.7.

A program was written to read in the file with the sampled voltage from the opto-switches, then using the A to D callibration the output voltages of the devices were calculated. The gradients of the voltage-displacement characteristics of the opto-switches were used to calculate the displacement of the string.

To test the accuracy of the callibrations, the two opto-switches were attached side by side to the guitar's neck so that they both measured the vertically polarised component of

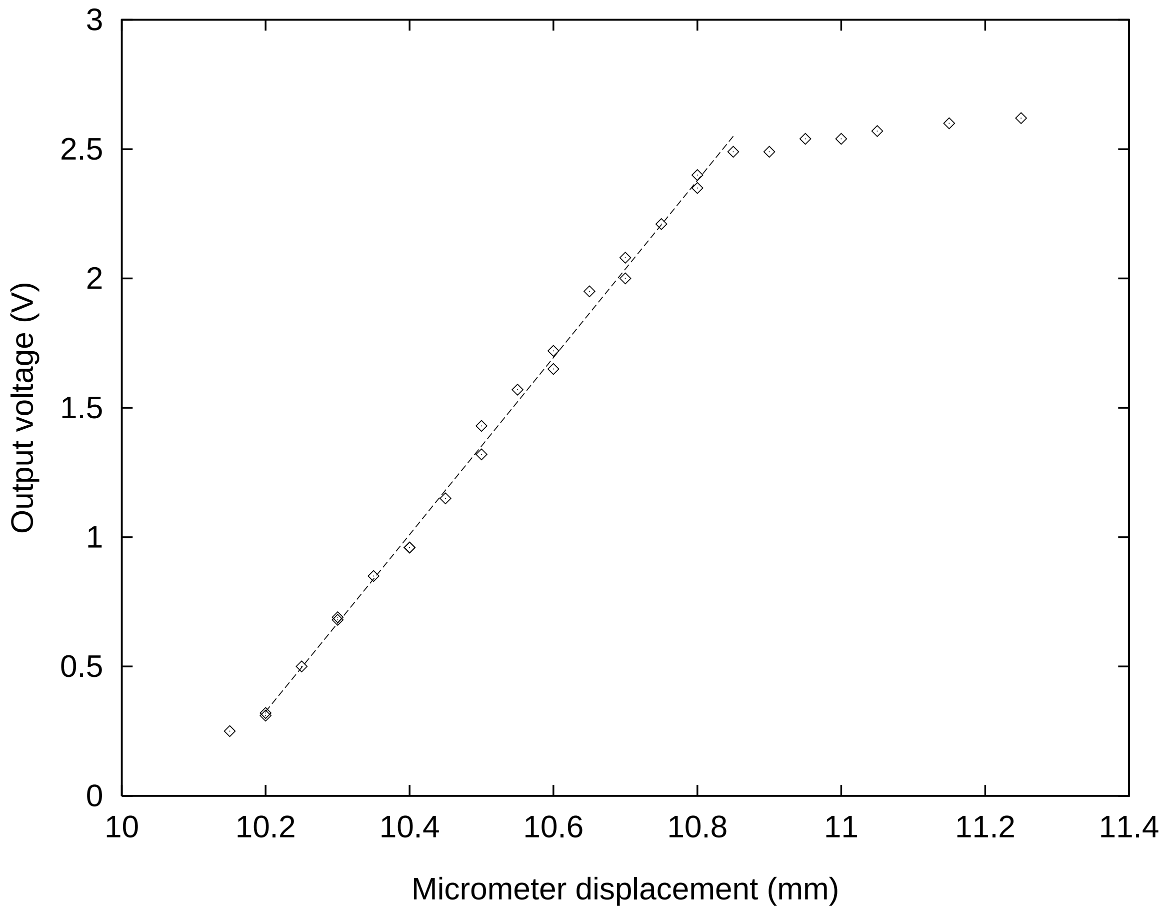


Figure 6.6: Voltage-displacement characteristics of one of the slotted opto-switches and straight line fitted to linear region.

	Optoswitch 1	Optoswitch 2
Gradient 1	3.49	4.56
Gradient 2	3.42	4.83
Gradient 3	3.35	4.85
Average	3.42	4.75

Table 6.3: Gradients (in V/mm) obtained for repeated measurements of the voltage-displacement characteristics of the two opto-switches.

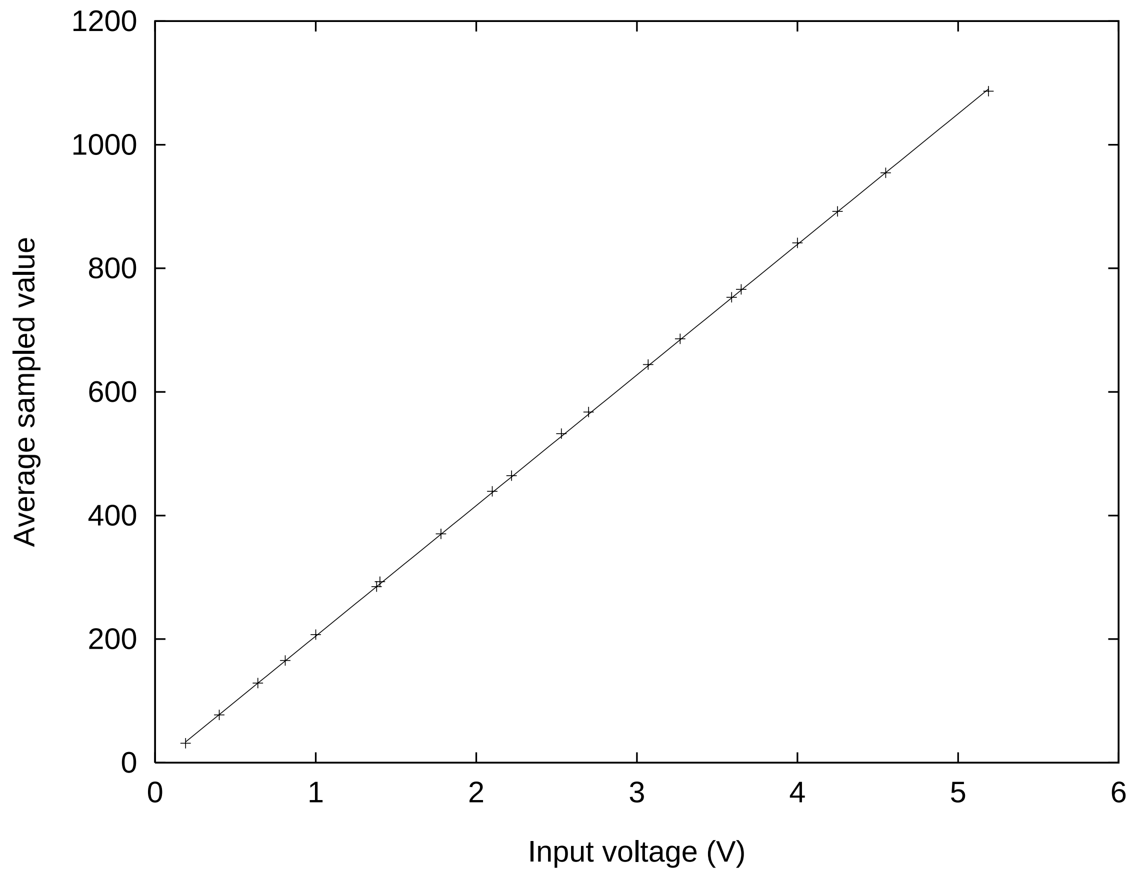


Figure 6.7: Callibration of computer's analogue to digital converter.

the string's motion. The difference in distance from the string's end to the two devices was measured so the small difference in string displacement measured by the two devices could be calculated. The string was held near the bridge and moved up and down a small distance so that the displacements measured by the two devices would be within the linear region. The outputs of the devices were sampled on the computer and the scaling program was used to calculate the displacements measured by the two devices. The resulting displacements were plotted together and compared with the expected result. The distances from the two devices to the string end were measured to be 9.4 and 10.4 cm. This meant that the displacement measured by the switch nearer the string end should be just over 90% of the value measured by other switch. The data showed this to be true to within an accuracy of about 2%. For the purposes of obtaining a measure of the relative displacements of the string in the horizontally and vertically polarised directions this accuracy was felt to be sufficient.

6.3.3 Measuring the vertical and horizontal string displacements

The two opto-switches were mounted on small wooden blocks to provide a larger surface with which to attach them to the guitar. The total mass of each device (opto-switch and mounting block) was measured to be less than 3 g; perturbation of the guitar's body modes was therefore small. The first device was attached to the neck of the instrument, at a distance of 9 cm from the end of the string, to measure the vertically polarised string motion. The second device was attached near the bridge of the guitar, at the same distance from the string's end, to measure the horizontally polarised string motion.

The devices were attached in such a way that when the string was at its equilibrium position it was already casting a small shadow on the light-sensitive surface. In this way the string's equilibrium displacement corresponded to a point on the linear region of the displacement-voltage curve. By restricting the maximum displacement of the string to around 0.7 mm we could be confident that the string remained in the linear portion of the curve. The voltage sampled on the computer was used to check for this; as long as the voltage did not fall to near its minimum value (maximum shadow), the string remained in this linear region for the half-cycle of vibration measured by the opto-switch.

The output voltages of the two devices were sampled as the string was driven into vibration by passing the sinusoidal current along its length. The frequency of the voltage applied to the string was swept through the resonance frequency of the string mode of interest by connecting the ramp voltage output of a Brüel & Kjær X-Y plotter to the frequency control input of the heterodyne analyser. The slowest sweep-rate possible was used because of the string modes' high Q-values. Measurements were made for a total of 12 different modes of the low E string. The string was tuned to four different pitches and measurements were made for the first three modes for each case. The sampling rate used was chosen to be at least twice as large as the mode frequency of the string.

The sampled output voltages were converted to values of string displacement in the horizontal and vertical directions. The results obtained are shown in Figure 6.8 and indicate that, although the string motion in the vertically polarised direction was the dominant component in most cases, the horizontally polarised motion was significant and, in one or two cases, actually larger than the vertical motion. In other words, the experimental conditions were not always a good match for the assumptions used in the numerical model. The horizontal string motion tended to be greatest at the frequencies where the guitar body had a strong resonance. At 97 Hz, very close to the Helmholtz body resonance, the horizontal string motion is almost half as large as the vertical motion. At frequencies a little above or below the Helmholtz mode the horizontal string motion is only one fifth that of the vertical motion. At 196 Hz, just above the $T(1,1)_2$ mode, the horizontal motion is again large, but is greatest at a frequency of around 240 Hz, just above the T(2,1) mode. This mode causes the bridge to pivot about its midpoint with one end rising as the other falls. This will cause the low E string termination at the bridge to have a significant component of 'side-to-side' motion as well as up and down. This sideways motion will couple relatively strongly with the string giving rise to the large horizontally polarised component of motion.

Very little experimental work has been performed on string instruments where the driving force has been applied to the string and the response of the body measured. Ideally, a system that drives the string at a point and can keep the horizontally polarised motion to a minimum is needed to accurately mimic the conditions established in the modelling scheme. Despite the shortcomings of the experimental arrangement described above, it was felt to be useful collect data in this manner for comparison with results from the model and with the results obtained using the wire pluck. (Section 6.2).

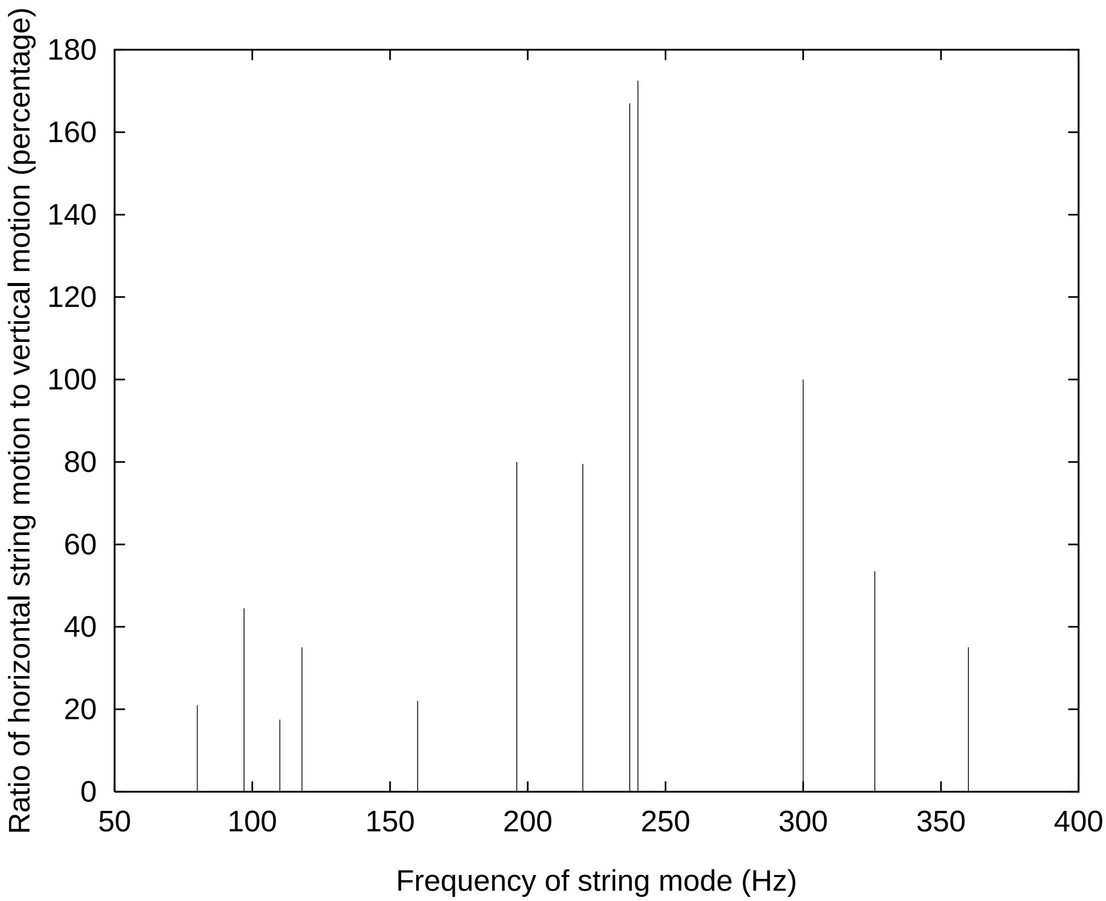


Figure 6.8: Graph showing relative amplitudes of horizontally and vertically polarised string motion for a number of string partials. Lines with amplitudes greater than 100 indicate string modes for which the displacement measured in the horizontal direction was greater than that measured in the vertical direction. For six of the twelve string partials, the horizontally polarised motion was at least a factor of two smaller than the vertically polarised motion.

6.3.4 Measurements of the coupling between string and body

Measurements were made of the guitar body's response to a driving force applied to the low E string. The accelerometer was attached to the flat portion of the bridge, adjacent to the point where the string is attached to the tie block. The frequency of the driving signal was varied in the region 50 to 250 Hz so that coupling between the lowest two string modes and the first two body modes could be investigated. The frequency of the string's fundamental mode was varied in small steps, starting a little below the body's Helmholtz resonance, and increasing so that the behaviour of the coupling as the string mode passes through the body mode could be observed. The $T(1,1)_2$ body mode was at a frequency almost exactly one octave above the Helmholtz resonance. For this reason, the second string overtone coupled strongly with the higher body mode at the point at which the fundamental string mode was tuned to couple strongly with the Helmholtz mode.

As the frequency of the string fundamental approaches that of the body mode the coupling becomes stronger, and when the two modes coincide a characteristic split resonance is seen. Figure 6.9 shows two of the measured acceleration responses. One has the string mode below the $T(1,1)_2$ body mode at 195 Hz showing weak coupling, the other has coincident string and body modes producing a split resonance. The acceleration responses predicted by the model for the same two cases are shown in Figure 6.10.

The agreement between measured response and modelled response is good, with the important features of the modelled response reflected in the real measurements. However, one feature of the modelled data that is not backed up by the experiment is the sharp peak that is seen in the middle of the strongly coupled split resonance (also seen in Figure 6.2). This may be explained by considering the coupling between the string and the different top-plate modes. The central, sharp peak only appears when coupling between the string and several top-plate modes is calculated in the model. The modelled response of the string coupling to just the fundamental top-plate mode produces the characteristics split resonance, with no sharp peak between the two broader resonances. When other body modes are included in the calculation, the string mode couples weakly to these, producing a sharp peak at around 195 Hz. A summation over all body modes yields the complete response curve, with the weakly coupled string mode at 195 Hz superimposed on the strongly coupled double resonance. The likely reason for the absence of the sharp peak at 195 Hz in the measured response (Figure 6.9) is

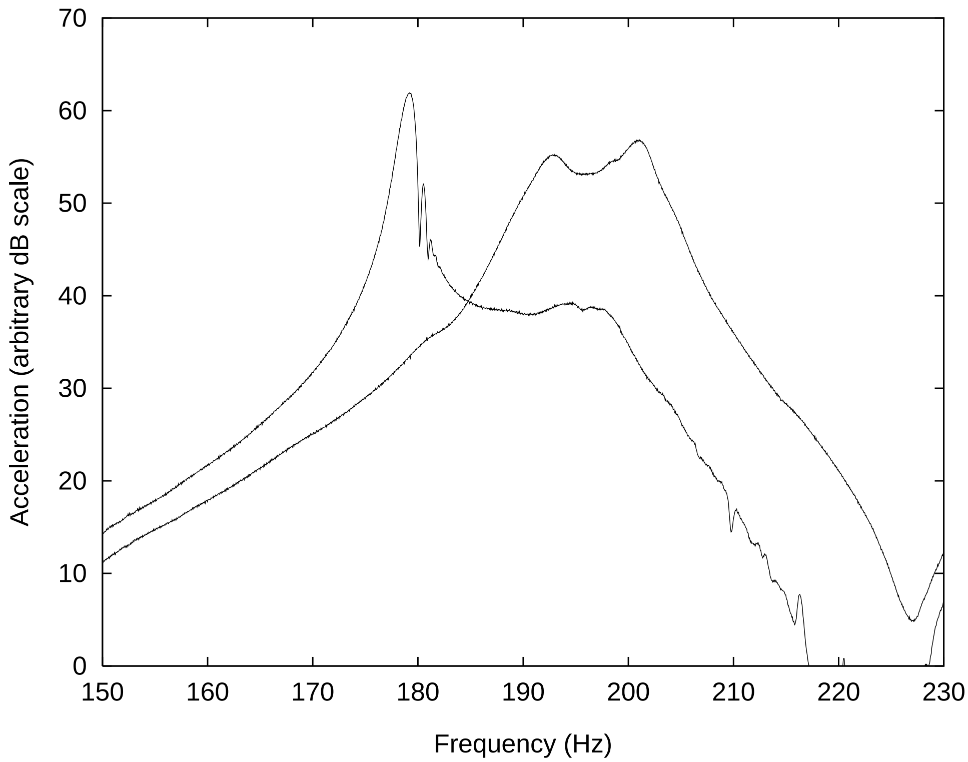


Figure 6.9: Measured string-body responses for string partials at 179 and 198 Hz. Note split resonance due to strong coupling between body mode at 195 Hz and coincident string partial.

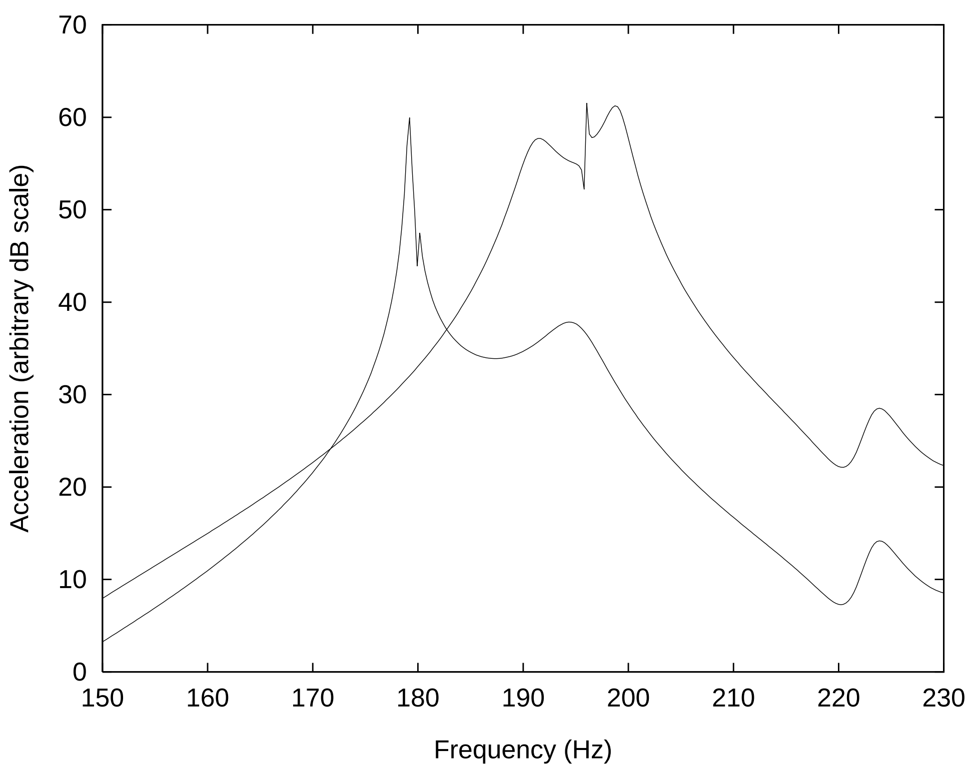


Figure 6.10: Modelled string-body responses for the two cases shown above in Figure 6.9 showing weak and strong string-body coupling.

the speed at which the driving frequency was varied. The slowest, linear sweep-rate obtainable was 1.67 Hz/sec. At this rate the system does not have enough time to respond and generate such sharp features in the response. This accounts for absence of the sharp peak at approximately 195 Hz as well as the much lower Q-value measured for the lower-frequency string mode. An additional measurement was made using the logarithmic frequency scale on the heterodyne analyser which allowed a slower frequency sweep-rate of less than 1 Hz/sec to be used. The response for the strongly coupled case is shown in Figure 6.11. The double resonance produced by the strong coupling between the vertically polarised string mode and the T(1,1) mode, and the central third peak corresponding to the weakly coupled horizontal string mode are visible. The measured Q-value of the central third peak is again lower than expected because of the relatively rapid rate at which the frequency was varied. Other measurements showing the weakly coupled string resonance superimposed on the strongly coupled double resonance have been presented by Gough (1980, 1983).

A series of measurements was then performed where the amplitude of the driving signal was held constant and the coupling between 5 different notes on the low E string were measured. The tension of the string was held constant and a capo was used to stop the string at different frets to produce the different string fundamentals. The responses of notes between F (stopped at the 1st fret) and A (stopped at the 5th fret) were measured. Figure 6.12 shows the measured results and Figure 6.13 shows the model's predicted responses for the same series of notes. As before, the sharpness of the peaks associated with the string modes is lower in the measured response than in the calculated response, but the experimental data compares favourably with the model's predictions. The height of the string peaks in the low-frequency region differs a little from the predicted response, but in the region of the body mode at 190 Hz the agreement is much better. Among the important features of the response is the drop in peak height at the point where string and body modes coincide. The maximum peak amplitude, and hence the loudest sound, is produced when the string mode is slightly above or below the frequency of the body mode. At the point of strongest coupling the amplitude falls and the damping is considerably increased, reflected in the lower Q-value of the two peaks in the split resonance.

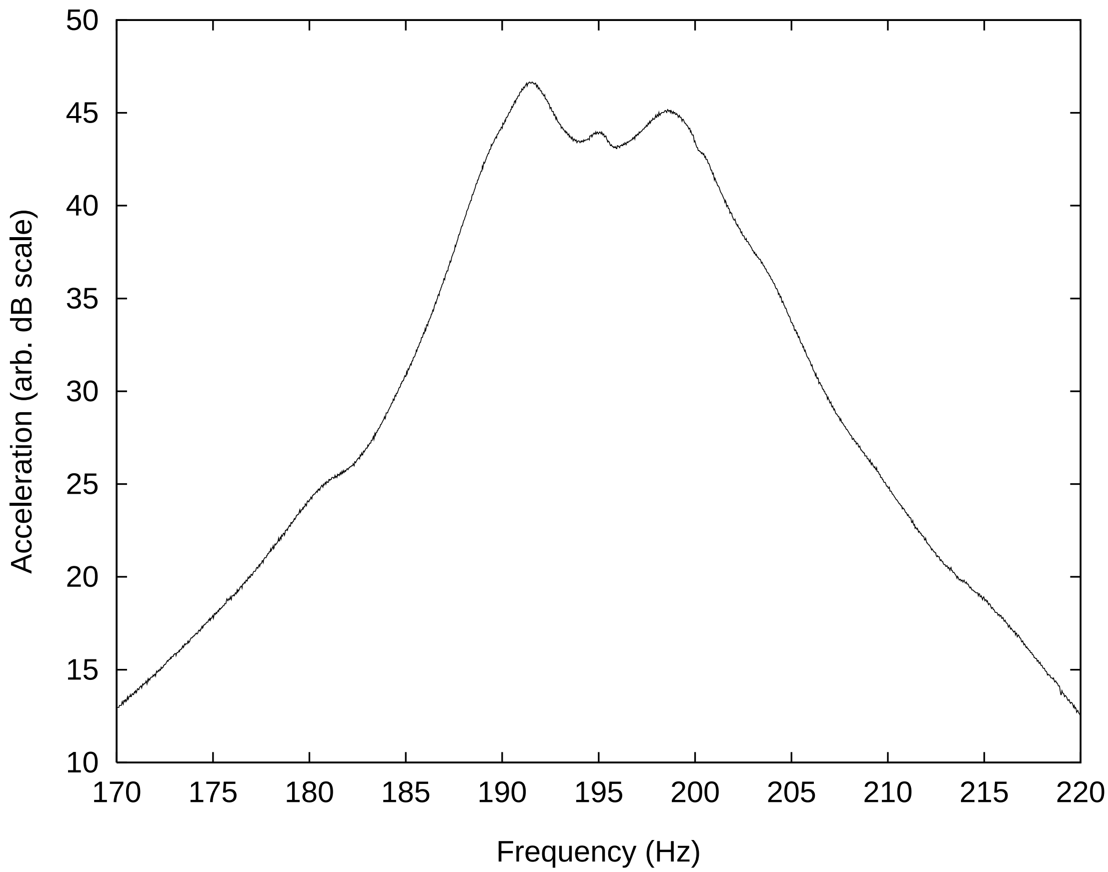


Figure 6.11: Detail of the strongly coupled case showing two large peaks, corresponding to the double resonance produced by the strong coupling between the string mode and the $T(1,1)_2$ mode. The additional small central peak corresponds to the string mode which couples weakly to other body modes.

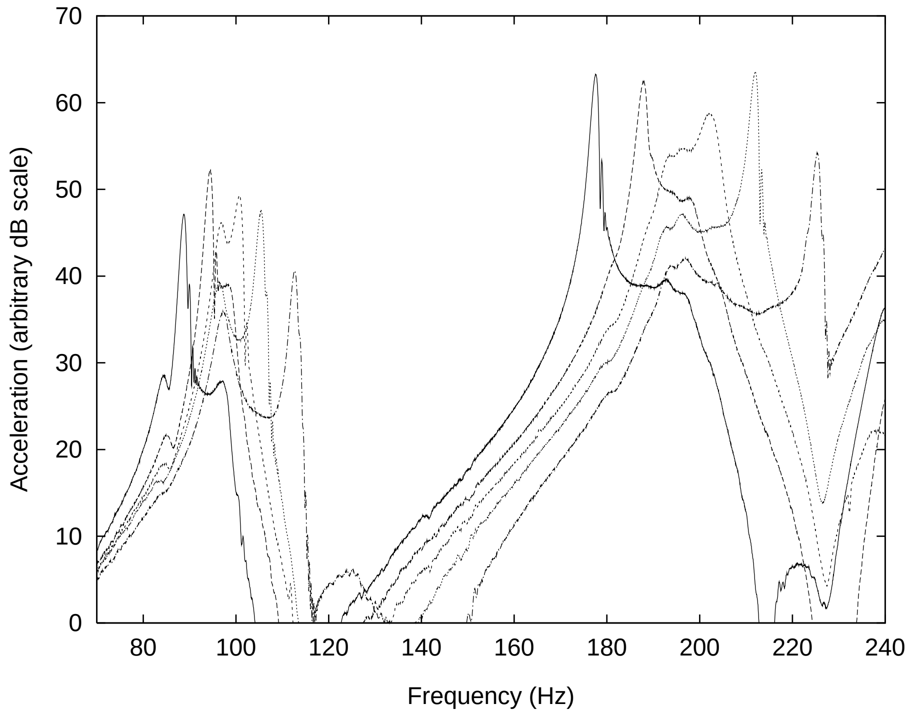


Figure 6.12: Response of the guitar body to a sinusoidal force applied to the string showing coupling between first two string partials and body modes at 100 and 200 Hz. Fundamental frequency of string increased in steps.

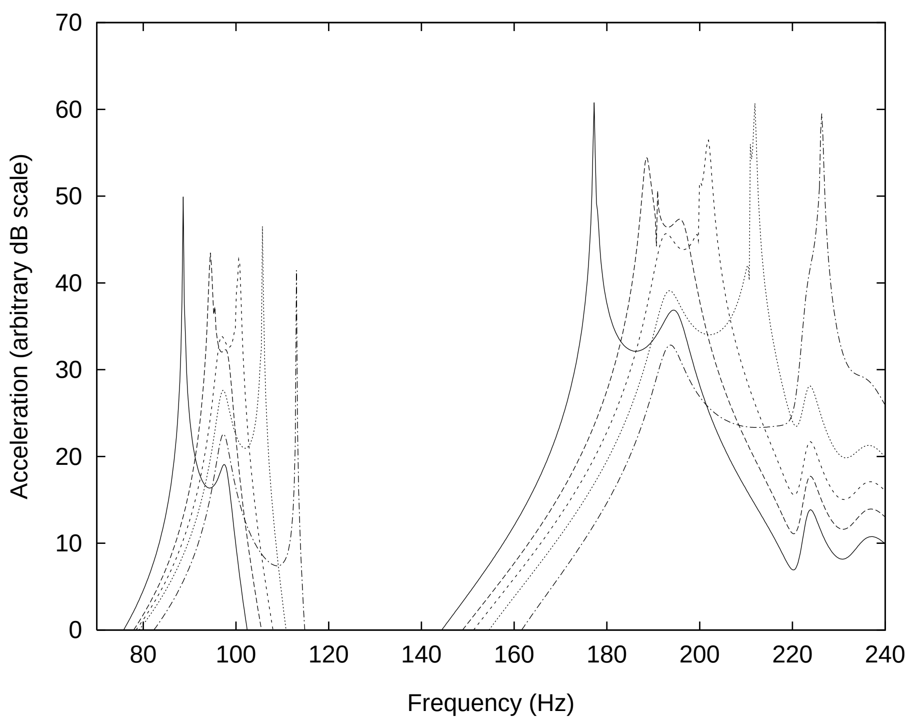


Figure 6.13: Responses of the coupled string-body system calculated by the model, equivalent to those shown above in Figure 6.12.

Chapter 7

Psychoacoustical listening tests

7.1 General aims

The final stage in numerical musical acoustics research should always be to listen to sounds produced by the models. Subjective impressions of these sounds can then be judged, to enable links to be made between the musical and perceptual significance of the sounds and the objective, physical parameters of the numerical model. In this case the final output of the model is the sound pressure response of the coupled string-body system, as detected at some distance from the guitar body. By performing a reverse Fourier transform on the pressure response, the time-domain signal can be obtained. We can, in effect, listen to the modelled sound of a plucked note.

The aim of this work was to determine the relative importance of the different acoustical parameters of the guitar body on the tone quality of notes played on the instrument. Typically, data relating to ten different body modes was used to synthesise the sounds. Each mode is defined by four parameters (resonance frequency, Q-value, effective mass and effective area) giving a total of around 40 parameters to be investigated. Clearly a structured approach was needed rather than a more general searching of the parameter-space, which would have been much more time consuming.

The advantage of using computer models to investigate the relationship between sound quality and construction of an instrument is that the construction, as defined by the mode parameters, can be altered in a controlled way. The construction parameters can be varied individually in a manner that would be impossible on real instruments. However, there are a lot of potentially interesting constructional variations to try out using the model. Each one may lead to a significant change in perceived tone quality, but the number of sounds used in each test must be limited to ensure that the test is not too long and the subject does not become tired.

Once the particular sounds to be presented in a test have been chosen, it is important to choose the questions that will be presented to the subjects with great care. If questions are phrased in terms of preference, i.e. whether one sound is judged to be 'better' or 'more pleasant' than another, it may be difficult, or even meaningless, to try and make overall sense of a large number of responses. If subjects are asked to try and describe the differences in perceived sound quality one is likely to end up with a large collection of words and phrases whose interpretation is open to question. For example, in the final two listening tests presented in this chapter, subjects were asked to comment on the tone quality of certain sounds. For one test sound, the following collection of words was used: clear, nasal, buzzy, woody, thin, twangy, shallow. I leave it to the reader to decide whether any useful overall meaning can be inferred from these judgements!

7.2 Methods

The calculation of so-called 'difference limens' is one technique which is often employed in psychoacoustical tests (e.g. Green et. al., 1984). The difference limen, or just-noticeable difference, corresponds to the threshold of change, for one parameter, above which the difference in sound can be reliably detected. This requires the use of an adaptive test procedure in which the test parameter is increased or decreased by amounts which change according to the subject's previous responses (for a description of some common test procedures, see Levitt (1971)). Typically, three test stimuli are presented, two of which are identical. The subject is required to identify the 'odd one out'. A relatively lengthy test is required to allow the difference limen for the parameter under investigation to be reliably estimated. This technique was judged to be unsuitable for my purposes because I wished to examine the effect on tone quality of changes to 15 or 20 different mode parameters, which would have required each subject to perform a large number of lengthy listening tests.

In the analysis of musical timbre, two psychoacoustical testing methods are commonly used: multi-dimensional scaling (MDS) and semantic differentials (SD). The most common MDS technique is one in which three test stimuli are presented and the subject is asked to choose the most similar and/or dissimilar pairs. Multi-dimensional analysis of the similarity ratings is then used to find a small number of orthogonal dimensions in which the data can be represented. Groups of similar sounds will appear as clusters on the multi-dimensional representation, and dissimilar sounds will be found at opposite ends of the axes. The interpretation of the axes in terms of the perceptual characteristics of the sound is, however, often difficult, and this is one disadvantage of the MDS approach. The technique is also best suited for groups of similar sounds; the use of test sounds which have many varying attributes will make it difficult for subjects to judge the similarities between sounds, and may lead to a poor multi-dimensional representation of the sounds.

One of the main disadvantages of using multi-dimensional scaling concerns the large number of test stimuli that are often required. All sounds in the test group must be compared by the subject with all others in the group. If the stimuli are presented in groups of three (triads), with the subject choosing similar pairs, a group of 15 test sounds requires the use of 455 stimulus triads. An alternative MDS method was used by Grey (1977) who investigated the timbre of 16 different sounds. Subjects were asked to rate the similarity between pairs of sounds on a 0–30 scale, but this still required the use of 240 test pairs. I wished to examine the effect on tone quality of changes to the four mode parameters for a number of body modes. The MDS approach would have required a very large number of test stimuli, resulting in lengthy tests, and the method was rejected for this reason.

The use of semantic differentials (SD) is one common alternative to the MDS approach (e.g. Von Bismarck, 1974). Subjects are asked to rate the test sounds on a number of scales which are defined by end points consisting of two opposite timbral attributes (e.g. bright—dull). The number of judgements that a subject is required to make is usually considerably smaller for the SD method than for the MDS technique. The main disadvantage with the SD method is that the test results are strongly dependent on the selection of the scales. The use of inappropriate scales, or scales that may be interpreted in different ways by different people, will limit the value of the results.

I was primarily interested in the differences in tone that result from changes to individual body-mode parameters. I therefore decided to ask subjects to rate the magnitude of the difference in perceived tone for pairs of synthesised sounds. For the first listening test, a single question was asked: "Can you perceive a difference between these two sounds ?". In the second test, the judgements were extended from this simple two-alternative system and subjects were asked to judge the magnitude of the perceived difference in tone for the pairs of sounds on an integer 0–3 scale. In the third and fourth listening tests, the judgements of the perceived magnitude of the difference in tone were retained, but subjects were also asked to comment on the nature of the perceived differences in tone. I decided not to use a number of SD scales for each sound because the length of the test might have increased significantly if judgements on several scales were required for each test sound. As a quicker alternative, subjects were asked to place a tick in any of five columns to describe the differences in tone quality (see Section 7.5.1).

Measurements of the reliability of a subject's responses are also important and must be incorporated into the test. This will usually involve repeated presentations of the same sounds to the test subject. Checks can then be made to see if the answers given on the different repetitions of the same sound are consistent. This inevitably leads to a greater number of sounds that are presented in each test, underlining the importance of limiting the overall length of the test. It was decided that, for all of the listening tests, each test sound should be presented three times to obtain a measure of the subjects' consistency. This allowed between 20 and 30 constructional changes to be tested (i.e. a total of 60–90 test sounds) giving a total test time of around 20 minutes. Each test was divided into three parts to give the subject two short breaks.

7.3 First listening test

7.3.1 Aims

It is a widely-held view that the top plate is the most important acoustical element governing the sound of the instrument. The air cavity modifies the top-plate response, its most important effect being the coupling to the T(1,1) mode which produces two strong resonances, typically around 100 and 200 Hz. The back-plate fundamental mode may also couple to the top plate and air cavity to produce a third (1,1)-type resonance, usually above 200 Hz. In some instruments the influence of the back-plate fundamental mode appears to be negligible, and only a double (1,1) resonance appears in the low-frequency region. Christensen (1982) found that only two guitars from a group of nine showed pronounced influence of the B(1,1) mode.

For the first listening test it was decided to concentrate on the first three resonances of the guitar body, i.e. those produced by the coupling between the Helmholtz air-cavity mode and the fundamental modes of the top plate and back plate (first three peaks in Figure 7.1).

The first test was constructed as follows. For each set of body parameters used, four responses were calculated, each response corresponding to the sound of a single plucked note. The time-domain signals for the four notes were put together into a single file, so that the notes overlapped but the onset of each was separated by a time delay to simulate the slow strumming of a chord. Subjects were presented with pairs of chords. The first chord in each pair was the control chord and was calculated from a 'standard' set of body parameters. The second chord of each pair was calculated from a set of parameters which differed by only a single value from the 'standard' set; for example, the effective mass of a single top-plate mode was changed. The subject was simply asked whether they perceived a difference in tone between the two sounds or not. For some parameter changes, only a single note in the second chord would differ in tone. Subjects were instructed to answer 'yes' if they perceived any difference in tone in any of the notes making up the chord. Most of the parameter changes investigated were tried on two different chords: E major and D major. The notes making up the D major chord were higher in pitch and allowed any frequency dependence of the changes to be determined (string fundamental frequencies for E major chord: 82, 124, 165, 208 Hz, fundamental frequencies for D major chord: 147, 220, 294, 370 Hz).

The test was divided into three parts, each part consisting of the same thirty chord pairs, but placed in a random order. A check could then be made to see if the subject responded consistently when presented with a given chord pair on the three different occasions. As a further check, a small number of chord pairs were used where both chords in the pair were identical. Subjects who judged several of these identical pairs as sounding different could clearly not be relied upon to give useful judgements on the other chords, and their answers could be excluded from the final results.

The top-plate mode data from which the sounds were generated was taken from work done previously at Cardiff (Walker, 1991) using finite element analysis of a 2.6 mm thick spruce guitar top-plate. These values were used as the basis of the 'standard' parameter set used to synthesise the first chord (the control chord) in each test pair. To this set of top-plate mode parameters, data for the fundamental modes of the back plate and air cavity was added so that the response of the fully coupled instrument could be calculated. For the air cavity, values for the resonance frequency and Q-value of the uncoupled Helmholtz mode were required. Data on the frequencies of this mode are given by Christensen (1982) for five different guitars showing values ranging between 122 and 135 Hz. A value of 133 Hz and a Q-value of 30 was chosen to complement the top-plate mode data. For the B(1,1) mode, values for the resonance frequency, Q-value, effective mass and effective area were chosen so that a $T(1,1)_3$ peak of moderate amplitude was produced at a frequency above that of the $T(1,1)_2$ (frequency of $T(1,1)_2$ is 195 Hz, frequency of $T(1,1)_3$ is 215 Hz; see Figure 7.1). The complete data set used as the 'standard' parameter set is given in Table 7.1. I will refer to this parameter set from now on as TOP26. Other data required by the model includes the area of the soundhole, A_h, and the volume, V, of the cavity. The values used were A_h=5.7 $\times$ 10^{-3} m^2 and V=14 $\times$ 10^{-3} m^3.

Sixteen variations of the TOP26 body parameters were used to generate the different test chords presented as the second in each test pair. The variations used included changes to the frequency of the Helmholtz resonance, the frequency and effective mass of the B(1,1) mode, and the frequency, Q-value, effective mass and effective area of the T(1,1) mode. All sixteen variations were used to generate the E major chords and the first twelve variations were used to generate the D major chords. Two pairs of identical chords were added making 30 test pairs in total.

The notes synthesised for the chords covered a frequency range of 2.5 kHz, i.e. the sampling frequency was 5 kHz. Each note lasted for 3.3 seconds. Body-mode information up to 1 kHz and all string modes up to 2.5 kHz were included. To account for the effects of string stiffness, the input values for the string mode frequencies were made slightly inharmonic using Equation 2.1 (Fletcher, 1964) with the value of the constant B set to 1.5 $\times$ 10^{-5}. The Q-values were set equal for all string modes so that the temporal decay of the higher modes was more rapid. Preliminary tests were performed to investigate the effect of using different Q-values for the different string modes. Values varying by a factor of two over the first twenty string partials were measured in Chapter 6 (see Table 6.2). Tests using Q-values similar to these gave no perceptible difference in sound, and so the Q-values were kept equal for the string

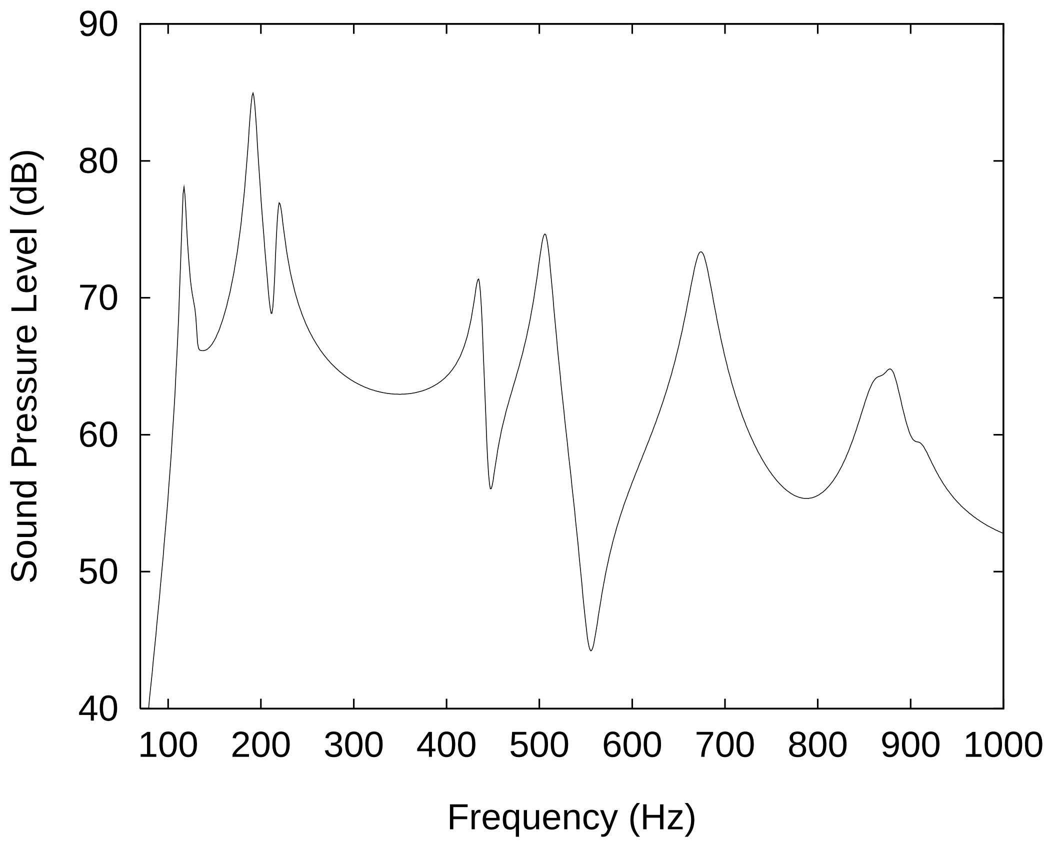


Figure 7.1: Calculated sound pressure response curve for the TOP26 mode data (see Table 7.1). Response shown is for the guitar body, driven at the bridge, without coupling to any strings. Frequencies of the body modes in the coupled response: $T(1,1)_1$ 112 Hz, $T(1,1)_2$ 195 Hz, $T(1,1)_3$ 215 Hz, T(1,2) 435 Hz, T(3,1) 505 Hz, T(1,3) 674 Hz, T(5,1) 880 Hz

Mode	Resonance frequency (Hz)	Effective mass (kg)	Effective area (m^2)	Q-value
T(1,1)	185.0	0.080	0.0381	20
B(1,1)	215.0	0.150	0.0250	25
T(2,1)	243.9	0.488	0.0000	36
T(1,2)	435.8	2.715	−0.1215	47
T(3,1)	506.8	0.256	−0.0265	35
T(4,1)	612.0	1.093	0.0000	22
T(1,3)	674.3	0.490	−0.0576	28
T(2,2)	737.7	2.049	0.0000	35
T(1,4)	863.6	0.272	0.0097	25
$T(5,1)_1$	878.6	0.646	0.0107	40
$T(5,1)_2$	908.0	0.437	0.0017	40

Table 7.1: TOP26 mode data used to synthesise sounds for the first two listening tests. Data for top-plate modes taken from finite element analysis of a 2.6 mm strutted, spruce plate (Walker, 1991). Mode frequencies given are for the uncoupled modes. The top-plate and back-plate modes were then coupled to the Helmholtz cavity resonance (uncoupled values: f_h=133 Hz, Q_h=30).

modes for the sake of simplicity.

During the test, the output of the computer's digital-to-analogue converter was low-pass filtered at 2 kHz, 96 dB/octave to prevent frequency aliasing problems. The output of the filter was amplified, and played to the subject using a pair of Sennheiser headphones.

The output of the numerical model is the sound pressure response, detected at a given distance in front of the guitar, resulting from a sinusoidal driving force of constant amplitude applied to a point on the string. The time-varying pressure wave arriving at the listening position is obtained by performing a reverse Fourier transform on the synthesised pressure response. One feature of the synthesised sounds obtained in this way was a small exponential rise in amplitude at the very end of the sound. The magnitude of the effect was dependent on the amplitude of the main portion of the sound, but was also affected by the amount of energy in the high-frequency region of the spectrum. The precise reason for the presence of this feature was not determined, but it was successfully suppressed by implementing a digital low-pass filter on the synthesised response before the reverse Fourier transform was calculated.

The sound synthesised by the model corresponds to the sound that would be heard if an impulse force was applied to the string. The plucking force applied to a string on a real instrument is not a true delta function in time, but may have durations of the order of tens of milliseconds. However, at the end of the plucking interaction, when the string leaves the finger, it accelerates rapidly producing a sharp spike in the force waveform, so the plucking force can be reasonably well approximated by an impulse.

The response of the modelled guitar to more realistic plucking forces was obtained by synthesising a simple time-varying force waveform and obtaining its Fourier transform. A convolution was then performed between the response of the guitar and the Fourier transform of the plucking force. The result was then reverse Fourier transformed to obtain the time-domain pressure signal. Fourier theory predicts that as the time duration of the exciting force is increased, the response at the lower frequencies is increased relative to the high frequencies. When the modelled guitar was excited with forces of finite time duration, there was a marked increase in the prominence of the (low-frequency) body transient, although for durations of 10–20 ms the difference was small. I decided that an impulse excitation of the response produced a more realistic sound, although for notes on the top string the body transient still seemed louder than for real guitars. All sounds for all the listening tests used an impulse excitation force, applied at a point on the string 9 cm from the bridge. All sound pressure responses were calculated using a force of 1 N with the listening position directly in front of the top plate of the guitar, at a distance of 2.5 m. The effects of fluid loading on the guitar were not calculated for the sounds used in the listening tests.

The numerical model was run on a Sparcstation 10 computer. Typical run-times for a single run of a 20 string mode, 10 top-plate mode, 16384 data point calculation were around three to four minutes.

7.3.2 Selection of parameter changes

Before selecting the sounds to be used in the test, I synthesised a number of test sounds so that I could listen to the differences in tone and check that the parameter changes used were suitable. As a first approach, many of the parameter values were doubled and the resulting sounds were evaluated to ensure that the changes in tone were not too extreme or too small. It is important that the differences between test sounds cover a range with some relatively obvious differences, some moderate differences as well as some more subtle differences. A test whose results show that, for example, people perceived no difference in tone for most of the test sounds will yield little information on the relative influence of the different mode parameters. A variety of sounds, with both small and large differences in tone are required.

After listening to some of the synthesised sounds it was felt that a change by a factor of two in the T(1,1) effective mass and area gave a very obvious change in tone. For the test, the factor was reduced to 1.5, although one sound was calculated with the factor of two change for the effective mass. For the T(1,1) Q-values, a factor of two change was found to give a rather small change and so this was increased to three for the test. For the mode frequencies, a doubling of the value (a change of one octave) would be too extreme and so a change of 26% (a musical interval of two tones) was found to be suitable. A discussion of the physical validity of the changes used is given in Section 7.4.2.

It should be noted that all parameter changes were applied to the uncoupled modes. Each time a response is synthesised, the coupling between the different parts of the guitar is calculated, hence changes to a single parameter of one uncoupled mode may result in changes to more than one mode in the coupled response. For example, a reduction in the frequency of the uncoupled T(1,1) mode influences the behaviour of all three of the coupled T(1,1) modes, as illustrated in Figure 7.2. In addition, changes to the effective mass and area of a mode may result in small changes in the frequency of the mode in the coupled response.

7.3.3 Results

A total of 19 subjects performed the first listening test. Four subjects were excluded from the final results because they judged more than one of the six identical chord pairs as 'different'. The results for the remaining 15 subjects are shown in Table 7.2. When compiling the responses of the 15 subjects, a simple check was made on the consistency of each subject's responses. For each test number, and for each subject, the results were used only where three identical responses were given to the three repeats of each chord pair. On average, each subject made consistent responses to around 20 of the 30 chord pairs. The total number of responses shown in Table 7.2 for each test number is therefore not the same.

Circles have been used in Table 7.2 to highlight results for which the difference between the number of people perceiving a difference and the number perceiving no difference is at least three. Results which are circled, and those which have the largest majorities (e.g. test pair numbers nine and twelve), can be treated with greater confidence.

Note that the test number referred to in the table is used only as a label to identify the different parameter changes; there is no implication that test number 1 is any more or less significant than test number 16. This applies equally to all other listening tests discussed in this chapter. Note also that all parameter changes for the first test refer to the fundamental modes of the top plate, back plate or air cavity.

For many of the physical changes investigated, the trend in results for the D and E chords is similar, indicating that the changes investigated resulted in similar changes in tone quality for notes on all six strings. Another general observation is that an increase and decrease of a certain parameter by the same amount does not necessarily lead to similar perceived changes in sound. This is particularly clear in test numbers seven and eight: the reduction of the T(1,1) effective mass by a factor of 1.5 leads to most people perceiving a difference in both E and D chords. A corresponding increase of the same parameter shows that most people perceived no difference. It is debatable which are the most important numbers to consider: the actual increase/decrease or the factor by which the parameter is increased/decreased. For an initial mass of 80 g, the actual increase is 40 g and the decrease is 27 g. The results show

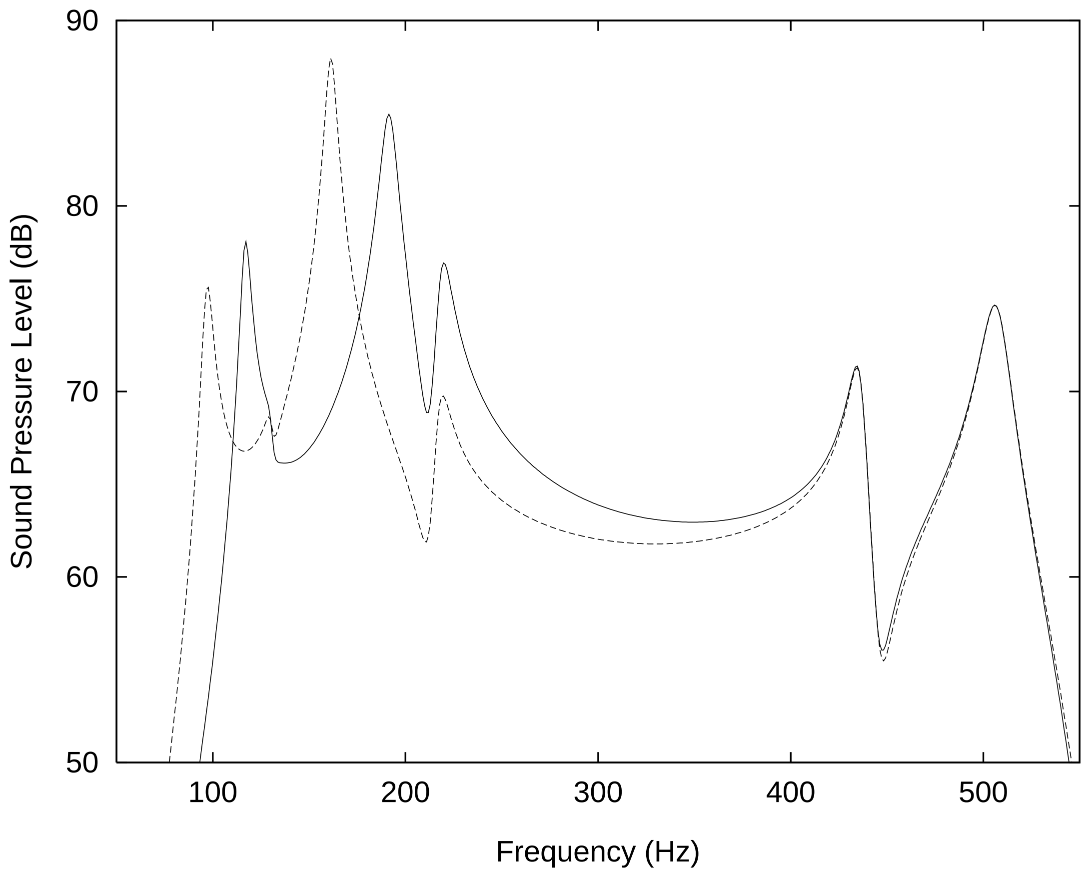


Figure 7.2: Sound pressure response of the TOP26 parameter set (solid line) compared with the response of the same parameters with a 26% reduction in the resonance frequency of the uncoupled T(1,1) mode (dotted line). Responses shown are for the guitar body without coupling to any string modes. Coupling between the fundamental modes of the top plate, back plate and air cavity causes all three of the T(1,1) modes to be affected by the change in mode frequency.

Test Pair No.	Parameter Change	E chords		D chords	
		No	Yes	No	Yes
1	$f_h + 26\%$	(5)	1	(12)	0
2	$f_h - 26\%$	3	(6)	(11)	0
3	$f_t + 26\%$	0	(12)	1	(11)
4	$f_t - 26\%$	0	(13)	2	(9)
5	$Q_t \times 3$	2	1	(9)	0
6	$Q_t \div 3$	(5)	1	(11)	1
7	$m_t \times 1.5$	5	3	3	1
8	$m_t \div 1.5$	1	(13)	4	(7)
9	$m_t \times 2$	0	(12)	0	(14)
10	$A_t \times 1.5$	1	(10)	2	(5)
11	$A_t \div 1.5$	1	(6)	2	2
12	$A_t \times -1$	0	(15)	0	(11)
13	$f_b + 26\%$	(7)	0	***	***
14	$f_b - 26\%$	2	1	***	***
15	$m_b \times 2$	(10)	2	***	***
16	$m_b \div 2$	(12)	0	***	***

Table 7.2: Results of the first listening test in which subjects were asked whether they could perceive a difference in tone between pairs of synthesised chords. A total of 15 subjects performed the test. Columns marked 'No' show the number of subjects who consistently perceived no difference in sound for the given test pair; the 'Yes' columns show the number who consistently perceived a difference in sound. Circles indicate results for which there was a difference of at least three between the number of people perceiving a difference and the number perceiving no difference. Note that all parameter changes refer to the fundamental modes of the air cavity, top plate or back plate.

that the decrease by 27 g gives a more obvious change in tone than the increase by 40 g, which underlines the importance of listening to sounds from the model before making any assumptions as to which parameter changes are perceptually most important.

From Table 7.2, the parameters changes which caused differences in tone that were most easily perceived include changes to the frequency, effective mass and effective area of the $T(1,1)_2$ mode (f_t, m_t and A_t). In test pair number two, a reduction in the frequency of the fundamental air-cavity mode (f_h) produced no change in sound for the D chord, but gave a significant change for the E chord. This was the only test number that produced different circled results for the two chords. The reason for the difference is that a change in frequency of the $T(1,1)_1$ mode, which is normally found at around 100 Hz, will affect only those notes at the bottom of the instrument's playing range. The lowest note in the D chord is 147 Hz, and is largely unaffected by the change in mode frequency. The bottom note of the E major chord (E_2 at 82 Hz) is more strongly affected, and a significant change in tone is perceived.

Changes to the Q-value of the fundamental top-plate mode (Q_t), and frequency and effective mass of the fundamental back-plate mode (f_b and m_b) produce negligible changes in sound quality. The weak influence of the Q-values on the tone quality is perhaps not surprising. The Q-values may vary quite considerably from day to day as the properties of the wood are altered by changes in atmospheric humidity. The Q-values of the modes will also be reduced when the instrument is held in the playing position, with the back and sides in contact with the players body. If these actions had any significant influence on the guitar's sound, players would have surely been aware of this fact.

The influence of the back-plate modes is not entirely clear. Most makers agree that the back plate, in comparison with the top plate, has a much smaller influence on the final sound quality. The nature of its influence seems to be more widely disputed. The results from this test confirm the fact that the fundamental back-plate mode has a much smaller role in determining tone quality than the fundamental mode of the top plate. Indeed, the results from this first test indicate that the B(1,1) mode has virtually no influence on tone quality. The role of the back plate was investigated in the second listening test and I will comment later on those results (see Section 7.4.4).

7.3.4 Summary of results

In the first listening test the effect on tone quality of changes to the fundamental modes of the top plate, back plate and air cavity was investigated. Notes were synthesised and arranged into chords. Subjects listened to pairs of chords and were asked whether or not they perceived a difference in tone between the two chords. The results show that changes to the frequency, effective mass and effective area of the fundamental top-plate mode have a profound influence on tone quality. Changes to the Q-value of the T(1,1) mode appear to have negligible influence on tone quality. Variations in the frequency of the Helmholtz mode have moderate influence for low-frequency notes, and changes to the frequency and effective mass of the B(1,1) mode have virtually no influence on the instrument's sound.

7.4 Second listening test

7.4.1 Aims

The main aim of the second listening test was to obtain a better measure of the relative effects on tone quality of each of the parameter changes. Subjects were asked to judge the magnitude of the difference in sound on an integer 0–3 scale, instead of simply judging each pair of sounds as 'the same' or 'different'.

For the second test, synthesised notes were combined into phrases rather than chords to prevent the possibility of later notes in a given chord being masked by the decaying sound of earlier notes. To ensure that changes in tone for any of the notes could be clearly perceived, the notes were put together to mimic the playing of a short scale so that none of the notes overlapped.

A larger number of top-plate modes was investigated in the second test. The first three top-plate modes that produce significant monopole radiation are the T(1,1), T(1,2) and T(3,1). Changes to the resonance frequency and effective mass for these modes were tested. Changes to the frequency and effective mass of the B(1,1) mode, the frequency of the Helmholtz mode and the volume of the cavity were also tried. All changes were applied to the same TOP26 parameter set.

One other aim of the second listening test was to determine whether the parameter changes have a similar effect on the tone quality of notes in different parts of the instrument's playing range. For many of the parameter changes investigated, three phrases were synthesised using notes whose fundamental frequencies covered different regions of the spectrum. For other parameter changes, the test phrases were synthesised with notes whose fundamental frequencies were closest to the frequency of the body mode being altered. The fundamentals were chosen to be in the ranges 124–185 Hz ('low'), 330–494 Hz ('mid') and 523–784 Hz ('high'). These cover a significant portion of the range of fundamental frequencies available on a guitar, which is approximately 80–1000 Hz. The low-frequency phrase has string fundamentals that lie between the $T(1,1)_1$ and $T(1,1)_2$ modes. The mid-frequency phrase has notes whose fundamentals pass through the T(1,2) mode and extend almost to the T(3,1) mode. The high-frequency phrase has string fundamentals that lie above the T(3,1) mode, and represents notes at the top end of the guitar's top E string. The relationship between the fundamental frequencies of the string notes and the frequencies of the body modes is illustrated in Figure 7.3. By examining the change in tone of a particular parameter change applied to the three different test phrases, effects that are perceived for all notes can be distinguished from effects that are perceived only for notes of specific pitch.

In other respects the second test followed the same outline as the first. It was divided into three parts, each part having a different ordering of the same 34 pairs of phrases. Three identical pairs were included in each part of the test and subjects' responses were checked to ensure that they gave a low score to these pairs. Each test phrase lasted just over two seconds. The mid- and high-frequency phrases used five notes, but the low-frequency phrase used only three because the starting transients of some notes were more complex, and a longer note-duration was felt to be necessary. A moderately higher sampling rate of 8 kHz was chosen for the second test, and the output from the computer was low-pass filtered at 3 kHz.

Subjects were asked to judge the difference in tone for each pair of sounds by entering an integer score of 0, 1, 2 or 3. In order to give the subjects an idea of the 'callibration' of the scale, two example sounds were played which represented the extreme ends of the range. Two identical phrases were given as the zero-difference example; the choice of the test example used for the difference-of-three example inevitably introduced a small degree of bias. It would, however, be time-consuming to allow subjects to listen to all the test sounds, before making any judgements, to allow them to make their own assessment of the range of differences in tone to expect. Some reference sounds are certainly needed at the start of the test. If no such

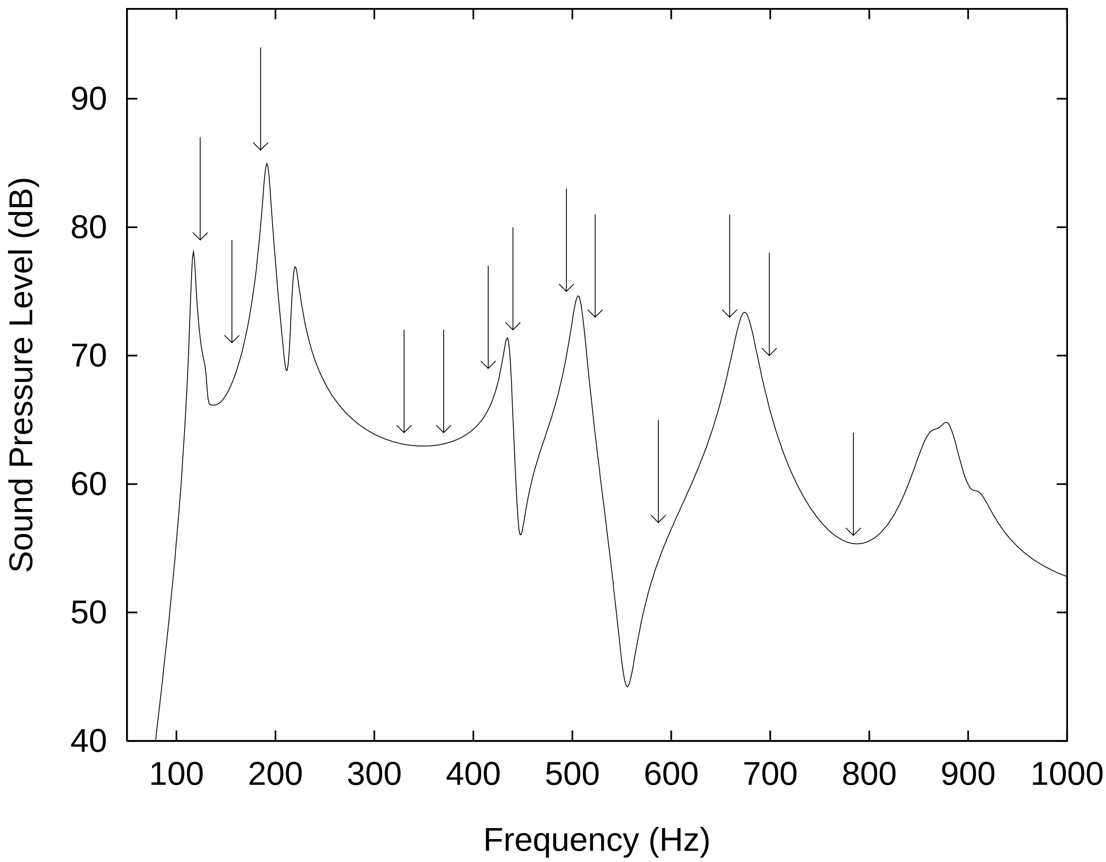


Figure 7.3: Relationship between fundamental frequencies of the string notes and frequencies of the body modes used to synthesise sounds for the second listening test. Response shown is the sound pressure response for the TOP26 mode parameters. The fundamental frequencies of the string notes used in the test phrases are indicated by the arrows. Three notes were used in the low-frequency phrase; five notes were used to make up the mid- and high-frequency phrases.

reference is given, the difference scale used by subjects would be likely to vary during the test. Subjects might judge one of the early test pairs as having the maximum difference of three. A later test pair, which is perceived as having a greater difference in tone than the previous high-scorer, could not be given a score greater than three. In effect, the standard of the 'greatest difference sound' (i.e. the three level) would change as the test progressed. In order to provide a constant reference for the level-of-three difference, a test pair had to be chosen which, to my ears, represented the greatest difference in tone quality. This is not entirely satisfactory since judgements of the greatest difference may vary somewhat between individuals, but this method was chosen as being preferable to the 'variable reference' approach. The example chosen to represent the maximum difference in tone was an increase in the T(1,1) effective mass for notes in the low-frequency region.

7.4.2 Discussion of validity of parameter changes

To ensure that the parameter changes used in the test are physically realistic, comparisons with data taken from real instruments are needed. The data presented in Chapter 5 was used as one of the guidelines. Data available from the literature was also used, although very few papers exist which provide values of effective masses or effective areas of the body modes of the guitar.

Effective mass and effective area

The values for the effective masses of many top-plate modes are seen to be significantly different when measured at different string positions. The results from Chapter 5 indicate that when the driving position is moved by as little as 1 or 2 cm the effective mass may undergo a change of up to 100% or more. The percentage by which the effective masses in the tests were changed was between 50 and 400. The 400% change was applied to the T(1,2) mode because the nodal line of this mode usually lies very close to the bridge making the value for its effective mass particularly sensitive to changes in driving position. It is conceivable that, for many guitars, the effective mass of the T(1,2) could change by a factor of four if the driving point was moved by one or two centimetres.

Christensen (1984) measured the sound pressure responses of five different guitars and obtained data for the top-plate modes of each by using curve-fitting techniques. Values for the resonance frequency, Q-value and ratio of effective area divided by effective mass (A/m) are given for the first few top-plate modes of the five guitars. Excluding work at Cardiff, Christensen's data on the ratio A/m is the only data available on the effective masses and areas of the body modes of classical guitars. Separate values of effective area and effective mass measured by Christensen cannot be deduced because his data is given as a ratio of the two. For the five guitars he measured, the range of values for the ratio A/m is between 7 and 14 cm^2/g for the first top-plate mode, between -1 and 4 for the second and between -1.5 and -0.6 for the third top-plate mode. In other words, differences of a factor of two or more, for the same mode on different instruments, are common. In the listening tests, the changes applied to the effective areas and masses seem therefore to be comparable to differences found with real instruments.

Note that I have followed Christensen's use of the ratio of effective area to effective mass (A/m) as a convenient measure of a mode's sound-producing capacity. The sound pressure radiated by a single, uncoupled mode is proportional to A/m (Equations 4.8 and 4.17). When coupling between the different parts of the guitar is included, this measure of a mode's contribution to the sound pressure is not completely accurate, but it provides a simple means with which to compare the relative influence of the top-plate modes on the guitar's sound pressure response. For the remainder of this thesis I will use the term 'strongly radiating mode' as a convenient shorthand for 'mode with a large value of A/m'.

Resonance frequency and Q-value

There is a large amount of data, published by a variety of authors, on classical guitar mode frequencies. This has been collected in Table 7.3 to obtain a measure of the typical variations in mode frequency between instruments. Note that the frequencies quoted in Table 7.3 are the frequencies of the fully coupled instrument. Mode frequencies for guitars whose back plate had been damped, or whose soundhole was sealed, were not included. The data used to compile this table was taken from work by Christensen (1982, 1984), Jansson (1971), Marty (1987), Lai and Burgess (1990), previous work at Cardiff by Richardson and Roberts (1983) as well as my own measurements on guitars BR1 and BR2. The maximum variation in mode frequency between different instruments seems to be around 30% for these five modes.

The frequency variation used in the listening tests was 26% (or two tones). This change

Mode	Frequency (Hz)
$T(1,1)_1$	91–117
$T(1,1)_2$	185–234
T(2,1)	213–292
T(1,2)	360–460
T(3,1)	415–537

Table 7.3: Typical mode frequencies for the classical guitar. Data taken from a variety of sources (see text).

was applied to the *uncoupled* mode frequency. The actual change in frequency of the coupled mode will therefore be less than 26%, the magnitude of the change being influenced by the coupling between top plate, back plate and air cavity. The change in mode frequency used in the listening tests is therefore comparable with the differences found between instruments. Day-to-day changes in mode frequency, arising due to changes in atmospheric conditions, will be rather less than two tones (see page 217). The changes that can be engineered by the maker by 'tuning' the modes of the free plate are unlikely to be greater than one or two tones, unless the pattern of construction differs radically from conventional techniques.

Changes in the Q-value of a mode may arise due to changes in atmospheric conditions, in particular the humidity. The response curves for guitar BR2 (see Chapter 5) were not all measured on the same day and the variations in the Q-value for particular modes reflect this. Changes in Q-value of almost a factor of two are seen. Similar variations between instruments are seen in the data presented by Christensen (1982, 1984). Q-values for the first top-plate mode are seen to lie in the range 12–30. Changes in Q-value were not investigated in the second test, but the first and third listening tests used a three-fold change in the Q-value of the T(1,1) mode. This change is slightly greater than the inter-instrument differences measured by Christensen, and is probably greater than changes arising from variations in atmospheric conditions. The results for the first and third tests indicate that only very small changes in tone result from the changes in Q-value. The implications for real instruments suggest that the smaller changes in Q-value actually encountered are not likely to produce significant changes in tone.

7.4.3 Results

A total of 23 subjects performed the second listening test. Their answers were checked to ensure that they were both consistent and accurate. The subjective nature of the test means that the term accuracy cannot usually be meaningfully applied, but the inclusion of a number of test pairs with identical phrases allowed some measure of a subject's accuracy since the true difference score for these should be exactly zero. Three identical pairs were used in each of the three parts of the test. The total score for these nine pairs was used to judge which subjects' answers should be excluded from the final results. The maximum possible total score for the identical pairs was 27 since each test pair had a maximum difference score of three. All but three of the 23 subjects obtained a total score of five or less for the identical pairs, with sixteen of these scoring three or less. Only one subject obtained the 'accurate' score of zero. The three subjects who scored six, seven and eight were judged unsuitable and their results were excluded. The average score obtained by the other twenty subjects for the identical test pairs was 0.27 (out of a maximum of three).

Of the 20 subjects who remained, the consistency of their answers was checked by measuring the maximum deviation between the scores given for the three repetitions of each test pair. For true consistency, this score would be zero. Although I had performed many listening tests while preparing the test sounds, my answers showed deviations between repeats of the same sounds, despite my familiarity with the test. I obtained consistent scores (zero deviation) for 18 out of the 31 test sounds and a deviation of one for eleven of the pairs. One deviation score of two and one of three were also obtained. The other subjects obtained deviations of zero or one for around 85% of the test pairs, with occasional scores of two or three. Although a deviation of three means that a subject has used both ends of the scale (0 and 3) to judge the difference in tone for two presentations of the *same* test sound, such inconsistencies are no doubt caused by temporary lapses in concentration. These are to be expected, and it was decided not to exclude subjects on the basis of one or two inconsistent answers, so the number of subjects used to obtain the final averaged scores remained at 20. The difference scores given by each subject were averaged over the three repetitions of each test phrase to obtain a single measure of the perceived difference in tone for each subject judging each test sound.

The deviations described above provide a measure of each subject's consistency. It is also useful to measure the deviations between different subjects to see whether there is general agreement in the subject group on the perceived magnitude of a particular change in tone quality. For each of the test pairs, an average difference score for the 20 subjects was calculated, which represents the magnitude of the perceived change in tone. Standard deviations within the group were also calculated. The average difference scores and the standard deviations are plotted in Figures 7.4 and 7.5. Figure 7.4 shows the perceived differences in tone for test sounds synthesised in all three frequency ranges; Figure 7.5 shows the results for phrases tested in just one frequency range. The average difference scores and their standard deviations are also given in Tables 7.4 and 7.5. Each test pair number is associated with a single change to the mode data, and the definitions are given in the two tables.

To facilitate the interpretation of the data in Tables 7.4 and 7.5, bold type has been used to highlight results which showed smaller standard deviations. In other words, results in bold indicate greater agreement within the subject group. The results in each table have been split into two, according to the size of the standard deviation. The half with the lowest deviations are highlighted in bold.

7.4.4 Analysis

The results in Figure 7.4 show that the differences in tone caused by each parameter change are not strongly dependent on the fundamental frequencies of the notes used to synthesise the test sounds. Results for the low-, mid- and high-frequency ranges (data plotted with solid, dashed and dotted lines) are similar for most test pairs. The division of results into test pairs that cause large differences in tone and those that cause small differences in tone is therefore not greatly influenced by the frequencies of the notes used to synthesise the test sounds. Test pair numbers eight and nine show the greatest difference between the results in the three frequency ranges. The reason for the higher score obtained by these two test pairs in the low-frequency range is discussed below in Section 7.4.5.

There are two ways in which the results can be interpreted. The mode-parameter changes achieving the highest average difference scores can be identified, regardless of their standard deviations. This will highlight changes to the top-plate modes which produce the greatest differences in tone, but may overlook significant inter-subject variations. Alternatively, the

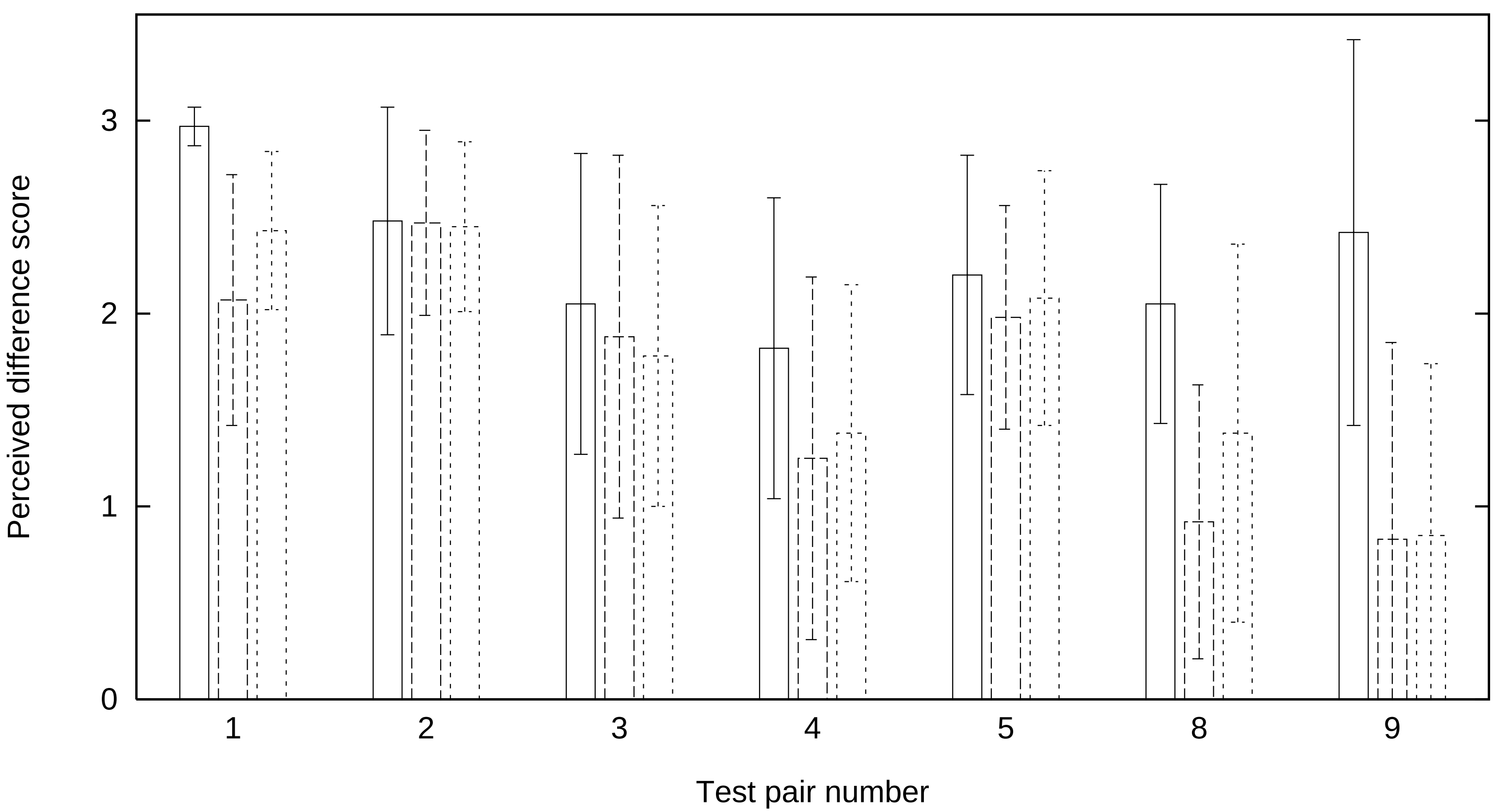


Figure 7.4: Results for listening test two showing the magnitude of the perceived differences in tone for changes to individual top-plate modes. Parameter changes associated with each test pair number are given in Table 7.4. Different line types indicate sounds synthesised with string fundamentals in the frequency ranges 124–185 Hz (solid), 330–494 Hz (dashed), and 523–784 Hz (dotted). Averages calculated for 20 subjects. Numbers used to identify the test pairs are purely arbitrary; no sequence should be inferred.

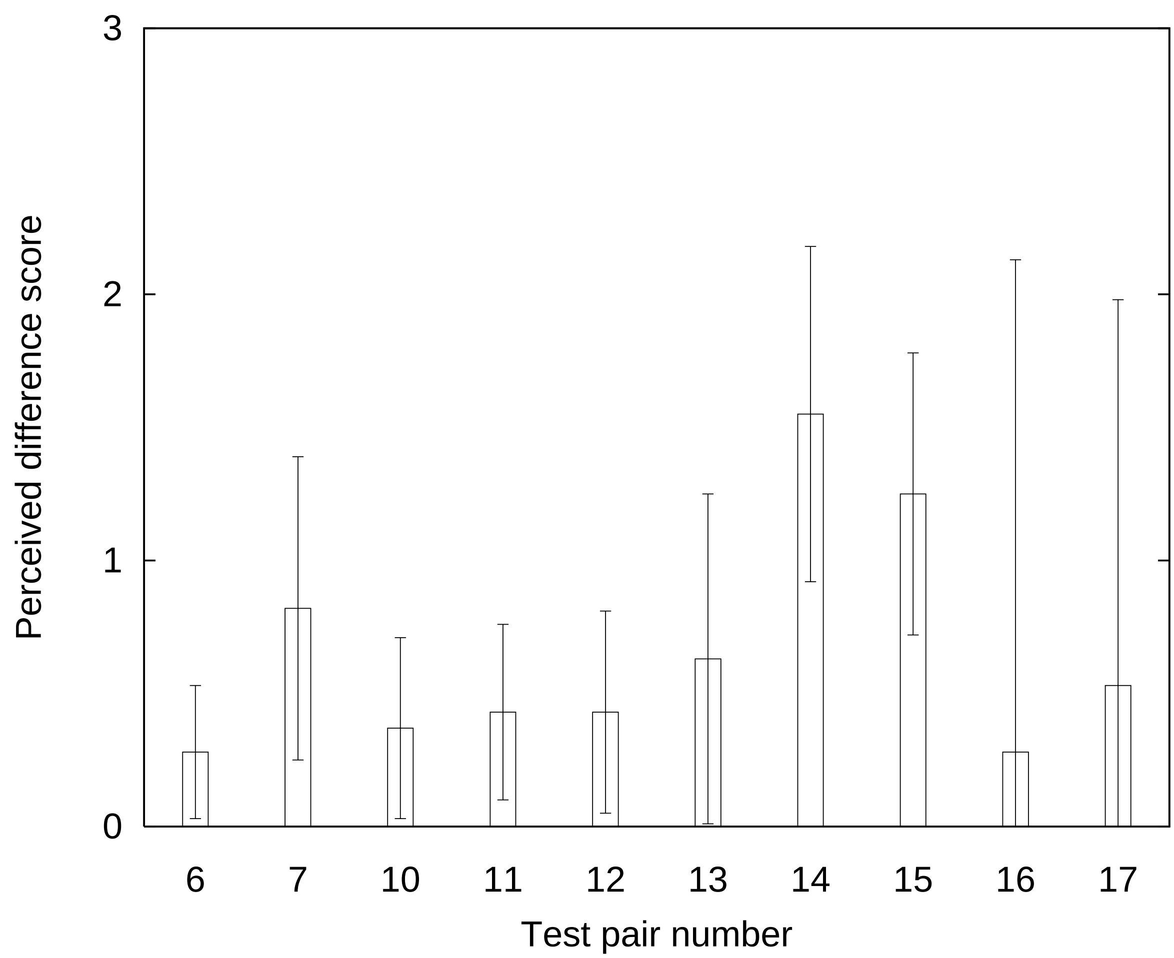


Figure 7.5: Results for listening test two showing the magnitude of the perceived differences in tone for changes to the body modes. Parameter changes associated with each test pair number are given in Table 7.5. Numbers used to identify the test pairs are purely arbitrary. Test pairs 6 and 7 were synthesised using notes in the mid-frequency range, all others were synthesised in the low-frequency range. Averages and standard deviations calculated for 20 subjects.

Test pair number	Parameter change	Frequency range	Average perceived difference score	Standard deviation
1	m_t (1,1) $\times$1.75	Low	**2.97**	**0.10**
		Mid	**2.07**	**0.65**
		High	**2.43**	**0.41**
2	m_t (1,1) $\div$1.75	Low	**2.48**	**0.59**
		Mid	**2.47**	**0.48**
		High	**2.45**	**0.44**
3	f_t (1,1) +26%	Low	2.05	0.78
		Mid	1.88	0.94
		High	1.78	0.78
4	f_t (1,1) −26%	Low	1.82	0.78
		Mid	1.25	0.94
		High	1.38	0.77
5	m_t (1,2) $\div$4	Low	**2.20**	**0.62**
		Mid	**1.98**	**0.58**
		High	**2.08**	**0.66**
6	f_t (1,2) +26%	Mid	**0.28**	**0.25**
7	f_t (1,2) −26%	Mid	**0.82**	**0.57**
8	m_t (3,1) $\div$2	Low	**2.05**	**0.62**
		Mid	0.92	0.71
		High	1.38	0.98
9	f_t (3,1) +26%	Low	2.42	1.00
		Mid	0.83	1.02
		High	0.85	0.89

Table 7.4: Results for listening test two showing the magnitude of the perceived difference in tone (on a 0–3 scale) for changes to individual top-plate modes. Average scores calculated for 20 subjects. The twelve results with the lowest standard deviations, indicating greater agreement between subjects, are highlighted in bold. Frequency ranges for string fundamentals: 124–185 Hz (low), 330–494 Hz (mid), 523–784 Hz (high). Resonance frequencies of the uncoupled top-plate modes: T(1,1) 185 Hz, T(1,2) 436 Hz, T(3,1) 507 Hz. Average score for the identical test pairs: 0.27

Test pair number	Parameter change	Frequency range	Average perceived difference score	Standard deviation
10	m_b (1,1) $\times 3$	Low	**0.37**	**0.34**
11	m_b (1,1) $\div 3$	Low	**0.43**	**0.33**
12	f_b (1,1) $+26\%$	Low	**0.43**	**0.38**
13	f_b (1,1) -26%	Low	0.63	0.62
14	f_h (1,1) $+26\%$	Low	1.55	0.63
15	f_h (1,1) -26%	Low	**1.25**	**0.53**
16	Cavity volume increased by 4×10^{-3} m^3	Low	0.28	1.85
17	Cavity volume decreased by 4×10^{-3} m^3	Low	0.53	1.45

Table 7.5: Results for listening test two showing the magnitude of the perceived difference in tone (on a 0–3 scale) for changes to the fundamental modes of the back plate and air cavity. Average scores and standard deviations calculated for 20 subjects. The four results with the lowest standard deviations, indicating greater agreement between subjects, are highlighted in bold. Low frequency range: string fundamentals between 124 and 185 Hz. Resonance frequencies of the uncoupled modes: B(1,1) 210 Hz, Helmholtz mode 133 Hz. Average score for the identical test pairs: 0.27

results with the lowest standard deviations can be selected and their average scores compared. This will highlight the judgements of difference in tone for which there was relatively good agreement between subjects. I will consider each approach in turn.

If the results from Figure 7.4 are selected purely on the basis of the average scores, test pair numbers one and two are the two highest-scorers, achieving averages of more than two in all three frequency ranges. These test pairs correspond to changes in the effective mass of the T(1,1) mode. Changes to the resonance frequency of the T(1,1) (test pair numbers three and four) achieve lower scores of between one and two. Test pair number five, corresponding to a reduction in the effective mass of the T(1,2) mode, achieves high scores (two or more in all frequency ranges). Changes in frequency of this mode (test pair numbers six and seven, Figure 7.5) score much lower (less than one).

Figure 7.5 indicates other low-scoring test pairs. Numbers 10–13, corresponding to changes to the fundamental back-plate mode, achieve average scores of much less than one, confirming the finding in the first listening test that changes to the B(1,1) mode have little influence on tone quality. It is possible that a B(1,1) mode with greater effective area or lower effective mass could have some effect on tone quality, but the values chosen for the TOP26 parameter set are large enough to produce a strong $T(1,1)_3$ resonance (see Figure 7.1). Given Christensen's (1982) finding that only a small proportion of guitars show a pronounced $T(1,1)_3$ peak in the sound pressure response due to the B(1,1) mode, and given the magnitude of the changes tried in the second listening test, it seems reasonable to conclude that the effect of the B(1,1) mode on tone quality can be largely ignored.

Changes to the volume of the air cavity (test pair numbers 16 and 17) also achieve very low scores, although the standard deviations for these two are very high showing considerable disagreement between subjects. This may indicate that the effect on tone quality is subtle and that the perceived difference score depends more critically on the sensitivity of the subject's hearing. On a real instrument, an increased cavity volume would cause a decrease in the frequency of the fundamental air mode as well as a decrease in the strength of coupling between top plate, back plate and air cavity. In this test the two effects were tested separately. Most subjects detected little difference in tone for the change in the cavity volume, but the change in frequency of the Helmholtz mode (test pair numbers 14 and 15) caused greater differences in tone, with scores between one and two.

Results for which there was relatively good agreement between subjects (i.e. those highlighted in bold in Tables 7.4 and 7.5) include test pair numbers 10, 11, 12 and 15. Subjects therefore tend to agree that changes to the effective mass or resonance frequency of the fundamental back-plate mode produce little change in tone. There was also good inter-subject agreement for test pair numbers one, two, five, six and seven. The perception that changes to the effective masses of the T(1,1) and T(1,2) modes have a strong influence on tone is therefore widely held. By comparing test pairs five, six and seven (Table 7.4) we can conclude that most subjects agree that changes to the T(1,2) mode's effective mass have a much stronger influence on tone quality than changes to its resonance frequency. Changes to the effective mass of the T(1,1) mode also appear to have a stronger influence on tone than changes to the mode's resonance frequency, but the standard deviations for test pair numbers three and four are rather high, and we cannot be confident that all subjects agree with this conclusion. The reason for the greater disagreement between subjects when asked to judge the difference in tone caused by changes in the frequency of the T(1,1) mode is discussed in Section 7.4.7.

7.4.5 Coincidence of string and body modes

Figure 7.4 shows that, for many of the test pairs, the scores in the low-, mid- and high-frequency ranges are very similar. The changes in tone are perceived almost equally for notes at the top and bottom of the guitar's playing range. Test pair numbers one and four achieve a higher score in the low-frequency range, but the difference is clearest with test pair numbers eight and nine, involving changes to the T(3,1) mode. Both test pairs achieve high scores in the low-frequency range, but relatively low scores in the other two ranges. This effect occurs when the frequency of a string mode almost coincides with that of a strongly radiating body mode. Changes of any kind to the body mode may then cause a large change in amplitude for the coincident string mode. This is illustrated in Figure 7.6, which shows the change in the sound pressure response of the first two notes in the low phrase when the T(3,1) mode frequency is altered.

The T(3,1) mode frequency is moved from 507 Hz to 639 Hz, a change of two tones (26%). When the frequency of the T(3,1) mode is at 639 Hz, partials of the first and second notes in the low phrase almost coincide with it, as shown by the dotted line responses in Figure 7.6. The extra 'boost' received by the coincident string partials is around 8 dB. The two partials

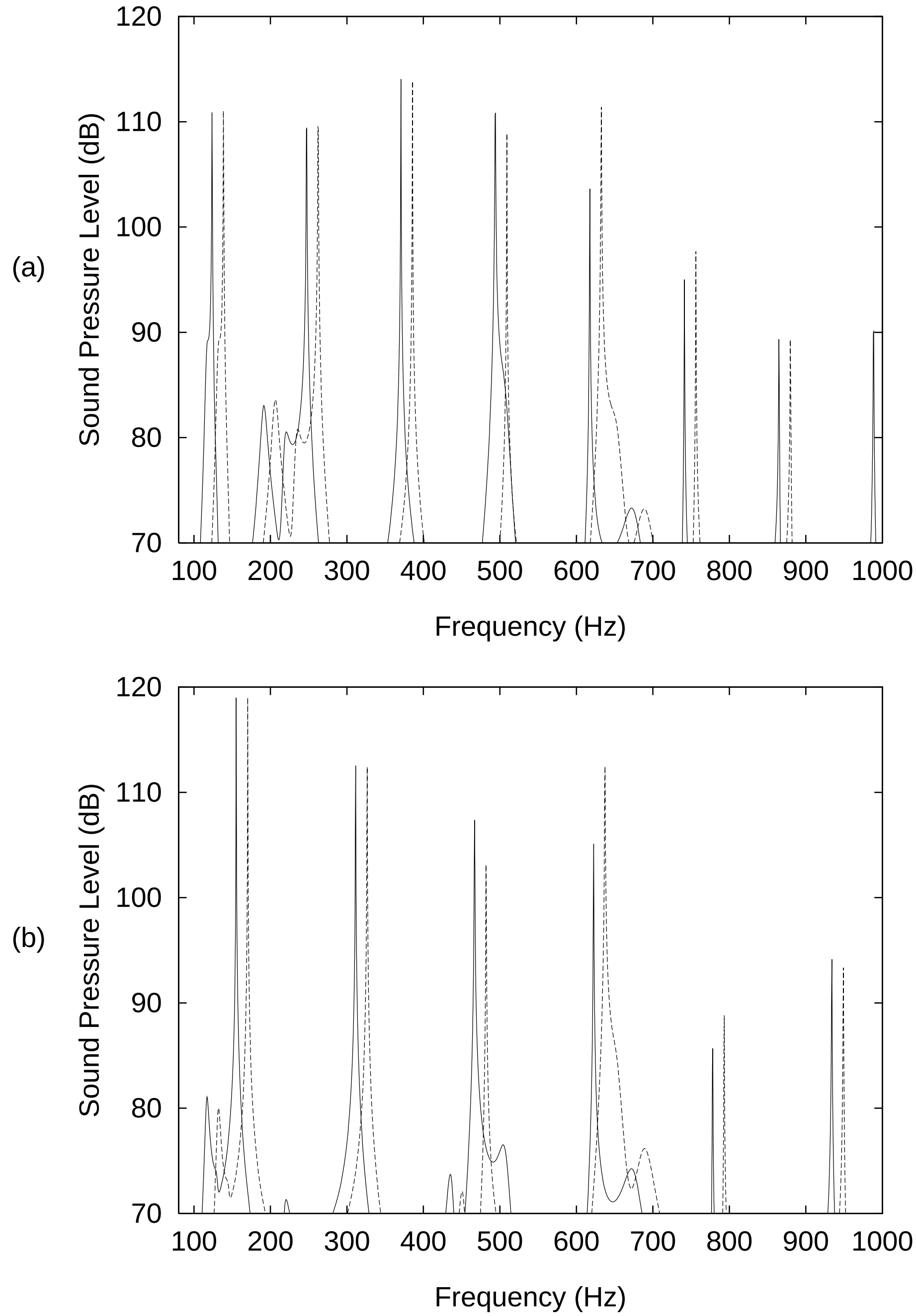


Figure 7.6: Change in the radiated spectrum of string partials for two plucked notes caused by a 26% increase in frequency of the T(3,1) mode. Solid line responses have T(3,1) mode at 507 Hz, dotted line responses have the mode at 638 Hz. Dotted line response in both (a) and (b) has been displaced by 15 Hz to allow comparison of string peak amplitudes. Fundamental frequencies: (a) 123.5 Hz, (b) 155.6 Hz. Changes in the amplitude of the string partials are limited to the two or three partials in the immediate vicinity of the body mode. Largest changes occur for the fifth partial in (a) and the fourth partial in (b) because their frequencies coincide with the higher-frequency T(3,1) resulting in stronger string-body coupling.

on either side undergo small changes in amplitude, but other partials are unaffected. The effect on the sound pressure response of a change in body-mode frequency is confined to the local frequency region close to that body mode.

The T(3,1) mode has a low effective mass, and couples relatively strongly with the two coincident string partials, as indicated by the broader string peaks at around 640 Hz. This causes the coincident string partials to have a relatively rapid decay, so when the the T(3,1) mode is moved up in frequency, the coincident string partials seem significantly louder and will also decay faster, accounting for the high score achieved by test pair number nine in the low-frequency range. For the phrases in the mid- and high-frequency ranges, no string partials coincide with the body mode, and little change in sound is perceived.

The reason for the large perceived change in tone caused by the change in effective mass of the T(3,1) mode (test pair number eight) is slightly different, and is illustrated in Figure 7.7. When the effective mass is lowered, the largest change in amplitude for the string partials does not occur in the vicinity of the body mode itself but at frequencies above the body resonance. For the low-frequency note (Figure 7.7a), the string partial closest to the T(3,1) mode receives a 2 dB increase in amplitude as the mode's effective mass is lowered and the string-body coupling is increased. The largest changes occurs for string partials at frequencies greater than 900 Hz. Nine of these high-frequency partials suffer a reduction in amplitude of between 5 and 6 dB. This can be explained by considering the role that a mode's effective mass has in determining its response at high frequencies and the relative phases of the sound radiation produced by the different body modes.

The response of a mode to driving forces above the mode's own resonance frequency is determined by the effective mass of the mode. Since the input data for the model has no body modes at frequencies above 1 kHz, the calculated response of the body to the high-frequency partials is dominated by lower-frequency modes with low values of effective mass. The T(1,1) mode has the lowest effective mass, and makes the most significant contribution to the radiation of high-frequency string partials. The T(3,1) has a negative effective area, so it will radiate sound with opposite phase to that radiated by the T(1,1) mode. In the high-frequency region, a decrease in the effective mass of the T(3,1) mode causes its contribution to the sound pressure to increase and it radiates a greater out-of-phase sound pressure, thus reducing the overall sound pressure level. The reduction in the sound pressure level of the

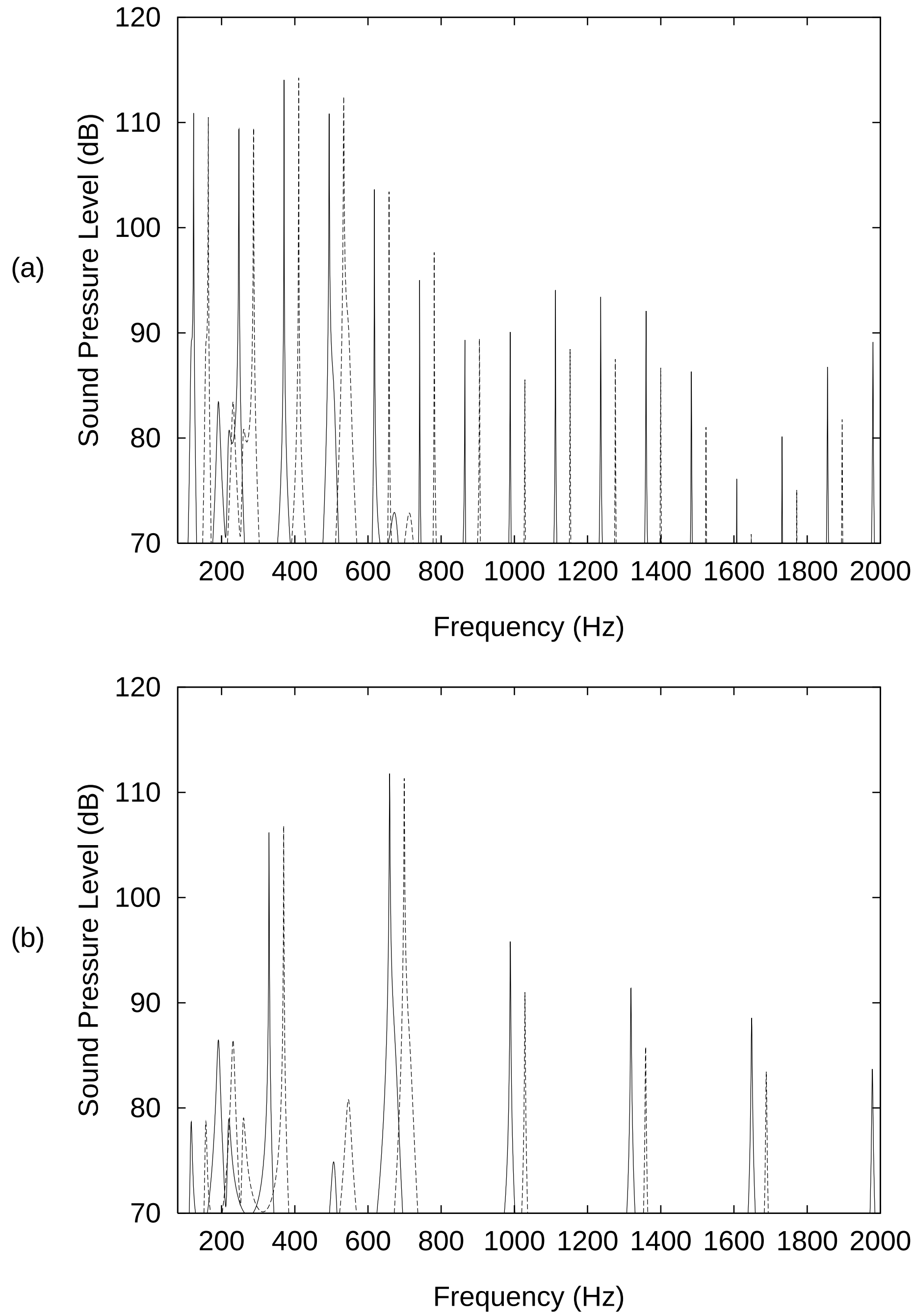


Figure 7.7: Change in the radiated spectrum of string partials for two plucked notes caused by a 50% decrease in the effective mass of the T(3,1) mode. Solid line responses have the T(3,1) effective mass equal to 256 g; dotted line responses have it at 128 g. Dotted line response in both (a) and (b) has been displaced by 15 Hz to allow comparison of string peak amplitudes. String fundamental frequencies: (a) 123.5 Hz, (b) 329.6 Hz. All string partials above 900 Hz in both (a) and (b) are reduced by 5–6 dB (dotted line responses). Changes to the effective masses of body modes are therefore seen to affect large numbers of string partials.

string partials is apparent at frequencies above 1 kHz. At frequencies below this, but above the resonance frequencies of both the T(1,1) and T(3,1) modes, the sound pressure level remains approximately the same. This is because there are four other radiating modes in this region, and although their value of A/m is smaller than that of the T(3,1), they are being driven close to their natural resonance and their contribution to the sound is therefore greater. When all the mode's are being driven at frequencies above their natural resonance, the influence of the modes with large values of A/m is more strongly felt.

This is an adequate explanation for the reduction in amplitude seen for many of the high frequency partials in Figure 7.7a, and it accounts for the large perceived difference achieved by test pair number eight in the low frequency range. However, the same physical effect is observed in Figure 7.7b with a note from the mid-frequency test phrase, which scored considerably lower in the listening test. The partials at frequencies above 900 Hz are again reduced in amplitude by around 5 dB when the effective mass of the T(3,1) is lowered, yet in the listening test the change was judged to give a relatively small difference in tone quality. One explanation for this is that the change in tone for the low-frequency note (Figure 7.7a) is perceived as larger because the number of partials affected is greater than in Figure 7.7b. Another possible explanation is that, for the low-frequency note, the ear detects the sudden reduction in amplitude for the eighth and higher partials, with partials one to seven remaining virtually unchanged. For the mid-frequency note (Figure 7.7b) only the first two partials remain unchanged, with the third and higher partials suffering a 5 dB reduction in amplitude. It is possible that this is perceived more as a reduction in volume, since nearly all partials are affected, rather than a change in tone, and that the reduction in volume is judged to be a smaller difference than the corresponding change in timbre of the lower note. In other words, the results suggest that the subjects may perceive the change in the *relative* amplitude of string partials to be greater than the same change in amplitude applied to all string partials.

7.4.6 Global and local changes

The previous example illustrates that the link between changes in the radiated spectrum of string partials and the corresponding perceptions are not always obvious. However, Figures 7.6 and 7.7 underline an important difference between changes caused by altering a mode's effective mass and changes caused by altering the mode's resonance frequency. Changes to the resonance frequency of a body mode affect the amplitudes of only one or two string partials for any given note. Such a change in tone is a *local* effect, i.e. one which is restricted to a narrow frequency region close to the body mode. Changes to the effective mass of a body mode are seen to have a *global* influence on the guitar's response, since they affect the amplitudes of many string partials at frequencies above that of the body mode. The initial value of A/m will determine the degree of the change in tone. When the frequency or effective mass of a strongly radiating mode (mode with a large value of A/m) is altered, relatively large changes in tone will result.

The examples given above illustrate the global influence of a mode's effective mass on the sound pressure response and the local influence of the mode's resonance frequency. However, the results for test pair numbers eight and nine have shown that the differences between global and local changes to the sound pressure response are not always reflected in the perceived difference scores for the low-, mid- and high-frequency ranges. Changes to a mode's effective mass, which affect partials over a wide range of frequencies, do not necessarily cause changes in tone that are perceived equally for notes in the low-, mid- and high-frequency ranges. One reason for this is due to coincidence of string and body modes, as discussed above in relation to test pair number nine. Changes to the resonance frequency of the T(1,1) mode (test pair numbers three and four) are, however, perceived almost equally in the three frequency ranges, despite the local nature of the change to the sound pressure response. This is due to perceptions of the body transient component of the sound.

7.4.7 The body transient

The body transient, made up of the combined impulse responses of the body modes, contributes a characteristic 'thump' or 'knocking noise' to the transient portion of each plucked note. The T(1,1) mode, having the largest value of A/m, tends to dominate the sound of the body transient, hence when its frequency is altered, the change in the sound of the body transient is easily perceived. Figure 7.8 illustrates the change in the sound pressure response caused by a 26% reduction in the resonance frequency of the T(1,1) mode for a note with a fundamental frequency of 330 Hz. This corresponds to the first note of the mid-frequency range for test pair number four, which achieved an average difference score of 1.38. Figure 7.8 shows that the amplitudes of the string partials are unaffected by the change in frequency of

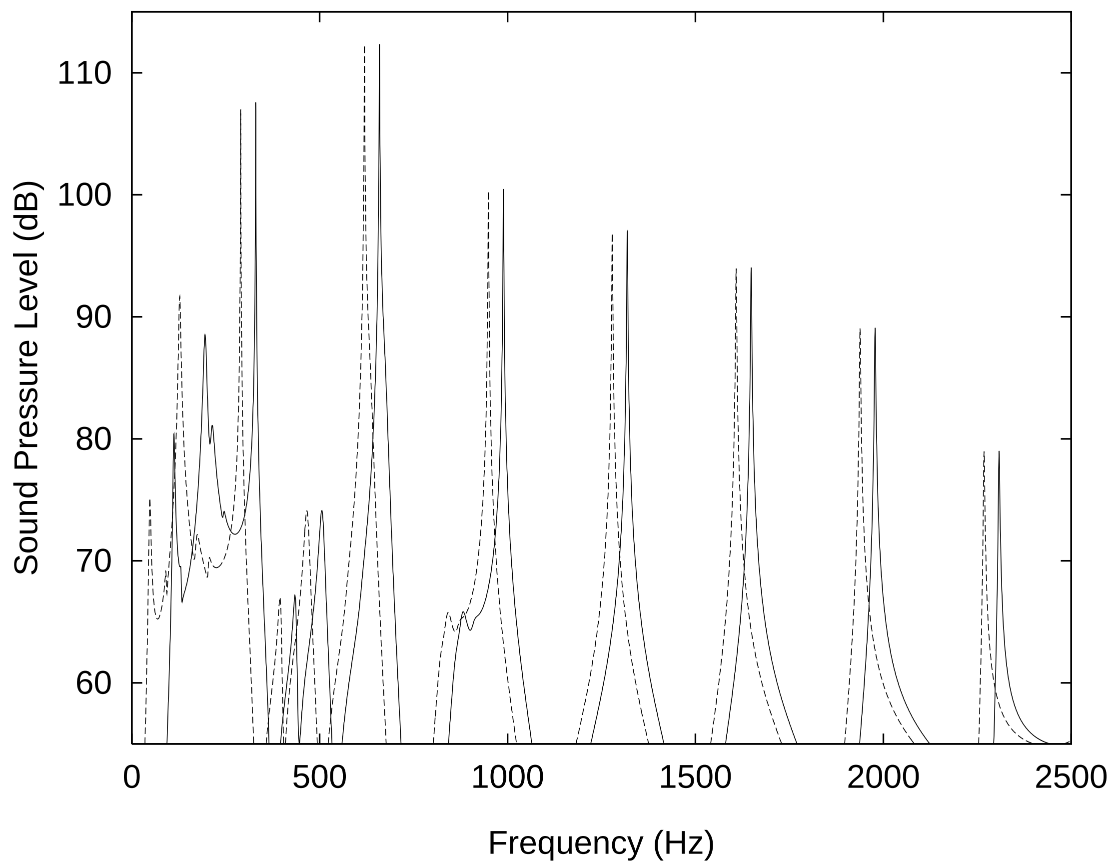


Figure 7.8: Change in sound pressure response due to a 26% decrease in the frequency of the T(1,1) mode. Responses calculated for a string, with fundamental frequency 330 Hz, coupled to the TOP26 mode data set. Dotted line, which shows the response with the T(1,1) at the lower frequency, has been displaced to the left by 40 Hz. Note that the amplitudes of the string partials are unaffected; only the frequency and amplitude of the T(1,1) body modes at around 100 and 200 Hz are affected. The perceived change in tone quality must therefore be due to the change in the sound of the low-frequency component of the body transient.

the mode. The perceived change in tone must therefore be due to the change in the body transient. The figure shows that the frequencies and amplitudes of all three T(1,1) modes are affected. This change in the body transient component of the sound will be the same for notes in all three frequency ranges and explains the similarity of results for test pairs three and four in the low-, mid- and high-frequency ranges.

Both test pairs three and four achieved large standard deviations (0.8–0.9) in all three frequency ranges. As one of the subjects who performed the test, I know that it is difficult to choose a number to represent the overall difference in tone because the 'string part' of the notes in the two phrases sounds identical, but there is an obvious change in pitch of the background 'thump' produced by the body transient. Evidently, some subjects focussed on the 'string part' and returned a low score, while others considered the change in pitch of the body transient to be worthy of a high score. The large standard deviation for this test pair reflects this.

The moderate difference in tone perceived when the Helmholtz frequency is altered may be due to a combination of the 'body transient' and 'coincident string partial' effects. As described earlier in Section 7.3.2, changes applied to the uncoupled Helmholtz mode will influence all three of the coupled T(1,1) modes. When the Helmholtz frequency is altered, perceived changes in the sound of the body transient may be due to a change in frequency or amplitude of either the $T(1,1)_1$ or $T(1,1)_2$ modes. Changes in the radiated level of some string partials may occur when the partial coincides with either the $T(1,1)_1$ mode or the $T(1,1)_2$ mode, causing a noticeable change in tone.

Higher-frequency modes, such as the T(1,2), make smaller contributions to the body transient because of their lower value of A/m. Changes in the frequency of these modes are less easily detected. The very low score observed for test number six indicates that virtually no change in body transient is perceived when the frequency of the T(1,2) mode is raised. The slightly higher score in test number seven, where the T(1,2) frequency is lowered, is probably due to a change in the radiated level of a coincident or near-coincident string partial.

7.4.8 Summary of results

The results from the second listening test have made it possible to distinguish between changes to mode parameters that have a significant influence on tone quality, and changes that have a small or negligible influence on tone. Alterations to three top-plate modes (T(1,1), T(1,2), T(3,1)) and to the fundamental modes of the back plate and air cavity were investigated. The parameters that have the most significant influence on tone quality are the effective masses of the T(1,1) and T(1,2) modes. Changes to the effective masses of these two modes have a stronger influence on tone quality than changes in the mode frequencies. This is true for notes with fundamentals at both low and high frequencies.

Changes to the frequency of the T(1,1) mode have a significant effect on tone quality, but this is largely due to perceived changes in the body transient. The radiation of string partials is not affected by a change in frequency of the T(1,1) mode, unless the partials coincide with the body resonance.

The parameters which appear to have little influence on tone include the effective mass and frequency of the B(1,1) mode, the frequency of the T(1,2) mode and the volume of the air cavity. The frequency of the Helmholtz cavity-mode was seen to have a moderate influence on tone quality.

Sections 7.4.5–7.4.6 have highlighted the important differences between the way in which tone quality is affected by changes to a mode's effective mass and changes to its resonance frequency. Changes in the effective mass of a top-plate mode have a global effect on the sound pressure response, causing changes in the amplitudes of a large number of string partials. The tone quality of notes with fundamental frequencies well above the mode's own resonance frequency may be strongly affected by changes to the mode's effective mass. In contrast, changes to the resonance frequency of a top-plate mode are seen to affect only the amplitudes of string partials in the local frequency region close to the body mode. The radiated spectrum of string partials is therefore unaffected by changes to a mode's resonance frequency unless the frequency of one of the string partials is close to the mode's own resonance. When the frequency of a string partial coincides with a body mode, large changes in tone result from alterations to either the mode's effective mass or resonance frequency.

7.5 Third listening test

7.5.1 Aims

One of the aims of the third listening test was to investigate the relative influence of the effective mass and effective area parameters. When the guitar body is driven by a sinusoidal force, the sound pressure level radiated by a single, uncoupled mode is proportional to the ratio A/m (Christensen, 1984; see also Equations 4.8 and 4.17). A two-fold reduction in the mode's effective mass therefore has the same effect as a two-fold increase in its effective area. When the effects of coupling are considered this is no longer true. The effective mass is crucial in determining the degree of coupling between string and body. Changes in the effective mass of a mode affect both the amplitude and the decay of a note. The effective area controls the volume source of the mode and affects only the amplitude of the radiated sound. Sounds were presented in the third test that compared the influence of changes of equal magnitude to the effective mass and effective area of the T(1,1) and T(1,2) modes.

Another effect investigated in the third test was the influence of the relative radiating phases of the top-plate modes. The T(1,2) mode usually has a negative effective area (see Tables 8.1 and 8.2). When driven above its natural resonance, it radiates sound with opposite phase to that of the T(1,1) mode and causes a loss of high-frequency response. Between the T(1,1) and T(1,2) modes the contributions to the radiated sound are in phase, and a region of strong response is created. One of the changes investigated in the third test was a reversal of the sign of the effective area of the T(1,2) mode. Both the T(1,1) and T(1,2) then have positive effective areas, thus reinforcing the guitar's high-frequency response and creating an anti-resonance between the two modes (see Figure 7.9).

Other changes investigated in the third test included a reduction by two tones (26%) in the resonance frequency of the T(1,1) and T(1,2) modes, and a three-fold reduction in the Q-value of the T(1,1) mode. Another group of test sounds were prepared in which the number of body modes contributing to the radiated sound was reduced. This allowed comparisons to be made between the sound radiated from the full compliment of body modes and the sound radiated from just the (1,1) triplet of modes or all modes up to and including the T(1,2). One other parameter change was tried in which the effective area of all modes except the T(1,1) was increased by 50%. This allowed comparisons between the standard parameter set and

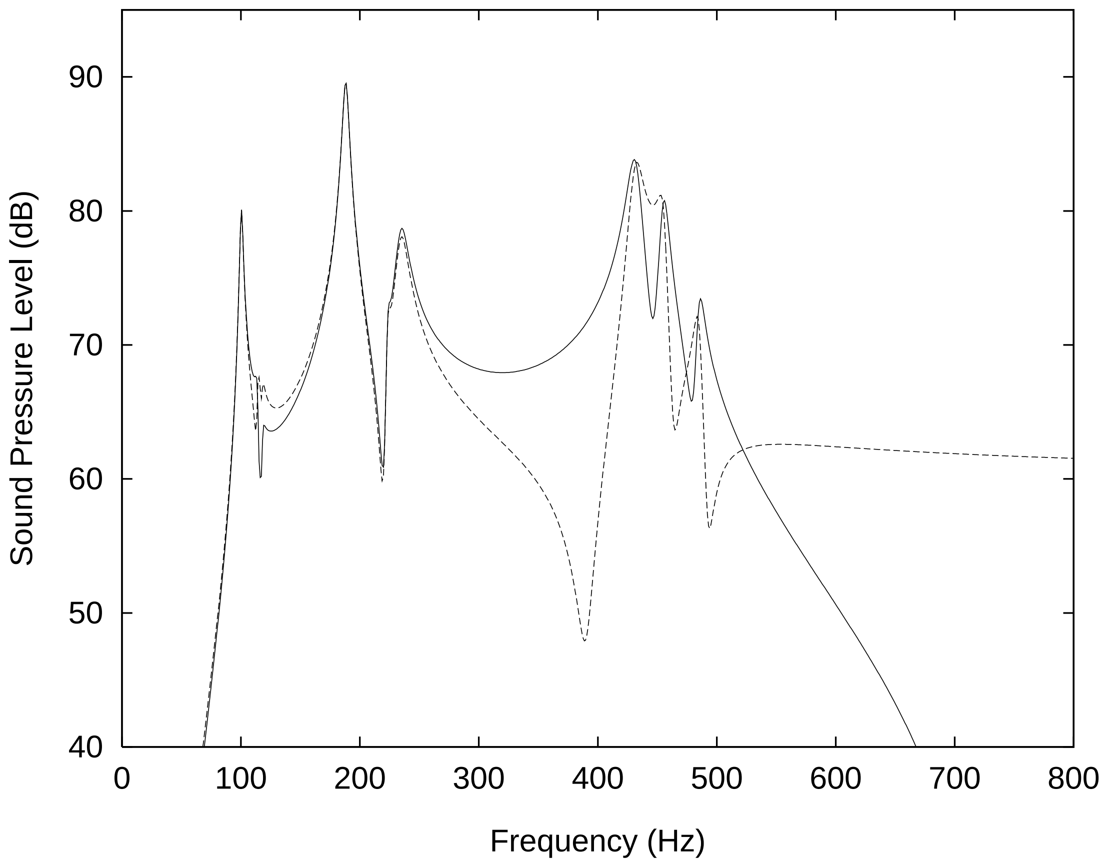


Figure 7.9: Effect on the guitar body's sound pressure response of a reversal of the sign of the T(1,2) mode's effective area. The resonance frequency of the T(1,2) is 430 Hz. The solid line corresponds to the case where the T(1,2) has negative effective area. When the effective area is made positive (dotted line) the response in the region 230–430 Hz is reduced and an antiresonance is created at 390 Hz due to the out-of-phase radiation of the T(1,1) and T(1,2) modes. Above 550 Hz, where both modes radiate with the same phase, the response is increased. Response calculated using data obtained for guitar BR2 from curve-fitting work (see Table 7.6). Driving position was at the A string position.

one in which the radiating power of the T(1,1) modes, relative to the other top-plate modes, was reduced.

One further aim of the third listening test was to measure not only the magnitude of the differences in tone caused by changes to the mode data, but also the *nature* of the differences in tone. Subjects were asked to make judgements relating to attributes such as 'loudness' and 'brightness' (see below).

The sounds used in the third test were synthesised using the mode data for guitar BR2, obtained from the curve-fitting work in Chapter 5. Small alterations were made to the BR2 parameters derived from the curve-fitting work because preliminary tests revealed that one or two notes in the test phrases used sounded rather 'nasal' and unpleasant. A reduction in the effective area of the T(1,2) mode and a small increase in the effective mass of the T(1,1) mode (to quell a mild wolf-note) produced satisfactory results. The data set used to synthesise the test sounds is given in Table 7.6. A different set of 'standard' mode parameters was chosen to synthesise the sounds in the third test so that comparisons could be made between the differences in tone that result from a given parameter change applied to both the TOP26 and BR2 data sets. This makes it possible to quantify the effect on tone quality of applying the same physical changes to two different guitars. In the fourth listening test (Section 7.6), sounds were synthesised using a third set of of mode parameters, those obtained for guitar BR1. Comparisons between the results obtained for the three different sets of mode data will be made in Section 7.6.2.

The way in which the sounds were presented in the third listening test was similar to the previous test. Individual notes were put together into short musical phrases. Two different phrases were used so that the fundamental frequencies of the notes covered two frequency ranges: 'low' (124–185 Hz) and 'mid' (330–494 Hz). The low-frequency phrase used the first five notes of the B major scale, starting on the note B_2 and the mid-frequency phrase used the first five notes of the E major scale, starting on the note E_4. Pairs of phrases were presented to the subject, and they were asked to judge the difference in tone of the two sounds on an integer 0–3 scale. The first phrase in each test pair was the control, synthesised from the BR2 mode parameter set. The test was split into three parts, each having a random ordering of the same 24 test pairs. The answers given by each subject for the three repetitions of each test pair were compared to check for consistency, and average difference scores were calculated

Mode	Resonance frequency (Hz)	Effective mass (kg)		Effective area (m^2)		Q-value
		A_2 string	E_4 string	A_2 string	E_4 string	
T(1,1)	182	0.078	0.068	0.047	0.043	37
T(2,1)	223	0.201	0.250	0.010	−0.018	64
$T(1,2)_1$	430	0.650	0.546	−0.183	−0.120	35
T(3,1)	455	0.600	0.678	−0.070	−0.080	55
$T(1,2)_2$	485	0.700	0.806	−0.025	−0.050	65
T(2,2)	550	0.350	0.250	0.000	0.000	45

Table 7.6: Data for six top-plate modes of guitar BR2, used to synthesise sounds for the third and fourth listening tests. Values of effective area and effective mass are given for two driving points, both situated on the bridge near the string termination positions. Data given is for the uncoupled modes; coupling to the Helmholtz cavity mode (uncoupled resonance frequency 122 Hz, Q-value 50) and fundamental back-plate mode (uncoupled resonance frequency 229 Hz, effective mass 51 g, effective area 0.023 m^2, Q-value 20) was calculated by the model.

for each test pair. Two test pairs in each set of 24 were identical. The average difference scores obtained by each subject for these identical pairs was used as a 'quality control'. Most subjects obtained average scores of 0 or 0.33 for the identical pairs and it was not necessary to exclude any subjects from the final results because of average scores for the identical pairs which were too high.

Example sounds representing the extremes of the difference scale were again used to give subjects an idea of the range of sounds to expect. The test pair chosen for the 'large difference' end of the scale was test number nine, using notes in the low-frequency range. One of the identical pairs was used for the 'small difference' end of the scale.

In addition to judging the magnitude of the difference in tone, subjects were asked to describe the nature of the perceived difference, either by ticking one of five boxes or by describing the difference in words. The five boxes provided were for the judgements 'louder', 'quieter', 'brighter', 'more muffled' and 'more uneven'. These descriptions were chosen from preliminary trials of the test sounds using a small number of subjects. It was felt that the meaning of these five words was relatively unambiguous, and that the judgements louder/quieter and brighter/more muffled could be treated as antonyms. Subjects were not told this, although the interpretation of 'louder' and 'quieter' as opposite attributes was obvious. Subjects' answers indicated that the judgements 'brighter' and 'more muffled' were also used as opposite timbral attributes.

A list of suggestions for other words with which to describe the differences in tone quality was also provided, collected from the preliminary tests, but subjects were free to pick words from the list or choose their own descriptions. The list was provided with the intention of limiting the number of descriptive words used by the subjects so that comparing the results of different subjects would be made a little easier.

7.5.2 Results: magnitude of the changes in tone

The third listening test was performed by 18 subjects. Before discussing the results relating to the nature of the perceived changes ('louder', 'brighter' etc) I will discuss the results relating to the magnitude of the perceived differences in tone. The perceived difference scores, averaged over all subjects, are plotted in Figure 7.10 and are also given in tabular form in Table 7.7. The average scores for the two identical test pairs were 0.04 ± 0.11 and 0.07 ± 0.18. Standard deviations are in the range 0.4–0.6 for most test pairs indicating fairly large variations in the perceptions of different subjects. Bold type has been used in Table 7.7 to highlight results for which there was better than average agreement within the subject group (i.e. lower than average standard deviation).

The results in Figure 7.10 show that the difference in tone caused by each parameter change is not strongly dependent on the fundamental frequency of the notes used to synthesise the test sounds. Results for the low- and mid-frequency ranges (data plotted with solid and dashed lines in Figure 7.10) are similar. The division of results into test pairs that cause large differences in tone and those that cause small differences in tone is therefore not greatly influenced by the frequency of the notes used to synthesise the test sounds. Test pair number three shows the greatest difference between the results in the low- and mid-frequency ranges. The reason for this is discussed below.

If the results from Figure 7.10 are selected purely on the basis of their average scores, and the standard deviations are ignored, the five highest-scoring test pairs are found to be test pair numbers four, five, seven, nine and eleven. Test pair number nine, the highest scorer, corresponds to the exclusion of top-plate modes above the T(1,1). The others correspond to changes to the effective mass and effective area of the T(1,1) and T(1,2) modes (including a reversal of the sign of the T(1,2) mode's effective area). Changes to a mode's effective area are found to have a slightly stronger influence on tone than an equivalent change to the mode's effective mass. Changes to the effective masses are seen to have a stronger influence on tone than changes to the resonance frequencies of the modes, confirming the results from the second listening test. The lowest scorers from Figure 7.10 are identified as test pair numbers three and six, corresponding to a three-fold reduction in the Q-value of the T(1,1) mode and a 26% reduction in the frequency of the T(1,2) mode.

I will now consider the results for which there was relatively good agreement between subjects, i.e. the results highlighted in bold in Table 7.7. Test pair numbers four, seven, nine and eleven were judged by most subjects to have a large difference in tone; all achieved average scores greater than two. Subjects therefore tended to agree that the exclusion of modes above the T(1,1), and alterations to the effective areas of the modes, had the strongest influence on tone quality. Results highlighted in bold and scoring between one and two (judged by most subjects to have a moderate difference in tone) include the reduction of the effective mass of

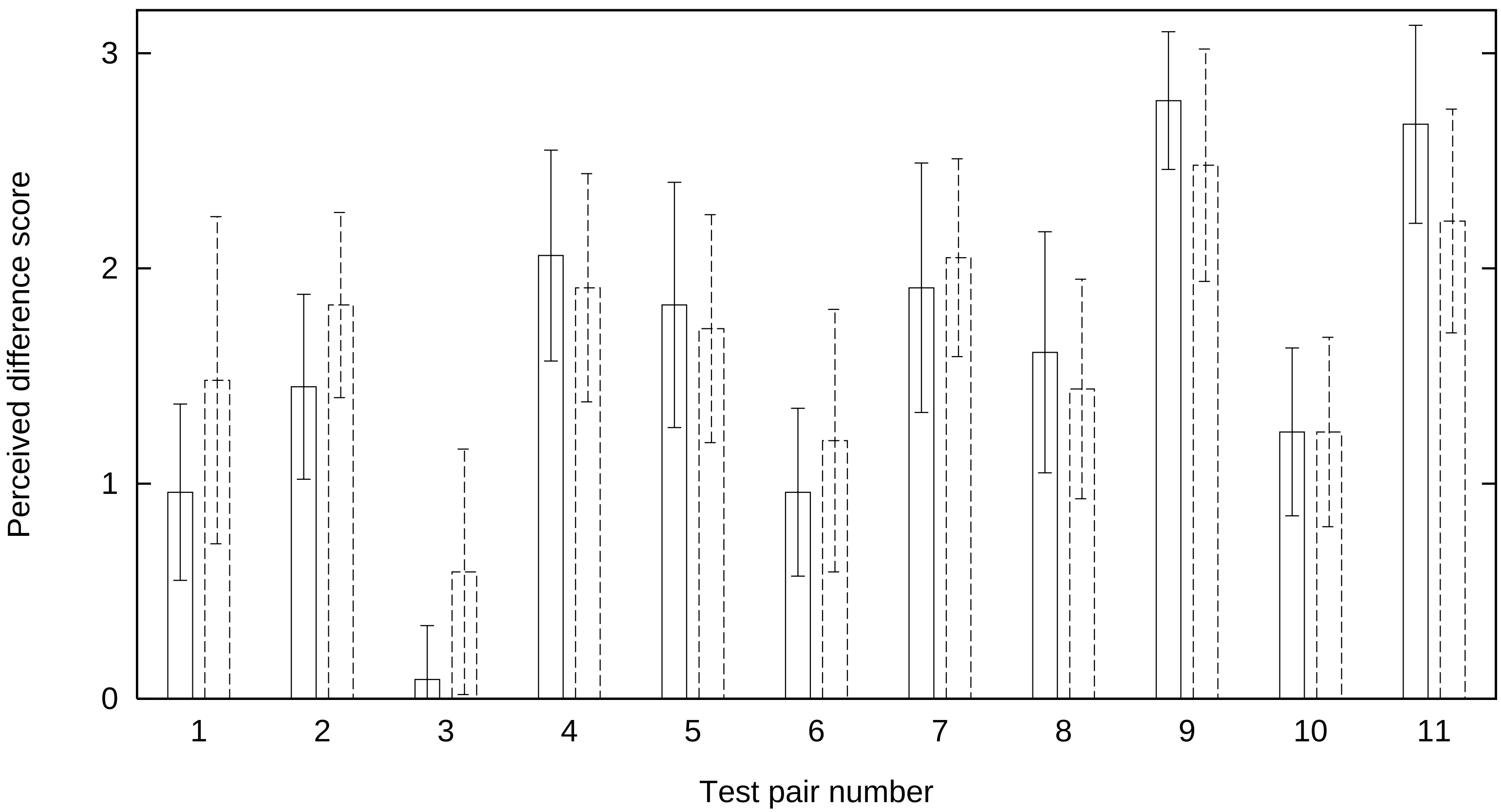


Figure 7.10: Results for listening test three showing the magnitude of the perceived differences in tone for changes to the top-plate modes. Parameter changes associated with each test pair number are given in Table 7.7. Different line types indicate sounds synthesised using string fundamentals in the frequency ranges 124–185 Hz (solid) and 330–494 Hz (dashed). Averages calculated for 18 subjects. Numbers used to identify the test pairs are arbitrary; no sequence should be inferred. The five highest-scorers, test pair nos. 4, 5, 7, 9 and 11, correspond to changes to the effective masses and areas of the top-plate modes and the exclusion of modes above the T(1,1).

Test pair number	Parameter change	Frequency range	Average perceived difference score	Standard deviation
1	f_t (1,1) -26%	Low	**0.96**	**0.41**
		Mid	1.48	0.76
2	m_t (1,1) $\div 1.75$	Low	**1.45**	**0.43**
		Mid	**1.83**	**0.43**
3	Q_t (1,1) $\div 3$	Low	**0.09**	**0.25**
		Mid	0.59	0.57
4	A_t (1,1) $\times 1.75$	Low	**2.06**	**0.49**
		Mid	1.91	0.53
5	m_t (1,2) $\div 4$	Low	1.83	0.57
		Mid	1.72	0.53
6	f_t (1,2) -26%	Low	**0.96**	**0.39**
		Mid	1.20	0.61
7	A_t (1,2) $\times 4$	Low	1.91	0.58
		Mid	**2.05**	**0.46**
8	$A_t \times 1.5$ for all top-plate	Low	1.61	0.56
	modes except (1,1)	Mid	1.44	0.51
9	Top-plate modes above	Low	**2.78**	**0.32**
	(1,1) excluded	Mid	2.48	0.54
10	Top-plate modes above	Low	**1.24**	**0.39**
	(1,2) excluded	Mid	**1.24**	**0.44**
11	A_t (1,2) $\times -1$	Low	**2.67**	**0.46**
		Mid	2.22	0.52

Table 7.7: Results for listening test three showing the magnitude of the perceived difference in tone (0–3 scale) for changes to the top-plate mode parameters. Averages calculated for 18 subjects. The eleven results with the lowest standard deviations, indicating greater agreement between subjects, are highlighted in bold. Frequency ranges for string fundamentals: 124–185 Hz (low), 330–494 Hz (mid). Resonance frequencies of the uncoupled top-plate modes were: T(1,1) 182 Hz, T(1,2) 430 Hz. The average scores for the two control pairs (identical sounds) were 0.04 ± 0.11 and 0.07 ± 0.18

the T(1,1) mode and the exclusion of modes above the T(1,2). Results in bold which scored less than one (judged by most subjects to have small differences in tone) include reductions in frequency of the T(1,1) and T(1,2) modes and a reduction in the Q-value of the T(1,1) mode.

For many of the test pairs there are relatively small differences between the average scores achieved in the low- and mid-frequency ranges. Test pair numbers one, two and three show more significant variation between the scores for the two frequency ranges; these correspond to changes to the frequency and Q-value of the T(1,1) mode. The greatest difference between the results for the two frequency ranges is for the reduction in Q-value of the T(1,1) mode (test pair number three). In the low-frequency range, no change in tone is perceived but in the mid-frequency range, a small but significant change is perceived. The reduced Q-value does not affect the radiation of the string partials but results in a more quickly decaying body transient. The change in the sound of the body transient can only be perceived when the string partials do not mask the sound of the body transient. In the low-frequency range, the string partials are at similar frequencies to the T(1,1) mode and the change in decay of the body transient is masked, so no difference in tone is perceived. For notes at higher frequencies, the change in body transient is no longer masked and a small but noticeable difference in tone is perceived.

General conclusions

The division of the test pairs into high and low-scorers is clearly dependent on the numerical value of the parameter change implemented in each case. We can conclude that, for example, alterations to the Q-value of the T(1,1) mode by a factor of three produce very small changes in tone quality compared to alterations to the mode's effective area by a factor of 1.75. However, the numerical values of the test changes have been shown (Section 7.4.2) to be representative of the typical differences found between instruments of broadly similar construction. We can therefore draw more general conclusions that indicate the relative significance of the four mode parameters.

One of the main conclusions from the second listening test is that changes to the effective masses of the top-plate modes produce more significant changes in tone than changes in the mode frequencies. This was shown to be true for the T(1,1), T(1,2) and T(3,1) modes. Moreover, the effect does not depend on the fundamental frequencies of the notes used to synthesise the test phrase; results for the low-, mid- and high-frequency ranges all show that changes to the modes' effective masses produce greater changes in tone than changes to the mode frequencies. The coincidence of string partials in the low-frequency test phrase with the T(3,1) mode complicated the results slightly, but the results for the T(3,1) mode in the mid- and high-frequency ranges are consistent with the above finding.

The results of the third test confirm this result: changes to the effective masses of the T(1,1) and T(1,2) modes have a stronger influence on tone than changes to the mode frequencies. The results of the third test also indicate that changes to a mode's effective area produce slightly larger changes in tone than an equivalent change to the mode's effective mass. This result is true for both the T(1,1) and T(1,2) modes: test pair numbers four and seven achieve slightly higher scores than test pairs two and five (see Figure 7.10). Both these results are true for test phrases in the low- and mid-frequency ranges.

The effect of a change in Q-value has only been tested on the T(1,1) mode where it was found to have a very small influence on tone quality. The 'string part' of the sound is unaffected; the small difference in tone is due to a more rapidly decaying body transient. The T(1,1) mode has the greatest value of A/m, and therefore the potential for a change in Q-value to affect tone quality is greatest for the T(1,1) mode. Since a three-fold reduction in the T(1,1) mode's Q-value produces a very small change in tone, similar changes to modes with smaller values of A/m are likely to have a small or negligible effect on tone. We can therefore conclude that the relative influence of the four mode parameters on tone quality appears to be: $A > m > f > Q$.

7.5.3 Results: nature of the changes in tone

The results for the judgements of the nature of the difference in tone quality are given in Table 7.8. Totals, over all subjects, were calculated for the number of times the five judgements ('louder', 'quieter', 'brighter', 'more muffled', 'more uneven') were used to describe the difference in tone for each test pair. Note that the number of times a particular judgement was used does not indicate the magnitude of the change in tone, which is measured by the perceived difference scores. The number of times a given judgement is made is better understood as a measure of the confidence we can associate with that judgement. For example, test numbers two, four, five and seven all obtained high scores (30–40) for the judgement 'louder' showing that subjects tended to agree on this aspect of the tone quality. Greater confidence can therefore be attached to the link between these four parameter changes and the effect on perceived loudness.

As mentioned above, I will discuss the results by referring to the judgements 'louder' and 'quieter', and 'brighter' and 'more muffled' as opposite timbral attributes. Subjects were not told that these judgements formed opposite pairs, but it was felt that the interpretation of the words was sufficiently clear for the judgements to be treated as such. For the two opposite-pair judgements, circles have been used in Table 7.8 to highlight results for which there was a majority of at least 80% and a minimum score of five. For these circled results, the judgements made are found to apply equally to the phrases in the low- and mid-frequency ranges. For a particular parameter change, therefore, perceptions of loudness and brightness etc. seem to apply to notes throughout much of the guitar's playing range.

Perceived loudness

Reductions in the effective masses and increases in the effective areas of the T(1,1) and T(1,2) modes (test pair numbers two, four, five and seven) result in high scores for the judgement 'louder'. The link between a mode's value of A/m (its 'sound producing capacity') and perceived loudness is therefore confirmed. Notes in both the low- and mid-frequency ranges are perceived as 'louder' when the values of A/m of the T(1,1) and T(1,2) modes are increased. The global nature of changes to the effective masses of the top-plate modes has already been discussed (Section 7.4.6). It now appears that changes to a mode's effective area also has a global effect on tone, in which string partials over a wide frequency range are similarly affected. Figure 7.11 illustrates this for a note in the mid-frequency range. The fundamental frequency of the note is well above the T(1,1) resonance, but the increase in the mode's effective area causes all string partials to be radiated more strongly. The sound is therefore perceived as 'louder'.

Other changes that give a 'louder' sound are the reversal of the sign of the T(1,2) mode's effective area (test pair number eleven) and the exclusion of all modes above the T(1,1) (test pair number nine). These two can be explained by considering the relative phases of sound radiation from the different modes. As discussed earlier, modes with a negative effective area, when driven at frequencies above their natural resonance, radiate sound with opposite phase

Test pair number	Parameter change	Frequency range	Louder	Quieter	Brighter	More muffled	More uneven
1	f_t (1,1) −26%	Low	1	0	0	(9)	(13)
		Mid	1	1	0	(6)	(7)
2	m_t (1,1) ÷1.75	Low	(38)	0	(12)	1	(8)
		Mid	(40)	0	(27)	1	0
3	Q_t (1,1) ÷3	Low	0	1	0	1	1
		Mid	1	3	4	0	0
4	A_t (1,1) ×1.75	Low	(37)	0	(13)	3	6
		Mid	(39)	0	(28)	0	2
5	m_t (1,2) ÷4	Low	(39)	0	(14)	1	3
		Mid	(42)	0	(20)	1	(15)
6	f_t (1,2) −26%	Low	0	4	0	(18)	(23)
		Mid	0	3	1	(8)	(24)
7	A_t (1,2) ×4	Low	(37)	0	(18)	0	(12)
		Mid	(45)	0	(25)	0	(31)
8	$A_t \times 1.5$ for all top-plate modes except (1,1)	Low	2	(8)	1	(41)	6
		Mid	4	2	0	(41)	(11)
9	Top-plate modes above (1,1) excluded	Low	(14)	1	18	8	7
		Mid	(25)	0	(41)	0	3
10	Top-plate modes above (1,2) excluded	Low	6	3	9	14	(13)
		Mid	(10)	0	(23)	2	(10)
11	A_t (1,2) $\times -1$	Low	(32)	0	(21)	5	4
		Mid	(19)	0	(40)	0	3

Table 7.8: Results showing the number of times each of the five judgements were used to describe the difference in tone quality resulting from changes to the top-plate mode parameters. Totals given are for 18 subjects. Circles have been used to highlight results (for the louder/quieter and brighter/more muffled judgements) for which there was a majority of at least 80%, and a minimum score of five. For the 'more uneven' judgements, circles have been used to highlight the eleven greatest scores. Changes to the effective areas and effective masses of the top-plate modes (test pair numbers 2, 4, 5, 7 and 11) scored significantly in both the 'louder' and 'brighter' categories.

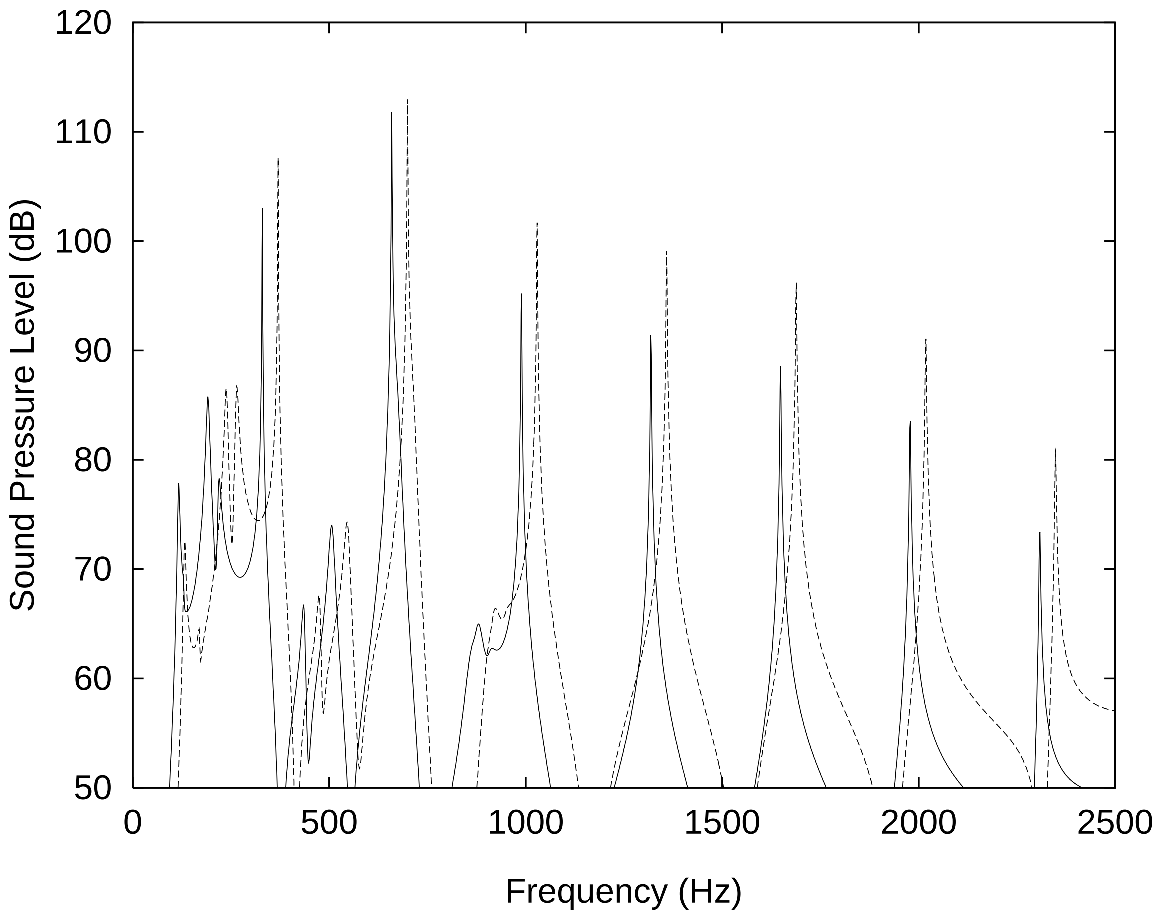


Figure 7.11: Change in sound pressure response of a plucked note (fundamental frequency 330 Hz) due to a 75% increase in the effective area of the T(1,1) mode. Dotted line response, which has the larger effective area, has been displaced by 40 Hz for clarity. The larger T(1,1) effective area causes an increase of 2–8 dB in the radiated amplitude of all string partials.

to that radiated by the T(1,1) mode. Since the T(1,1) mode has the largest value of A/m it will tend to make the most significant contribution to the high-frequency radiation[1]. Modes with negative effective areas will therefore reduce the high-frequency radiation produced by the T(1,1) mode. With test pair numbers nine and ten, when some or all of the anti-phase modes are excluded, the radiation of sound in the high-frequency region is increased and the sound is perceived as 'louder'. Test pair eleven, in which the sign of the T(1,2) mode's effective area is reversed, causes an increase in the guitar's high-frequency response but a decrease in the response between around 230 Hz and 430 Hz (see Figure 7.9). The gain in the high-frequency response seems to be perceptually more important than the loss of low-frequency response since the sound is perceived as 'louder'.

Perceived brightness

The judgement 'brighter' is commonly interpreted as meaning a sound with a greater number of high-frequency partials or a sound with a greater amplitude for the high-frequency partials. The two test pairs discussed above (numbers nine and eleven), caused the high-frequency response of the guitar to be increased. This resulted not only in the perception that the sounds were 'louder' but also that they were 'brighter'. The two judgements 'louder' and 'brighter' are seen to be chosen as a pair throughout the listening test. Results in Table 7.8 which are highlighted with a circle for the judgement 'louder' are almost always highlighted for the 'brighter' judgement because it is the high-frequency response of the instrument which is increased, giving an impression of greater overall loudness as well as an impression of increased 'brightness'.

The results are best understood by again considering the relative radiating phases of the top-plate modes. When some or all of the top-plate modes with negative effective areas are excluded (test pair numbers nine and ten), the high-frequency radiation which is of opposite phase to the T(1,1) mode is reduced, and the overall high-frequency response is increased. The corresponding sounds are perceived as 'brighter' since the amplitudes of the high-frequency string partials are increased. Similarly, when the effective area of the T(1,2) mode is made positive (test pair number eleven) and the value of A/m of the T(1,1) is increased (test pair

[1]Numerical work by Brooke (1992) has shown this to be true for a driving frequency of 990 Hz. The behaviour of the strongly radiating modes at higher frequencies is not yet understood, and the frequency at which the T(1,1) ceases to be the dominant radiator is therefore not known.

numbers two and four) the contributions to the high-frequency sound from the modes with positive effective areas are increased. The high-frequency response is therefore strengthened, and the sounds are again perceived as 'brighter' (and 'louder').

The results for test pair numbers five and seven are a little more surprising. The increases in the value of A/m of the T(1,2) mode were judged to produce a 'louder' and 'brighter' sound, indicating that the high-frequency response of the guitar was increased. The T(1,2) has a negative effective area and radiates high-frequency sound with the opposite phase to that radiated by the $\mathrm{T}(1,1)_2$. An increase in the T(1,2) mode's value of A/m leads to an increase in the out-of-phase radiation. A reduction in the guitar's high-frequency response, and a corresponding perception of a *less* bright tone might therefore be expected. The qualitative argument used above to describe the changes in the guitar's high-frequency response now needs to be refined to give a quantitative argument relating to the relative values of A/m of the different modes.

The residual value of A/m

The sound pressure radiated by a single, uncoupled mode, driven by a sinusoidal force of amplitude F and frequency ω is given by Christensen (1984) as:

$$p = F\frac{A}{m}\frac{\rho}{4\pi r}\frac{\omega^2}{(\omega_0^2 - \omega^2) + i\gamma\omega}\ , \tag{7.1}$$

where ρ is the density of air, r is the distance from the source, and A, m, ω_0 and γ represent the effective area, effective mass, resonance frequency and damping of the mode. When driven well above its resonance frequency, the sound pressure radiated by a single mode approaches a constant level, proportional to A/m. To obtain a measure of the high-frequency sound radiated by a combination of body modes, a summation of all the values of A/m is performed. If this is done on the BR2 parameter set used for the third listening test, a residual value for A/m of 0.16 is obtained. When the value of A/m for the T(1,2) mode is increased by a factor of four (test pair numbers five and seven) the residual value of A/m changes to -0.50, so the magnitude of the high-frequency sound pressure response is increased causing a perception of increased 'brightness'. The reversal of the sign of the residual value of A/m indicates that the net contribution of the modes with negative effective area has become larger than the positive contributions from the T(1,1) mode. Figure 7.12 illustrates the effect on the high-frequency response of a gradual increase in the effective area of the T(1,2). The response above 500 Hz is first reduced as the negative contributions from the T(1,2) cause the value of the (initially positive) residual A/m to become smaller. As the value of the T(1,2) effective area is further increased, the sign of the residual value of A/m changes, and its magnitude increases, causing the high-frequency response to increase. Small increases in the effective area of the T(1,2) will therefore cause perceptions of reduced 'brightness', while larger increases lead to perceptions of increased 'brightness'.

Other perceptions

Only one significant result was obtained for the judgement 'quieter': test pair number eight, in which the effective areas of all modes above the T(1,1) are increased by a factor of 1.5. These modes have negative effective areas and the increase in the magnitude of their effective areas increases the proportion of out-of-phase radiation at high frequencies. The overall level of high-frequency radiation drops, while the low-frequency response is slightly increased. The resulting sound is perceived as 'quieter', although the score is fairly low. A much higher score is obtained by this test pair for the judgement 'more muffled'. The reasoning is the same; the high-frequency response is reduced, so the sound is perceived as 'more muffled' (i.e. less bright) because the amplitude of the high-frequency string partials is decreased.

Other test pairs which result in the perception 'more muffled' involve changes to the frequencies of the T(1,1) and T(1,2) modes. Changes to the mode frequencies also have a tendency to give a sound which is judged as 'more uneven'. This may be due to strong coupling between the top-plate modes and certain notes used in the test phrases. If one note in the phrase has a partial that coincides with one of the body modes, the partial will couple more strongly to the mode and will be radiated more strongly. The change in the radiated spectrum of string partials will cause a changed perception of tone and the note may also be perceived as being being somewhat louder. The change in tone and loudness may be such that the note sounds considerably different than notes which follow and precede it in the test phrase, causing a sensation of 'unevenness'. The phrase may be perceived as having a note that 'sticks out' due to the changes in tone and loudness that accompany the coincidence of string partial and body mode. Increases in the effective area of the T(1,2) mode and decreases in the mode's effective mass also give rise to sounds which are perceived as 'more uneven'.

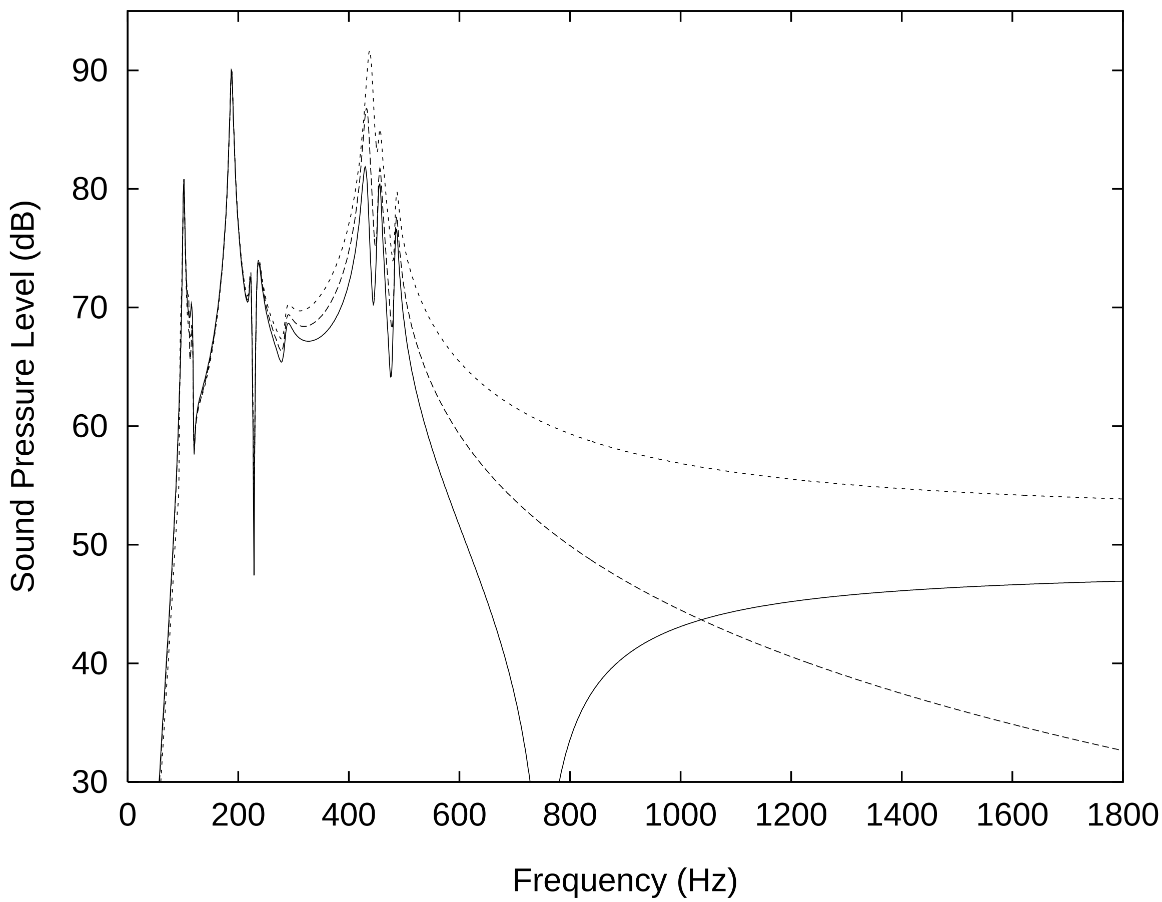


Figure 7.12: Influence on the guitar's high-frequency sound pressure response of successive increases in the value of A/m for the T(1,2) mode (uncoupled resonance frequency: 430 Hz). Effective area of the mode is initially 0.12 m^2 (solid line). An increase of 80% in the effective area leads to a *lower* response above 1 kHz (long-dashed line) because of greater out-of-phase radiation at high frequencies. A further increase of 80% leads to a *higher* response above 1 kHz (short-dashed line) because the radiation from modes with negative effective areas now exceeds that from the T(1,1) mode. Response calculated using the BR2 parameter set, driven at the top E string position.

This is again likely to be cause by strong coupling with certain notes, the increase in the value of A/m of the T(1,2) mode causing a greater change in sound for the coincident note.

Summary

Many of the results for perceived brightness and loudness can be understood by considering the sign and value of A/m of the top-plate modes. Increases in the value of A/m of a mode causes it to radiate more strongly, and the sound is perceived as 'louder'. The global nature of changes to the effective masses and effective areas of the modes mean that the guitar's response is changed not only at frequencies close to the mode's natural resonance, but also at frequencies well above the mode's resonance frequency. Notes with both low and high-frequency fundamentals are perceived as 'louder' when the value of A/m for the T(1,1) is increased.

For the T(1,1) and T(1,2) modes, increases in their value of A/m also lead to a 'brighter' sound. The perceived 'brightness' of the sound is directly connected with the guitar's high-frequency response, which can be understood by considering the net effect of the high-frequency monopole radiation from all top-plate modes. Calculating a residual value of A/m (a summation of the values of A/m for the top-plate modes) yields a single number which measures the net contribution of the low-frequency modes to the high-frequency sound. A larger value of the residual value of A/m leads to a greater contribution to the high-frequency radiation from the low-frequency modes, and therefore produces a 'brighter' sound.

The sign of the ratio A/m for the T(1,1) mode is always positive, but most top-plate modes above the T(1,1) have negative values of A/m. The magnitude of the T(1,1) mode's value of A/m is, however, usually sufficiently large that the residual value of A/m remains positive. In this case, further increases in the value of A/m for the T(1,1) (or other modes with positive effective areas) will lead to an increase in the residual value of A/m, leading to a stronger high-frequency response. Such changes are perceived as 'brighter' (test pair numbers two and four). Similarly, a reduction in the contribution from modes whose effective areas are negative will cause the residual value of A/m to increase, and the sound is perceived as 'brighter' (test pair numbers nine and ten).

When the residual value of A/m is positive, but the output from modes with negative effective areas is increased, the residual value of A/m will decrease, leading to a weaker response at high-frequencies. This is confirmed by test pair number eight in which the effective area of all modes except the T(1,1) is increased. The resulting change in tone is perceived as 'more muffled' (i.e. less bright) because of the weaker high-frequency response. If the sound radiated by the modes with negative effective areas is increased sufficiently, the residual value of A/m will become negative. After this point, further increases in the value of A/m of the modes with negative effective areas will cause the value of the residual A/m to *increase*, leading to a stronger response at high-frequencies. To calculate the change in response at high-frequencies, the residual values of A/m must be calculated before and after the applied change. An increase in the residual value, regardless of changes in sign, will lead to a stronger high-frequency response and will produce a 'brighter' sound.

This argument is, of course, a simplified version of what actually occurs on the guitar. The contributions of multipole radiation fields at high-frequencies are overlooked, and the strength of the monopole fields is assumed to be independent of driving frequency. At frequencies of around 1 kHz, radiation fields produced by many top-plate modes can be very well approximated by monopole sources (Brooke, 1992). For higher driving frequencies, further research is needed to provide a clearer picture of the way in which the radiation fields of the low-frequency modes vary with frequency. The quantitative description provided by the model is only a first-order approximation to the behaviour of the real instrument. In spite of this, the model is able to point out many features of a guitar's tone quality that are affected by the properties of the low-frequency body modes. In particular, the magnitude and sign of the ratio A/m for the different body modes are found to have important consequences for the guitar's response, not only in the low-frequency region, but also at high-frequencies. The perceived 'brightness' or 'dullness' of the instrument is therefore determined, to some extent, by the properties of the low-frequency modes.

Changes to the resonance frequencies of the top-plate modes have little effect on perceived 'loudness'. When individual string partials coincide with a strongly radiating mode, changes to the frequency of that mode will result in a change in perceived 'loudness'. However, the change in 'loudness' is confined to string partials in the frequency region local to that body mode. String partials at other frequencies are unaffected. The results of the third test indicate that the judgement most commonly associated with changes in mode frequencies is 'more uneven'. This is probably due to strong coupling between the body mode and a coincident string partial of one of the notes in the test phrase.

7.6 Fourth listening test: guitarists' test

7.6.1 Aims

A fourth listening test was performed using classical guitar players as the test subjects. No deliberate selection of any kind was used for the previous three listening tests; volunteer subjects were sought from staff and students at the departments of physics and astronomy, and computer science. A reasonably large proportion of those who volunteered for the first three tests had some musical interest or played an instrument, but several subjects described themselves as having a poor musical ability. The main aim of a fourth test, specifically aimed at classical guitar players, was to see if their judgements differed significantly from those of a group with mixed backgrounds. Only a small number of subjects were used in this test. Three classical guitar players from the Welsh College of Music and Drama performed the test, in addition to the three members of the Cardiff musical acoustics research group (Dr. B. Richardson, M. Pavlidou and myself).

A large proportion of the test sounds used in the fourth listening test were identical to those used in the third, so that direct comparisons could be made between the judgements and comments of the two subject groups. The mode data for guitar BR2 was used to synthesise the majority of sounds in the fourth test, but a small number of sounds were synthesised using the mode data for guitar BR1. This allows a comparison between the differences in tone perceived when a given parameter change is applied to two different sets of mode data. Comparisons will also be made with the results of the second listening test in which the sounds were synthesised using the TOP26 mode data.

The preparation of the test sounds, and the judgements that the subjects were asked to make, were the same as those in the third listening test. Identical test pairs were again incorporated into the set of test sounds and subjects' answers were checked to ensure that low scores were given for these pairs. The test was divided into three parts, each part having a different ordering of the same 26 test pairs.

7.6.2 Results: magnitude of the changes in tone

The perceived difference scores for the two identical test pairs, when averaged over the the six subjects, were exactly zero. The results for the average scores for the other test pairs are given in Figure 7.13 and Table 7.9. Bold type has again been used in the tabulated data to highlight results for which there was greater than average agreement between the subjects.

From Figure 7.13, one can identify the test pairs that achieve high scores on both low and mid-frequency ranges; these are test pair numbers numbers two, three, four, six, seven and eight. Test pair number eight, the highest scorer, corresponds with the parameter change in which all top-plate modes above the T(1,1) are excluded and only the T(1,1) mode triplet contributes to the radiated sound. All other high-scoring test pairs involve changes to the effective masses and effective areas of the top-plate modes. The test pairs achieving the lowest scores, numbers one and five, correspond to changes in the resonance frequencies of the T(1,1) and T(1,2) modes.

Overall, the results for the fourth test, in which the subjects were all guitarists, are very similar to those obtained in the third listening test, in which the subjects were a mixture of musicians and non-musicians. The main finding is that changes to the effective masses and effective areas of the modes have a stronger influence on tone than changes to the mode frequencies. This is apparent for test phrases synthesised in the low- and mid-frequency ranges, indicating the global nature of the changes in tone caused by altering the effective masses and areas of the modes.

Figure 7.14 compares the average scores obtained by the two subject groups for the test pairs common to the third and fourth tests. Given the relatively large difference in musical training of the two groups, the difference in the average scores is surprisingly small. For the test sounds common to both listening tests, the results from the fourth test confirm the earlier findings and indicate that the perceptions of the ‘specialist’ group are essentially the same as for the ‘mixed’ group.

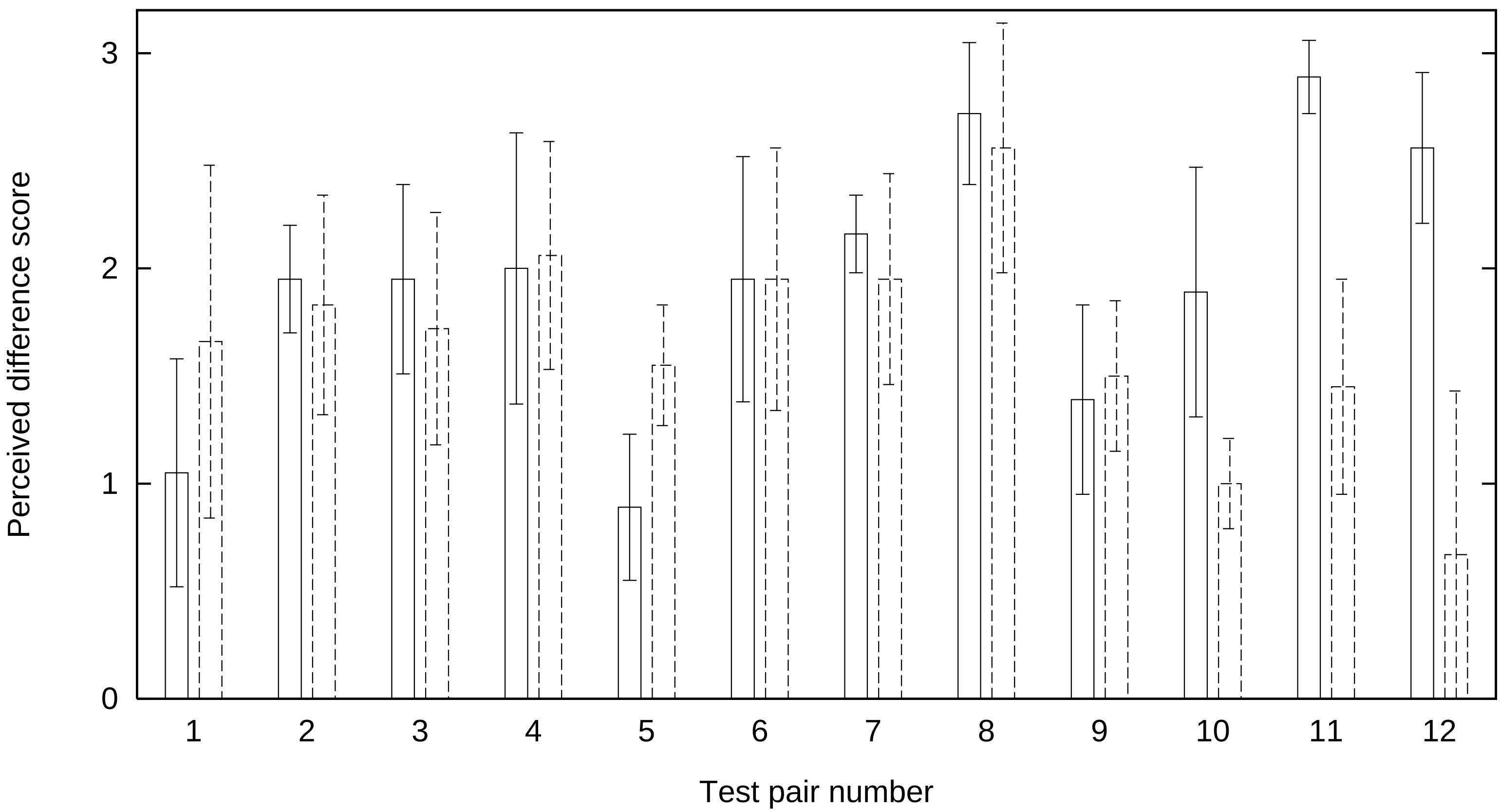


Figure 7.13: Results for listening test four showing the magnitude of the perceived differences in tone (0–3 scale) for changes to the top-plate modes. Parameter changes associated with each test pair number are given in Table 7.9. Different line types indicate sounds synthesised using string fundamentals in the frequency ranges 124–185 Hz (solid) and 330–494 Hz (dashed). Averages calculated for six subjects. Test pair nos. 1–9 synthesised using the BR2 mode data, nos. 10–12 used the BR1 mode data. Numbers used to identify the test pairs are arbitrary; no sequence should be inferred. Test pairs causing relatively small changes in tone include numbers 1 and 5 involving changes in mode frequencies.

Test pair number	Parameter change	Frequency range	Average perceived difference score	Standard deviation
1	f_t (1,1) −26%	Low	1.05	0.53
		Mid	1.66	0.82
2	m_t (1,1) ÷1.75	Low	**1.95**	**0.25**
		Mid	1.83	0.51
3	A_t (1,1) ×1.75	Low	**1.95**	**0.44**
		Mid	1.72	0.54
4	m_t (1,2) ÷4	Low	2.00	0.63
		Mid	2.06	0.53
5	f_t (1,2) −26%	Low	**0.89**	**0.34**
		Mid	**1.55**	**0.28**
6	A_t (1,2) ×4	Low	1.95	0.57
		Mid	1.95	0.61
7	A_t ×1.5 for all top-plate modes except (1,1)	Low	**2.16**	**0.18**
		Mid	**1.95**	**0.49**
8	Top-plate modes above (1,1) excluded	Low	**2.72**	**0.33**
		Mid	2.56	0.58
9	Top-plate modes above (1,2) excluded	Low	**1.39**	**0.44**
		Mid	**1.50**	**0.35**
10†	A_t ×1.5 for all top-plate modes except (1,1)	Low	1.89	0.58
		Mid	**1.00**	**0.21**
11†	Top-plate modes above (1,1) excluded	Low	**2.89**	**0.17**
		Mid	1.45	0.50
12†	Top-plate modes above (1,2) excluded	Low	**2.56**	**0.35**
		Mid	0.67	0.76

Table 7.9: Results for listening test four showing the magnitude of the perceived difference in tone (0–3 scale) for changes to the top-plate modes. Averages calculated for six subjects. The twelve results with the lowest standard deviation, indicating greater agreement between subjects, are highlighted in bold. Test pairs marked with a † used the BR1 mode data; all others used mode data for guitar BR2. High-scoring test pairs involve changes to the effective masses and effective areas of the modes, and the exclusion of modes above the T(1,1).

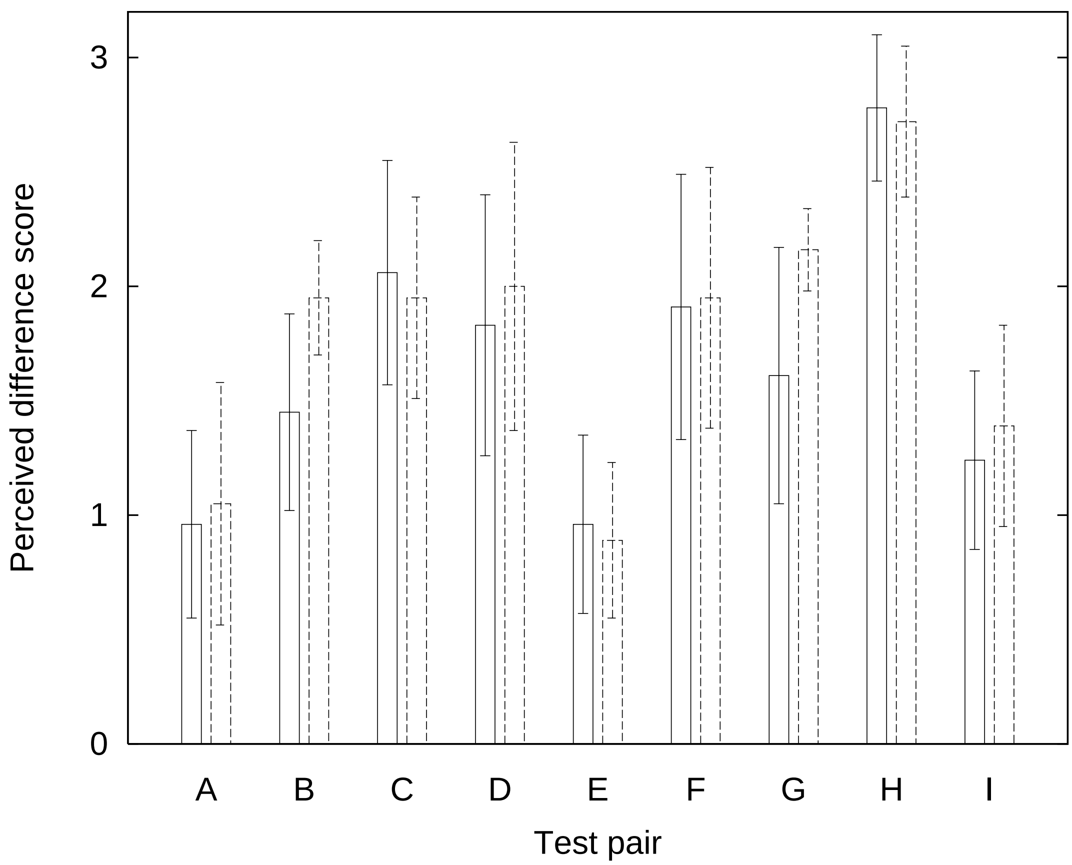


Figure 7.14: Comparison between average perceived difference scores of the subject group containing guitar players (dashed lines) and the subject group containing non-musicians and musicians (solid lines). The perceptions of the two groups are very similar for all test pairs. Results shown are for test pairs synthesised using notes in the low-frequency range. All test pairs involved alterations to either the resonance frequency, f_t, effective mass, m_t, or effective area, A_t, of one top-plate mode. Parameter changes associated with labels A–I are as follows. A: f_t (1,1) -26%, B: m_t (1,1) $\div 1.75$, C: A_t (1,1) $\times 1.75$, D: m_t (1,2) $\div 4$, E: f_t (1,2) -26%, F: A_t (1,2) $\times 4$, G: A_t $\times 1.5$ for all top-plate modes except (1,1), H: top-plate modes above (1,1) excluded, I: top-plate modes above (1,2) excluded.

Comparison between parameter changes applied to different sets of mode data

Test pair numbers seven, eight and nine involve parameter changes applied to the BR2 set of mode data. The same three parameter changes are used in test pair numbers ten, eleven and twelve, but applied to the BR1 mode data set. The results therefore give an indication of whether certain alterations to the mode parameters yield similar changes in tone quality when applied to different guitars (sets of mode data). With only three test pairs to compare, the evidence is not sufficient to justify generalisations. A comparison of the average scores for test pairs seven, eight, nine and ten, eleven, twelve shows that the magnitudes of the changes in tone show significant differences for the two sets of mode data. The implication is that a particular change to a single mode parameter may have a greater or lesser effect on tone quality, depending on the properties of the entire set of mode data.

It is possible to make a further comparison between results obtained using different sets of mode data by looking again at the results obtained in the second listening test, which used the TOP26 mode data, and the results from the third test, which used the BR2 mode data. There are four parameter changes common to the two tests, all but one of which were presented using notes in both the low- and mid-frequency ranges. Comparisons between the results obtained using the two sets of mode data are somewhat inconclusive because of the relatively large standard deviations. However, Figure 7.15 seems to confirm that the magnitude of the difference in tone, caused by a particular change to a single mode parameter, may differ significantly when that change is applied to different sets of mode data. This can be explained by using the modes' values of A/m as a measure of their 'sound-producing power'. If the effective mass of one mode is halved, but its original value of A/m is very low, the effect on tone quality will be slight. If, on another instrument, the value of A/m for the same mode is initially much higher, that mode has greater potential to influence the tone quality. If the effective mass is now halved, a more substantial change in tone will result. Similarly, differing values of A/m for modes other than the one to which the change is applied will cause the resulting change in tone to vary for different instruments.

It is important to realise that it is the magnitude of the change in tone quality, rather than the nature of the change, that may vary when an identical parameter change is applied to different instruments. In the next section I will discuss the results from the fourth test for

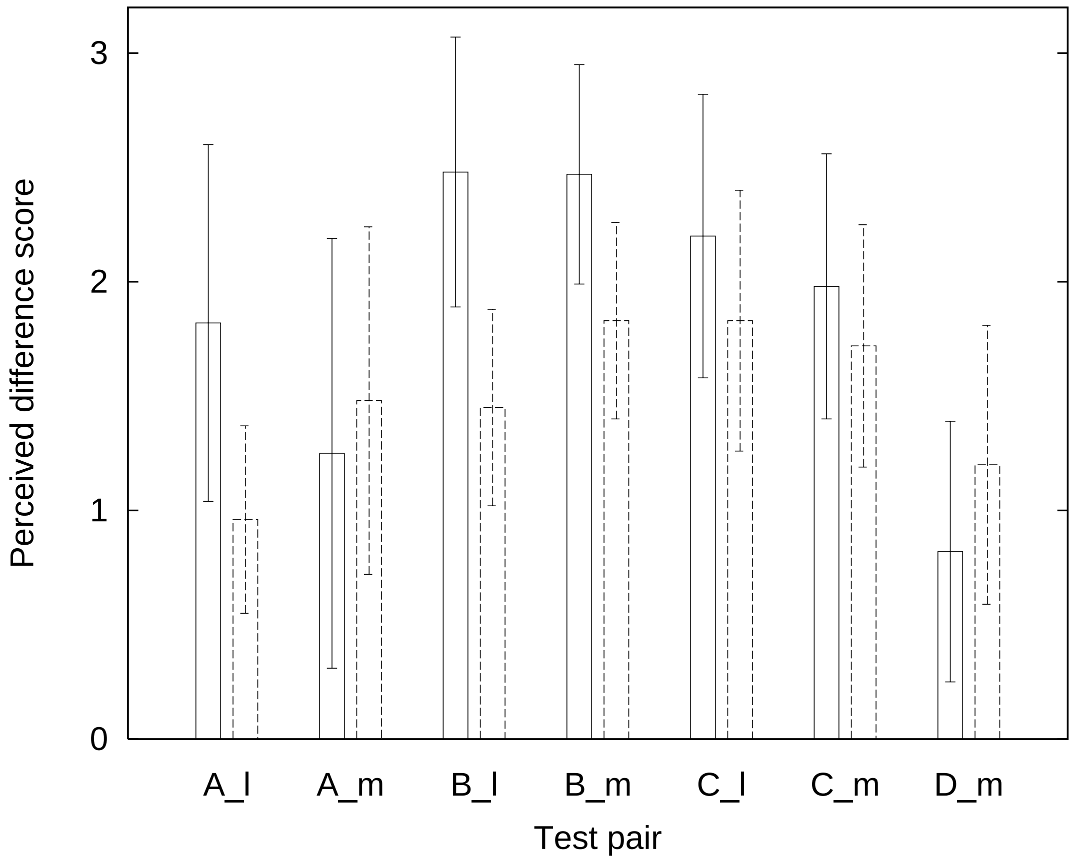


Figure 7.15: Comparison between average difference scores (and their standard deviations) obtained for tests two and three in which the test sounds were synthesised using different sets of mode parameters: TOP26 data (solid lines), BR2 data (dashed lines). Suffixes l and m are used to distinguish between test sounds which used notes with fundamentals in the low- and mid-frequency ranges. All test pairs involved alterations to either the resonance frequency, f_t, or the effective mass, m_t, of one top-plate mode. Parameter changes associated with labels A–D are A: f_t (1,1) -26%, B: m_t (1,1) $\div 1.75$, C: m_t (1,2) $\div 4$, D: f_t (1,2) -26%.

the nature of the perceived changes in tone. I will also show that the descriptions given by the subjects to describe the nature of the changes in tone seem to be similar for test sounds synthesised using different sets of mode-data.

7.6.3 Results: nature of the changes in tone

Results for the nature of the perceived differences in tone are given in Table 7.10. Totals for the six subjects were calculated for the number of times the five judgements ('louder', 'brighter' etc.) were used to describe the difference in tone for each test pair. I have again assumed that the judgements 'louder' and 'quieter' can be treated as opposite timbral attributes, and similarly with 'brighter' and 'more muffled'. For these judgements, circles have been used to highlight results for which there was a majority of at least 80% and a minimum score of five.

The main results are again very similar to those found in the previous test. When the value of A/m for the T(1,1) mode is increased (test pair numbers two and three), the sound is judged to be 'louder' and 'brighter'. This is apparent for notes with fundamentals in either the low- or mid-frequency ranges and emphasises the global nature of changes to a mode's value of A/m. The change is not confined to the region close to the mode's natural resonance, but the radiated level of the high-frequency partials is increased, accounting for the perception of increased 'brightness'.

Increases in the value of A/m of the T(1,2) mode are also perceived as giving 'louder' and 'brighter' sounds, despite the fact that the mode has a negative effective area. As discussed on page 183, this is because the change is sufficiently large that the residual value of A/m is increased, resulting in a stronger high-frequency response.

When all top-plate modes above the T(1,1) are excluded (test pair numbers eight and eleven), the sound is again judged to be both 'louder' and 'brighter'. This is because the excluded modes are those that have negative value of effective area. These normally radiate high-frequency sound with opposite phase to that of the T(1,1), causing a reduction in the high-frequency response. When they are excluded, the high-frequency response is strengthened, and the sound is judged to be 'louder' and 'brighter'. When modes above the T(1,2) are excluded (test pair numbers nine and twelve) the effect is less pronounced, because fewer modes are being excluded, but many people judged the sounds to be 'louder' and/or 'brighter'.

Changes in the resonance frequencies of the modes have little effect on perceived 'loudness',

Test pair number	Parameter change	Frequency range	Louder	Quieter	Brighter	More muffled	More uneven
1	f_t (1,1) −26%	Low	2	0	0	1	3
		Mid	1	0	0	0	2
2	m_t (1,1) ÷1.75	Low	(17)	0	(9)	0	1
		Mid	(16)	0	(8)	0	2
3	A_t (1,1) ×1.75	Low	(17)	0	(8)	0	1
		Mid	(11)	0	(10)	0	1
4	m_t (1,2) ÷4	Low	(13)	0	4	0	1
		Mid	(12)	2	(7)	0	(10)
5	f_t (1,2) −26%	Low	0	3	1	4	(7)
		Mid	0	(5)	0	(5)	(9)
6	A_t (1,2) ×4	Low	(13)	0	(6)	0	4
		Mid	(12)	1	0	3	(13)
7	A_t ×1.5 for all top-plate modes except (1,1)	Low	1	3	0	(17)	1
		Mid	0	(5)	0	(17)	(9)
8	Top-plate modes above (1,1) excluded	Low	(5)	0	(5)	1	0
		Mid	(6)	1	(11)	0	0
9	Top-plate modes above (1,2) excluded	Low	3	0	(8)	0	4
		Mid	1	1	(7)	0	(8)
10†	A_t ×1.5 for all top-plate modes except (1,1)	Low	(15)	0	(8)	0	0
		Mid	0	(5)	0	3	4
11†	Top-plate modes above (1,1) excluded	Low	(7)	1	(8)	1	0
		Mid	(12)	0	(12)	0	0
12†	Top-plate modes above (1,2) excluded	Low	(6)	0	(7)	0	0
		Mid	3	0	(7)	0	0

Table 7.10: Results showing the number of times the five judgements were used to describe the differences in tone quality resulting from changes to the top-plate modes. Totals given are for six subjects. Test pairs marked with a † used the BR1 mode data; all others used the BR2 mode data. Circles have been used to highlight results (for the 'louder'/'quieter' and 'brighter'/'more muffled' judgements) for which there was a majority of at least 80%, and a minimum score of five. For the 'more uneven' judgements, circles have been used to highlight scores greater than five.

although the mid-frequency phrase for test pair number five was judged to give a 'quieter' sound. Changes in the mode frequencies tend to give a sensation of a 'more uneven' sound because of changes in the coupling between the body mode and coincident string partials.

A comparison of the results for the parameter changes that were applied to the BR1 and BR2 mode data indicates that the perceptions are similar. There are only six test pairs to be compared (test pair numbers 7–12), each having been synthesised with notes in the low- and mid-frequency ranges. For the test pairs in which top-plate modes were excluded (numbers 8, 9, 11 and 12) the perceptions of a 'louder' and 'brighter' sound are common to both sets of mode data. The results for test pair numbers seven and ten, in which the effective areas of all modes above the T(1,1) are increased, show differences in tone that depend to some extent on the set of mode data used. Although both are perceived as 'quieter' in the mid-frequency range, test pair seven also scores significantly in the 'more muffled' category. In the low-frequency range, test pair seven is again judged as producing a 'more muffled' sound, but test pair number ten is judged as 'louder' and also somewhat 'brighter'.

Chapter 8

Discussion: mode properties and tone quality

8.1 Introduction

In this chapter I will use the results from previous chapters to outline a scheme by which certain aspects of the guitar's tone may be understood in terms of the properties of the guitar's modes of vibration. In this thesis, the body modes have been characterised using four parameters: the resonance frequency f, the effective mass m, the effective area A and the Q-value Q. The relationships between perceived tone quality and each of the four parameters are discussed.

I will start by examining the limitations of the current model and will continue with a brief summary of the results from the four listening tests. I will then discuss the influence of the four mode parameters on the sound pressure response of the guitar, paying particular attention to the amplitudes and decay rates of the string partials in the different regions of the spectrum. I will continue with a discussion of the ways in which alterations to the body modes affect perceptions of the guitar's tone quality.

Compromise is the fundamental concept when considering tone quality. One rarely has the freedom to choose from a menu of 'tone-quality options'. Many aspects of tone quality can be achieved only at the cost of other potentially desirable properties. For example, increasing the loudness of particular notes will often lead to a loss of sustain. Both are desirable qualities. The difficulty is in choosing the optimum balance between the two. The choice of the 'ideal' balance point is, of course, entirely subjective. This chapter therefore makes no attempt to define such an optimum in terms of preferred values of effective mass or resonance frequency for particular modes. The purpose of this discussion is to describe the effects on tone quality of specific changes to the mode parameters, and hence outline ways in which the the guitar's modal properties can be adjusted in order to control particular tonal qualities. The implications of this discussion for the guitar-maker are outlined at the end of the chapter.

8.2 Limitations of the model

Before discussing the relationship between tone quality and the mode parameters it is important to understand the limitations encountered when trying to model the response of the guitar using just four parameters to describe each body mode. The curve-fitting used in Chapter 5 has confirmed that the vibrational response of the guitar body is linear and can be accurately described by a summation of normal modes, each characterised by an effective mass m, a resonance frequency f and a Q-value Q. The comparison between synthesised velocity response curves for the coupled string-body system and the Fourier spectra of real plucked notes shows that the coupling between the string partials and the body modes is well described by the model, at least for frequencies below 1 kHz. At high frequencies there seems to be a tendency for the model to overestimate the height of the string partials in the response. Increased damping of the high-frequency partials due to fluid loading of the top plate may account for the lower measured response. More accurate modelling of the high-frequency radiation and the inclusion of fluid loading for the top-plate modes are needed to improve the predictive power of the model at high frequencies.

The main limitations of the model relate to the use of the effective area parameter, A. The use of this single parameter to model both the coupling between plate modes and the Helmholtz cavity resonance *and* the sound radiation from the body modes has already been shown to be unsatisfactory (Section 5.5). However, the effective area parameter provides a simple means of calculating the volume source of a mode whose amplitude of vibration is known at the driving point. Christensen (1980) and Caldersmith (1978) have already shown that the volume source of the driven T(1,1) mode accurately describes its interaction with the fundamental cavity mode. The use of a mode's volume source has also been shown to be effective in modelling the far-field sound pressure response of a combination of top-plate modes (Christensen 1984). Difficulties arise when a single value of effective area is used to calculate both the monopole sound radiation from a mode and its coupling to the Helmholtz cavity mode. The use of two separate values of effective area for each plate mode would provide a means by which the plate-cavity coupling and the sound radiation from the instrument could be calculated with greater precision. An alternative approach would be to use a separate parameter for 'radiation efficiency'. Two modes which produce the same volume source will not necessarily radiate with the same strength, since this depends on the amplitude distribution of the mode, and the proximity of antinodal areas to the edges of the top plate. By multiplying the calculated volume source of each mode by the corresponding value of radiation efficiency, a more accurate method of calculating the sound radiation could be achieved.

The effective area concept could be further extended to provide a means of calculating the variations in the sound radiation field in three-dimensional space. The current model assumes that the body modes radiate as monopole sources, so the radiation fields calculated do not have any directional dependence. By considering two monopole sources of opposite phase, separated by a distance d, dipole-type body modes can be modelled so that some directional dependence of the sound radiation fields can be included, as shown in Figure 8.1. For a symmetrically strutted guitar, modes such as the T(2,1) produce two volume sources of equal amplitude and result in a pure dipole radiation field (Figure 8.1a). For guitars with asymmetrical strutting, the T(2,1) mode produces two volume sources of different amplitude resulting in a radiation field with monopole and dipole components (Figure 8.1b). Two values of effective area would therefore be needed to calculate the two volume sources needed to model sound radiation from dipole-type modes. This idea could be extended to cover multipole radiation of order n by defining n volume sources each located at positions (x_n, y_n). The amount of extra processing time needed to calculate the sound radiation from multipole sources would be fairly small, but measurements of the n effective areas and the positions of the n sources would present some difficulties.

The incorporation of the above ideas, and the introduction of two or more values of effective area for each mode, provides a means to increase the predictive power of the model at little

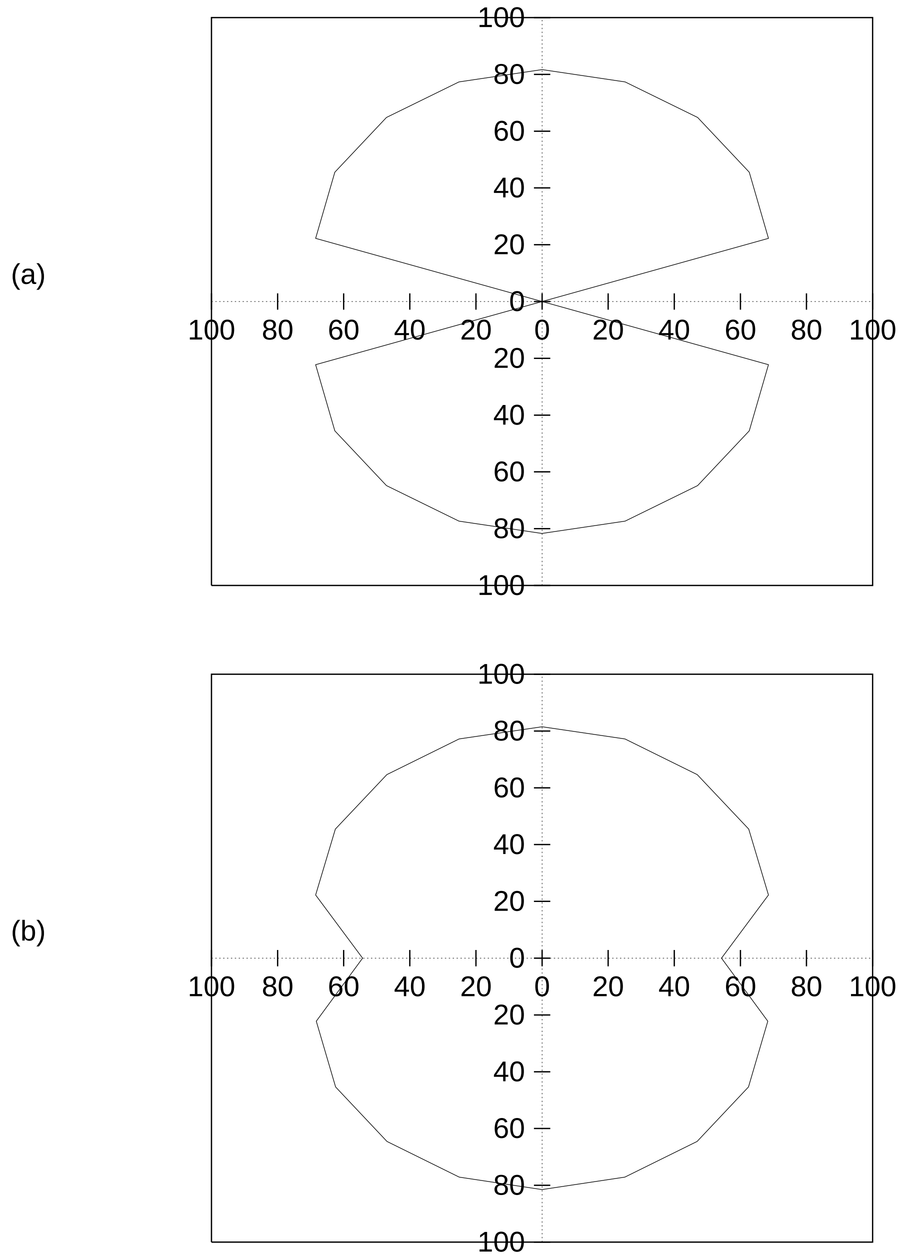


Figure 8.1: Polar plot showing the sound pressure levels, in dB, radiated by two volume sources of opposite phase, separated by a distance of 20 cm. Sound pressure levels calculated at 20 points around a circle of radius 2.5 m, centred on the guitar top plate for a 330 Hz string mode coupled to T(2,1) mode at 244 Hz. Radiation fields plotted as if looking down along the neck of the instrument; the x-axis corresponds to front-back axis of the guitar. Figure (a) corresponds to pure dipole field (two volume sources of equal magnitude); figure (b) corresponds to field with monopole and dipole components (one volume source 5% smaller than the other).

extra cost in processing time. I will, however, restrict this discussion to just four mode parameters, i.e. a single parameter for effective area.

Limitations relating to the way in which the string is modelled are mainly due to the inclusion of only one transverse mode of vibration. The modelled guitar body can be driven in one direction only, so the coupling between string and body is calculated only for string motion polarised in the 'vertical' direction (perpendicular to the top plate). On real instruments, the existence of both horizontally and vertically polarised transverse string modes results in more complex patterns of vibration of the string, some of which have significant consequences for the tone of each note. The coupling of the two polarisations of transverse vibration, via changes in string tension, leads to a precession of the string orbit (Gough, 1984) and a cyclical exchange of energy between the horizontal and vertical string vibrations, which results in a tone quality that is constantly evolving as the note decays. The current model does not include such evolutionary features of the guitar's tone quality.

For string notes which couple strongly with the body, for example, notes which contain string partials that coincide with the $T(1,1)_2$ mode, the current model may have a tendency to exaggerate the unpleasant tonal characteristics that result from such strongly coupled notes. The modelled string is confined to vibrate in the vertically polarised plane, which will result in the strongest coupling to the body. When such strongly coupled notes occur on a real instrument, the more weakly coupled horizontal string modes will not be significantly perturbed by the motion of the body and will help to fill in the sound and produce a somewhat longer sustain. The inclusion of the horizontally polarised string modes in the model could be achieved if values could be found for the effective masses of the body modes when driven in the direction parallel to and across the top plate, but the inclusion of coupling between the horizontal and vertical string modes via changes in string tension is a much more difficult task.

Before discussing the relationship between mode properties and tone quality I will summarise the important results obtained from the listening tests.

8.3 Summary of listening test results

The first listening test investigated the influence on tone quality of the four mode parameters (frequency, effective mass, effective area and Q-value) as applied to the fundamental modes of the top plate, back plate and air cavity. Subjects were played pairs of synthesised chords and were asked whether or not they perceived a difference in tone between the two chords. Although not every parameter change was investigated, the results show that the top plate is the primary influence on the tone quality. The results show that changes to the values of the resonance frequency, effective mass and effective area of the T(1,1) mode all have a significant influence on the sound quality. The influence of the Helmholtz air-cavity mode was found to be important only when the fundamental frequencies of the string notes were sufficiently low. Changes to the frequency of the Helmholtz mode show a moderate influence on the tone quality for notes in the E major chord, where the lowest fundamental frequency was 82 Hz. The same changes for the D major chord, where the lowest fundamental frequency was 147 Hz, were not perceived by any of the subjects. Changes to the Q-value of the T(1,1) mode, and the resonance frequency and effective mass of the B(1,1) mode were all found to have virtually no influence on tone quality.

In the second test the relative effects on tone quality of the various parameter changes were measured by asking subjects to measure the differences in tone on a 0–3 scale. Changes to the resonance frequencies and effective masses of three top-plate modes and the fundamental back-plate mode were investigated. Alterations to the frequency of the Helmholtz mode and the volume of the air cavity were also tested. Their effects on tone quality were investigated for synthesised notes with fundamental frequencies covering three regions of the spectrum. The results show that changes to the effective masses of the top-plate modes have a greater effect on tone quality than changes to the modes' resonance frequencies. This result is true for notes with both low- and high-frequency fundamentals. Changes to the effective mass of the T(1,1) mode were found to have the greatest influence on tone.

The results also confirm some of the earlier findings from the first listening test. Changes to the effective mass and frequency of the fundamental back-plate mode have little influence on sound quality, and changes to the frequency of the Helmholtz mode show moderate influence on tone quality for notes with low-frequency fundamentals.

Analysis of the results from the second test also showed that changes to the frequencies of the body modes only affect string partials in the local frequency region close to the body resonance. Changes to the effective masses of the body modes have a global effect on tone quality, in which string partials covering a wide frequency range are affected.

The third and fourth listening tests investigated the nature of the differences in tone brought about by the parameter changes, as well as the magnitude of the differences. The results confirm that the effective masses have a greater influence on tone quality than the mode frequencies, but also show that changes to a mode's effective area have a slightly stronger effect on tone than numerically equivalent changes to the mode's effective mass. The results confirm that the Q-value of the T(1,1) mode has very little influence on tone quality.

Changes to the value of A/m for the body modes were found to affect perceptions of 'loudness' and 'brightness'. Increasing the value of A/m for modes with positive effective areas, or reducing the output of modes with negative effective areas, results in a 'louder' and 'brighter' sound. A reversal of the sign of the T(1,2) effective area also produces a 'louder' and 'brighter' sound. Increasing the value of A/m for the modes with negative effective areas can cause perceptions of reduced 'loudness' and 'brightness' if the increases are relatively small. Larger increases cause the sound to be perceived as 'louder' and/or 'brighter'. Changes to the resonance frequencies of the modes have little effect on perceived 'loudness' or 'brightness' but have a greater influence on the 'evenness' of the guitar's tone.

8.4 Influence of the mode parameters on the response of the guitar

In the model described in Chapter 4, each body mode is characterised by four parameters: the effective mass, the effective area, the resonance frequency and the Q-value. The influence that changes to these four parameters have on the physical response of the guitar is now discussed. The implications of these physical changes for the tone quality will be described afterwards in Section 8.5.

Effective mass

The amplitude of vibration of a mode is primarily determined by the driving force applied and by the mode's effective mass at the driving point. The effective mass can be interpreted as representing the ease with which the mode can be driven into vibration. A lower effective mass leads to a larger amplitude of vibration. The resonance frequency and Q-value have some influence on a mode's amplitude of vibration. Changes in the resonance frequency of the mode will alter the vibration amplitude in the vicinity of the resonance, but alterations to the effective mass change the vibration amplitude at all frequencies.

It is important to remember that the effective mass of a mode may vary significantly for different driving positions. When a driving force is applied at a point close to a nodal line, the effective mass will be high and the mode is weakly driven. When the driving position is closer to an antinodal region, the effective mass will be lower, and the mode is driven more strongly. The coupling between the strings and any one body mode will therefore vary for each string. The T(2,1) mode, for example, usually has a nodal line running perpendicular to and through the centre of the bridge. The D and G string positions on the bridge will be close to the nodal line and the effective masses at these points may be as much as 1 or 2 kg, resulting in poor string-body coupling. The top and bottom E string positions are closer to the antinodal regions of the mode. The effective masses at these points are usually low (around 200 or 300 g), giving relatively strong string-body coupling. A plucking force applied to the top E string will drive the T(2,1) mode effectively but the same force applied to the G string will drive the mode rather ineffectively.

A change in a mode's effective mass affects the amplitude of vibration of the body, but will only affect the sound pressure response to a significant degree if the mode radiates relatively strongly, i.e. its effective area is reasonably large. For a strongly radiating mode, a reduction in its effective mass increases the string-body coupling and will result in stronger radiation of the string partials, as well as causing a more rapid decay. If the frequency of one of the string partials coincides with a body mode with low effective mass, the string-body coupling may be so strong that the string's energy is radiated very quickly giving a loud sound with short sustain. If the effective mass of the mode is sufficiently low, the coupling between string and body produces a split resonance, imparting a dissonant 'out of tune' character to the note.

One of the most important results regarding changes in a mode's effective mass is the effect it has on string modes at different frequencies. The results of the second and third listening tests show that changes to the effective mass of a mode have a global effect on tone quality. At frequencies above the natural resonance of a body mode, the vibrational behaviour of the mode is mass-controlled. The amplitudes of all string partials at frequencies above the body resonance are affected by changes to the mode's effective mass, as illustrated in Figure 8.2. The change in the T(1,1) effective mass affects the amplitudes of the seven string partials by between 2 and 10 dB. The solid line response, corresponding to the T(1,1) mode with lower effective mass, has a greater amplitude for most of the string peaks. Two string peaks have a lower amplitude; this is due to near-coincidence of the partials with body modes at 436 and 674 Hz. Both body modes have negative effective areas and radiate sound at the partial frequencies with opposite phase to the T(1,1). The negative-phase contributions from the two modes will dominate at the frequencies of the two partials because the modes are being driven close to resonance. When the effective mass of the T(1,1) is lowered, the positive-phase contributions it makes are increased and the net sound pressure level at the frequencies of the two partials is decreased.

The main effect on the sound pressure response of a change in the effective mass of a mode is a change in the amplitude of the string partials. Changes in the effective mass of the T(1,1) mode also affect the coupling to the fundamental modes of the air cavity and back plate and will cause changes in the resonance frequency of the coupled T(1,1) modes.

Effective area

The effective area of a mode determines the efficiency with which the mode converts its vibrational energy into radiated sound waves. A mode with a large effective area will couple strongly with the surrounding air and will tend to produce a high sound pressure level at a given listening point. The reason that it will not *necessarily* radiate strongly is because the radiated sound pressure depends on values of the effective area *and* effective mass, as already mentioned above. Put simply, a mode can only radiate strongly if it has vibrational energy available to convert into sound energy. Using the language of mode parameters, we say that a mode with a large effective area will radiate a high sound pressure level only if the mode's effective mass is sufficiently low. A change in a mode's effective area will therefore produce a significant change in radiated sound only if the mode's effective mass is relatively low. In order to simplify this aspect of the discussion, I will continue to use the ratio of effective area to effective mass, A/m, as a convenient single measure of a mode's sound-producing capacity, following Christensen (1984). I will also continue to use the phrase 'strongly radiating mode'

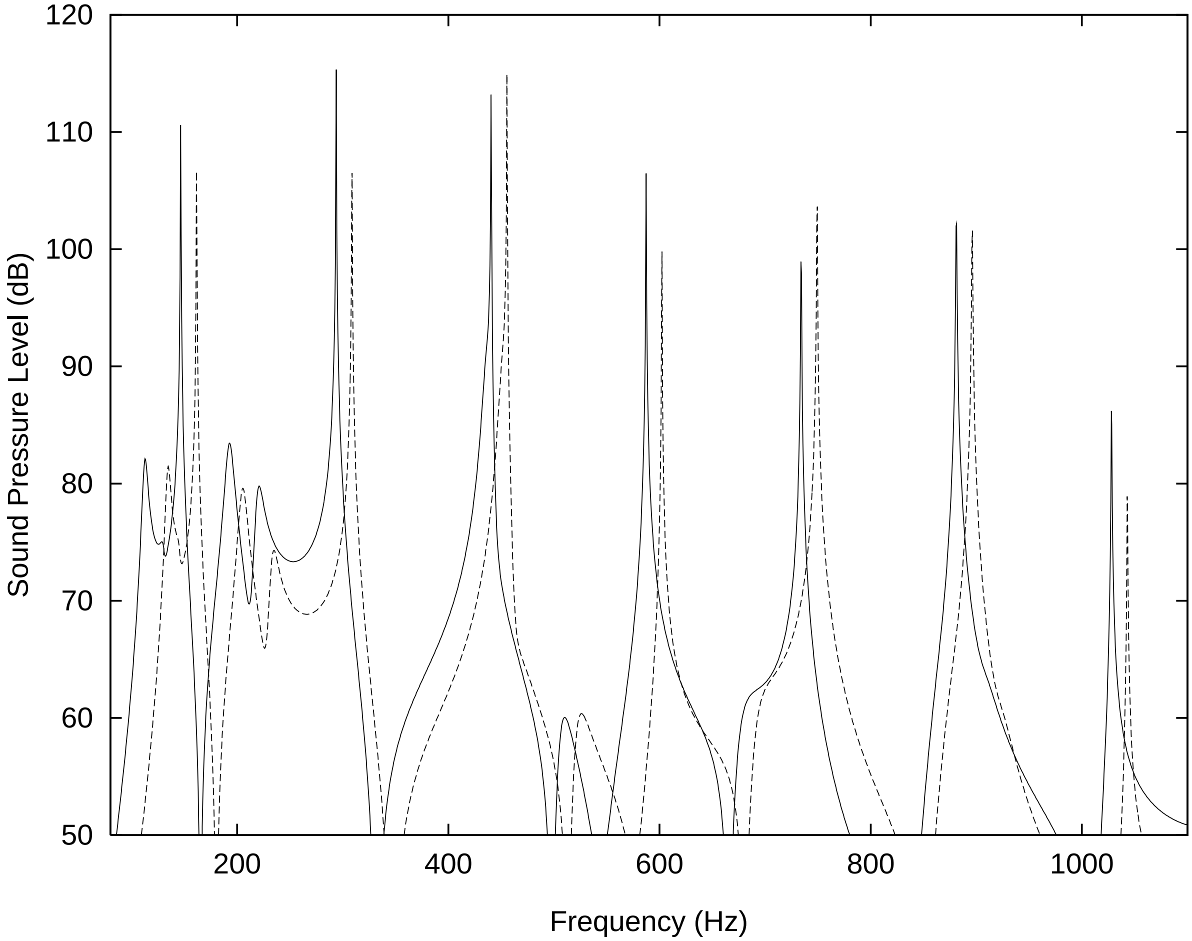


Figure 8.2: Effect on the guitar sound pressure response of a change in the effective mass of the T(1,1) mode at 190 Hz. Solid line: m=60 g, dotted line: m=100 g. The dotted line response has been displaced by 15 Hz to allow easier comparison of the relative heights of the string peaks. Note that the amplitudes of all string partials are affected by the change in effective mass of the T(1,1). Most string peaks in the 60 g response are higher by between 2 and 10 dB. The third and fifth partials are lower in the 60 g response by 2 and 5 dB respectively because of contributions of opposite phase from body modes at 436 and 674 Hz (see text).

as a convenient shorthand for 'mode with a large value of A/m'.

Changes to a mode's effective area, like changes to the effective mass, have a global effect on the guitar's sound pressure response. The amplitudes of string partials in the vicinity of the mode and partials at frequencies well above the resonance are all affected (see Figure 7.11). This gives the effective area and effective mass parameters a particularly important role in determining tone quality since they can affect the amplitude of the guitar's response over a wide range of frequencies. Changes to the effective area of the T(1,1) affect its coupling to the fundamental air cavity and back-plate modes, and will cause small changes in the frequencies and amplitudes of the coupled T(1,1) modes.

The simplified way in which the current model treats the process of sound radiation means that the physical mechanisms associated with fluid loading of the guitar body are overlooked. The problems associated with modelling the fluid loading of the body modes as that of a circular piston in an infinite coplanar baffle have already been discussed in Section 4.5. The difference between the value of a mode's effective area and the physical area of the plate that vibrates means that the fluid loading scheme could not be used successfully for modes other than the T(1,1) triplet. The increase in the radiated sound pressure that occurs when a mode's effective area is increased will tend to be over-estimated by the model because the fluid loading, which would increase with a larger effective area, is not included. The increased fluid loading of the mode would slightly reduce the mode's Q-value; this effect is also overlooked by the model. The trend in behaviour for real instruments, however, is the same: a larger effective area leads to a stronger sound pressure response over a wide range of frequencies.

Resonance frequency

Changes in the frequency of a mode are most noticeable when that mode is also a strong radiator, i.e. when it has a large value of A/m. The change in mode frequency may cause a change in pitch of the body transient, and it may also cause differences in the radiated spectrum of string partials. If a particular string mode lies close to a strongly radiating body mode, the string mode will feature prominently in the radiated sound. If the frequency of the body mode is changed, the string mode will no longer be close to it and the coupling between string and body will decrease. The amplitude of the string mode in the radiated sound spectrum may fall by as much as 10–15 dB, as shown in Figure 8.3(a). For string modes whose frequencies are not so close to the body mode in the initial case, the change in the body-mode frequency will have a much smaller effect, as in Figure 8.3(b). In general, a change in the frequency of a strongly radiating body mode will have a significant effect on the radiated levels of only a small number of string partials whose frequencies are in the local region close to the body mode.

Q-value

The radiation of string partials is only marginally affected by the Q-values of the body modes. Even for notes in which a string mode is almost coincident with the T(1,1) mode, a doubling of its Q-value changes the radiated sound pressure level of the string mode by less than 1 dB. The amplitudes of string partials at frequencies some distance from the body mode are unaffected by changes in its Q-value. In the case of strong coupling between the string and the T(1,1) mode, the mode's Q-value may be more important because the decay rate of the strongly coupled, split resonance is determined primarily by the Q-value of the body, not the string. Changes to the Q-value of a body mode will cause a change in the decay rate of the mode's contribution to the body transient, although this will only be significant for strongly radiating modes.

Summary

The amplitude of the guitar's sound pressure response is strongly influenced by the effective masses and effective areas of the body modes. The effective mass can be seen as determining the ease of input of energy into the body, while the effective area determines the ease of output of energy as sound. For the mode to contribute strongly to the radiated sound a good input (low effective mass) and good output (high effective area) are required. Altering the value of A/m for one body mode has a global effect on the response, influencing string partials close to the body mode as well as those at frequencies well above the body resonance. Changes to a mode's value of A/m can cause the amplitudes of some string partials to be increased while others are decreased, due to contributions to the radiated sound from modes with both positive and negative effective areas.

Reductions in the effective masses of the body modes increase the string-body coupling, which affects both the amplitude and decay of the string partials. For string partials which

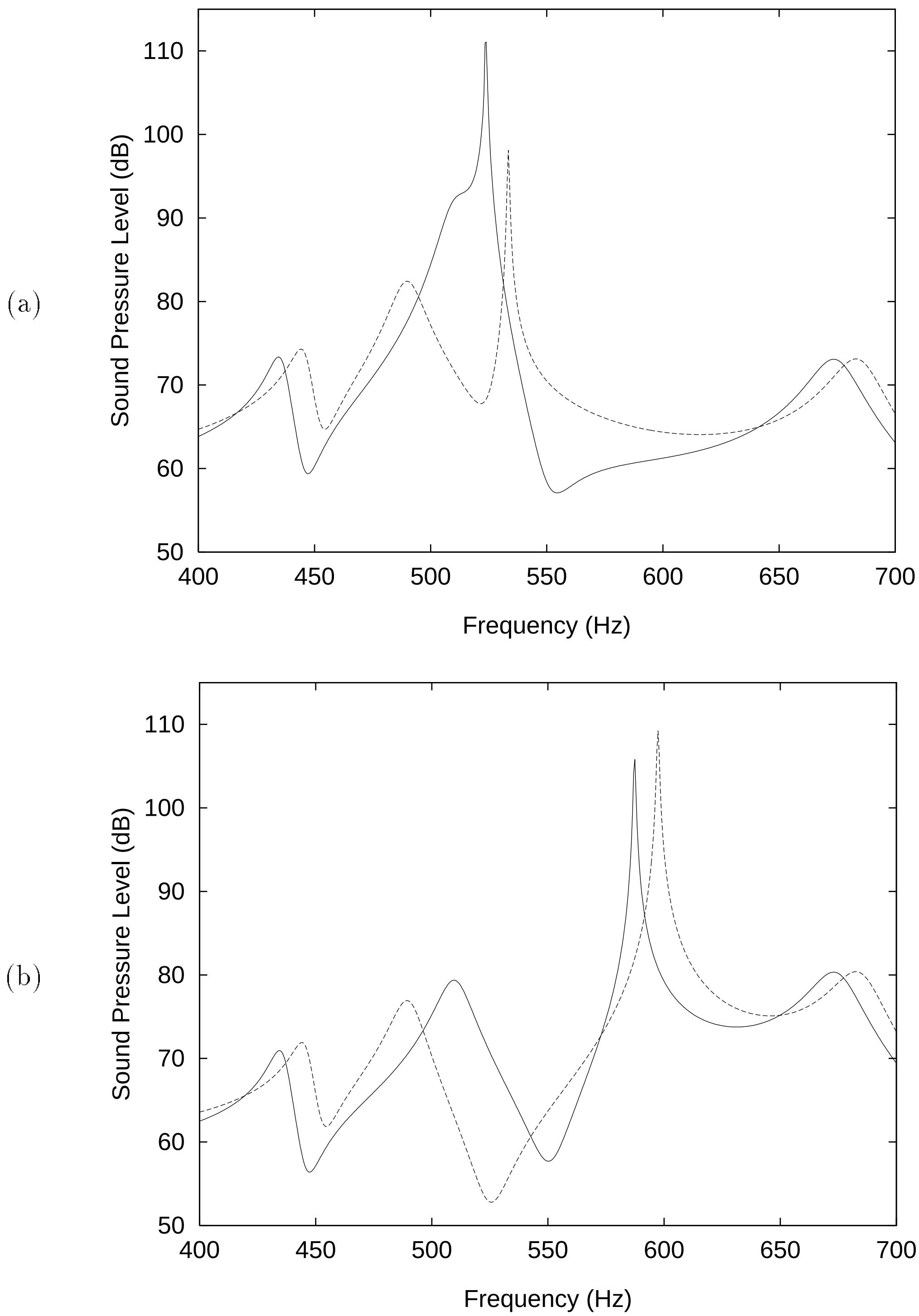


Figure 8.3: Change in the radiated sound pressure of two string modes due to a 30 Hz reduction in the frequency of the body mode at 510 Hz. The dotted lines correspond to the responses with the lower mode frequency, and have been displaced by 10 Hz to allow comparison of the string-mode amplitudes. String fundamental frequencies: (a) 523 Hz (note C_5), (b) 587 Hz (note D_5). In (a), the amplitude of the string mode drops by 13 dB. In (b), the fundamental frequency of the string is one tone higher but the change in amplitude is only 3 dB.

coincide with the $T(1,1)_2$ body mode, a loud note with rapid decay and strongly inharmonic partials may be produced.

Changes in mode frequency have an effect on the radiated sound that depends on two factors: the proximity of string modes to the body mode, and the value of A/m for that mode. Figure 8.3(a) shows that if a string mode coincides with a strongly radiating body mode, a large change in the radiated amplitude of the string mode may occur when the frequency of the body mode is altered. For a string mode that does not coincide with the body mode in the initial case, the change in sound pressure is considerably smaller, as shown in Figure 8.3(b). The influence on the sound pressure response of changes to mode frequencies is confined to the local frequency region close to the mode. Changes to the Q-values of body modes have a negligible effect on the radiated sound of the string partials, but may affect the decay rate of the body transient.

8.5 Influence of the body modes on tone quality

Now that the physical implications of changes to the four modal properties have been outlined, the effects on tone quality of specific changes to individual body modes can be discussed. The aspects of tone quality discussed relate to the absolute and relative amplitudes of the string modes in the sound pressure response and their rate of decay. The body transient will also make some contribution to the overall perceived tone quality of a note. It's influence is determined by the amplitude, frequency and Q-value of the strongly radiating body modes.

8.5.1 Fundamental air-cavity mode: $T(1,1)_1$

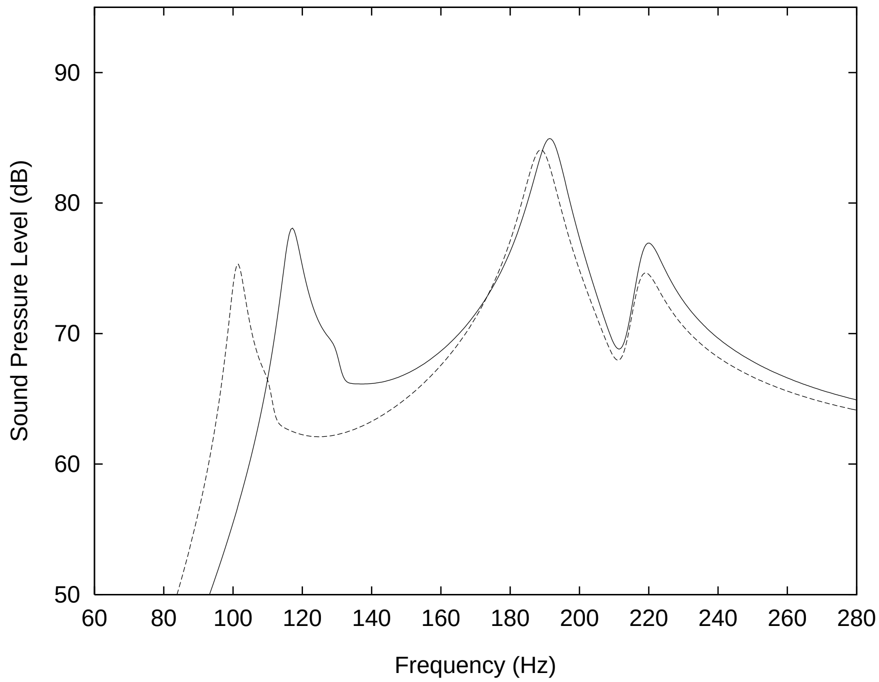
The fundamental air-cavity mode, or Helmholtz mode, has a frequency in the region of 120–130 Hz in the absence of coupling to the modes of the top and back plate. In the fully-coupled system, the Helmholtz mode interacts with the fundamental top-plate mode to produce two resonances: the $T(1,1)_1$ and $T(1,1)_2$. The lowest resonance, the $T(1,1)_1$, is usually found at frequencies close to 100 Hz. The data presented in Table 7.3 indicates the range of values found for the $T(1,1)_1$ frequency in different instruments; the lowest value is 91 Hz and the highest is 117 Hz. The reflex action of the Helmholtz mode helps to extend the bass response of the instrument below the frequency of the fundamental top-plate mode and results in a stronger string fundamental for notes at the lower end of the instrument's range. The fundamental frequency of the low E string of a guitar is 82.4 Hz, although it is relatively common for guitar music to call for altered tunings, in which case the lowest note may drop to a D at 74 Hz. Without the reflex action of the Helmholtz mode, these low frequencies would be poorly radiated.

As the frequency of the Helmholtz mode is lowered there is a trade-off between a stronger response in the vicinity of the $T(1,1)_1$ resonance, and a weaker response between the $T(1,1)_1$ and $T(1,1)_2$ resonances. This is illustrated in Figure 8.4, which shows the effect on the sound pressure response of a 20 Hz reduction in the Helmholtz frequency. The response of the guitar body is given in Figure 8.4(a) and the response of the low E string of the guitar coupled to the body is given in Figure 8.4(b). The reduction in the Helmholtz frequency gives a note with a stronger fundamental but a weaker second and third partial. This opens up the question of what a 'stronger bass response' actually means. It might seem reasonable that the response in Figure 8.4(b) with the lower Helmholtz frequency, and hence the stronger string fundamental, produces the sound with 'more bass'. On comparing the sound of the two notes, I perceived the sound with the *higher* Helmholtz frequency to be rather 'more bassy' (this judgement was confirmed by another listener). This is perhaps because the increased level of the second partial (at 165 Hz) has greater perceptual influence than the reduced level of the string fundamental, which is some 25 dB lower. This example serves to underline the importance of actually listening to the sounds produced by the model. The apparently 'logical' interpretation of the physical data may be misleading.

The amplitude of the $T(1,1)_1$ peak in the sound pressure response, as well as its frequency, will be important in determining the strength of the instrument's bass response. The peak amplitude may be difficult for the guitar maker to control as it is determined partly by the frequency and Q-value of the uncoupled Helmholtz mode, but also by the coupling between the Helmholtz mode and the fundamental modes of the top and back plate. The value of A/m for the $T(1,1)_2$ mode will have the greatest influence on the height (and frequency) of the $T(1,1)_1$ peak, so an instrument with a strong $T(1,1)_2$ peak will tend to have a strong $T(1,1)_1$ peak.

The influence of the $T(1,1)_1$ mode on the radiated spectrum of string partials is largely restricted to notes with low-frequency fundamentals. Changes to the frequency of the mode

(a)



(b)

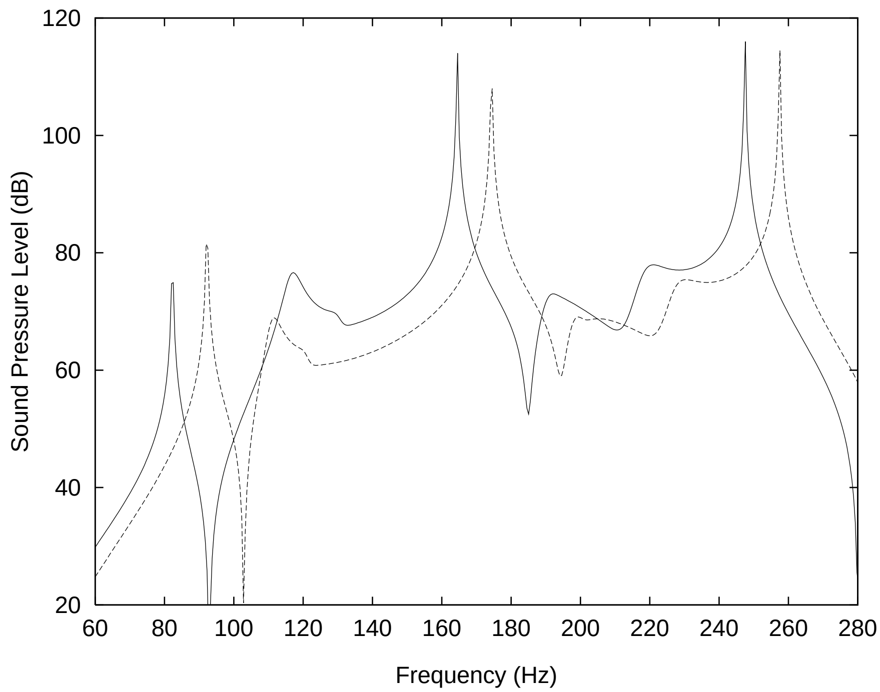


Figure 8.4: Changes in the radiated sound pressure resulting from a 20 Hz reduction of the Helmholtz frequency. The responses with the lower Helmholtz frequency are indicated by dotted lines. The response of the guitar body is given in (a). In (b), the response of a low E string coupled to the body is shown; the dotted line response has been displaced for clarity. In (a), the lower Helmholtz frequency gives a stronger response between 90 and 110 Hz, but a weaker response at frequencies above 110 Hz. In (b), the lower Helmholtz frequency increases the height of the string fundamental at 82 Hz by 6.5 dB, but reduces the height of the second and third partials by 6 and 1 dB respectively.

have little effect on the tone of high-frequency notes (Section 7.3.3). The main function of the $T(1,1)_1$ mode is to provide stronger radiation of string partials in the 80–200 Hz region, and thus to provide a better bass response. A lower frequency for the Helmholtz mode does not necessarily imply a stronger bass response. The action of the guitar at these low frequencies, and hence the perceived strength and quality of the bass response, is determined by the coupling between the T(1,1) mode and the Helmholtz mode.

8.5.2 The $T(1,1)_2$ mode

The fundamental top-plate mode, the T(1,1), couples to the fundamental air-cavity mode to give rise to the $T(1,1)_2$ mode in the fully coupled response. The $T(1,1)_2$ mode is characterised by a low effective mass and a large effective area. It's value of A/m is usually between three and ten times greater than that of the other top-plate modes making it the strongest radiator of sound energy of all the body modes.

One of the important features of its behaviour, confirmed by results from the listening tests, is that the $T(1,1)_2$ mode is the dominant radiator not only at frequencies close to its natural resonance (typically around 200 Hz) but also at frequencies well above its natural resonance. This result was first highlighted by Brooke (1992) who used the boundary element method (BEM) to show that, at a driving frequency of 990 Hz, the $T(1,1)_2$ mode radiated more strongly than modes with resonance frequencies much closer to 990 Hz. The results from the listening tests show that changes to the effective mass and effective area of the $T(1,1)_2$ mode have profound effects on tone quality, not only for notes close to the $T(1,1)_2$ resonance, but also for notes with high fundamental frequencies. The $T(1,1)_2$ mode therefore has a crucial tone-determining role for a large part of the guitar's playing range.

The frequency at which the $T(1,1)_2$ ceases to be the dominant radiator is not precisely known. Brooke (1992) used the BEM to calculate the radiation field of the $T(1,1)_2$ mode when driven at a little under 2 kHz. The BEM solutions are less reliable above 1 kHz, so some caution must be applied in interpreting the results. However, the mode is still seen to radiate strongly to the front of the instrument, although the increased directionality of the radiation field is apparent. In this thesis, the radiation fields of the modes are assumed to be frequency-independent. Up to a frequency of 1 kHz this is a reasonable assumption. Beyond 1 kHz it is not clear how the strength and directionality of the fields change, nor is it clear at what frequency multipole radiation from the high-frequency modes begins to be more significant than the radiation from modes such as the T(1,1)$_2$. More work is needed to investigate the frequency dependence of the radiation fields of the low-frequency (monopole and dipole) and high-frequency (multipole) modes.

Brightness

Although rather little data has been published on effective masses and areas of guitar top-plate modes, it seems that most modes above the T(1,1)$_2$ are characterised by negative values of effective area (Christensen, 1984; see also Tables 8.1 and 8.2). At high frequencies, the radiation from modes with negative effective areas is out of phase with the sound radiated by the T(1,1)$_2$. This suggests that the high-frequency response of the guitar, and hence aspects of tone quality such as 'brightness', are strongly affected by the relative radiation strengths of the T(1,1)$_2$ and higher-frequency modes. Guitars which have large values of A/m for the T(1,1)$_2$, but relatively small values of A/m for modes such as the T(1,2) and T(3,1) will have a large positive residual value of A/m and will radiate high-frequency partials relatively strongly, producing a 'bright' tone. This effect was confirmed in the third and fourth listening tests in which the sound radiated by the T(1,1) triplet alone was judged to be 'brighter' than the sound radiated by the full compliment of modes. In addition, the test sound which had the effective areas of the modes above the T(1,1) increased was judged to be 'more muffled' (i.e. less bright), since the increased out-of-phase radiation from modes above the T(1,1)$_2$ caused a reduction in the high-frequency response. Alterations to the value of A/m of the T(1,1)$_2$ mode will therefore have significant effects on the perceived 'brightness' of the guitar's tone.

Loudness

As well has having an important role to play in determining the perceived 'brightness' or 'dullness' of notes, the T(1,1)$_2$ is the mode which has the strongest influence on the loudness of the instrument. It has the largest value of A/m and therefore makes the greatest contributions to the radiated sound over a considerable range of frequencies.

Care must be taken when interpreting perceived loudness because it depends not only on the initial amplitude of the plucked note, but also on its rate of decay. For example, a note with a given initial amplitude and decay rate may be perceived as louder than another note which has a greater initial amplitude and a faster decay rate. The amplitude of string peaks in the guitar's sound pressure response does not therefore correspond directly with perceived loudness.

Altering the value of A/m for the $T(1,1)_2$ mode affects the loudness of notes with low and high-frequency fundamentals (Section 7.5.3). Changes to the effective mass, however, have a slightly different influence on loudness than changes to the effective area. The effective mass of the $T(1,1)_2$ mode affects the coupling between string and body and will therefore influence both the amplitude and decay-rate of many string partials. The effective area of the $T(1,1)_2$ affects only the radiated amplitude of the partials. The amplitude of a note can therefore be increased by a reduction in the $T(1,1)_2$ effective mass, or by an increase in the mode's effective area. The reduction in effective mass will also cause a more rapid decay (shorter sustain). Results from the third and fourth listening tests show that the perceived increases in loudness are very similar for changes to either the effective mass or effective area (Table 7.8).

The $T(1,1)_2$, having the largest value of A/m, has the greatest influence on the loudness and sustain of an instrument. Coupling between string partials and the $T(1,1)_2$ mode will be strong for most partial frequencies because of the mode's low effective mass. When the frequencies of the partials are close to other body modes coupling to these modes may be stronger than the coupling to the $T(1,1)_2$ because of the near-coincidence of body mode and partial frequencies. For the majority of partials, coupling to the $T(1,1)_2$ will be the strongest, and the loudness and sustain for most notes is therefore strongly influenced by the effective mass (and effective area) of the $T(1,1)_2$ mode.

The guitar wolf-note

When string partials coincide with the $T(1,1)_2$ mode, they will couple strongly with it because of its low effective mass. The resulting note will be loud but will decay rapidly. The strong coupling also perturbs the frequencies of both the body mode and the string partial, producing a split resonance. The resulting increase in string inharmonicity can give the note a rather unpleasant sound. All guitars will suffer from one or two so-called wolf-notes, which may occur when either the fundamental or second mode of the string coincides with the body mode. The frequency of the $T(1,1)_2$ determines the notes at which the strong coupling occurs, the effective mass of the mode determines the strength of the coupling and therefore the 'badness' of the note. The 'badness' of the note is also affected to some degree by the mass of the vibrating portion of the string. A lighter string will cause a more severe wolf-note.

Figure 8.5 illustrates the guitar wolf-note phenomenon and shows the response of string partials at a variety of frequencies coupled to the $T(1,1)_2$ mode. As the partial frequencies approach the body resonance at 190 Hz, the coupling increases and the amplitude of the coupled string peak increases. When the partials are about 15 Hz from the body mode, the coupling increases causing their amplitudes to fall slightly and their damping to increase. The decrease in the Q-value of the two peaks either side of the central peak is clear. When the partial frequency coincides with the $T(1,1)_2$ mode, the coupling produces the split resonance which is characteristic of the guitar wolf-note. The amplitude of the peak is lower than the neighbouring peaks and the Q-value (sharpness) of the peak drops significantly, highlighting the rapid decay of the strongly coupled note (see also Figure 3.3). The damping of the strongly coupled note is largely determined by the Q-value of the $T(1,1)_2$ mode. A high Q-value may be desirable as it minimises the damping of the note, and thus maximises the sustain.

The perceptions of loudness of notes close to the $T(1,1)_2$ mode was not investigated in the listening tests. The notes which lie either side of the most strongly coupled note have the greatest amplitudes and may be the instrument's loudest notes. The resonance frequency of the $T(1,1)_2$ will therefore determine the pitch of the guitar's loudest notes as well as the pitch at which the guitar wolf-note occurs.

If the $T(1,1)_2$ resonance falls between two successive semitones, the string partial can never coincide exactly with the $T(1,1)_2$ mode and the maximum strength of string-body coupling (and therefore the 'badness' of the wolf-note) will be slightly reduced. For example, a guitar with a $T(1,1)_2$ mode at 190 Hz will have its closest note at 185 Hz (F♯), with a frequency separation of 5 Hz. Another guitar with the $T(1,1)_2$ mode at 195 Hz will have G, at 196 Hz, as its closest note, with a separation of 1 Hz. The guitar with the smaller separation will couple more strongly with its closest note, and will therefore give rise to a stronger wolf-note, hence a separation between the $T(1,1)_2$ mode and the nearest note of a quarter-tone is the 'ideal' case. However, the above reasoning fails to take into account the significant variations in mode frequency that occur as a result of changes in atmospheric humidity. Over a period of several weeks, the frequencies of the $T(1,1)_1$ and $T(1,1)_2$ modes of guitar BR2 were measured to investigate the day-to-day variations in mode frequency. The mean frequency of

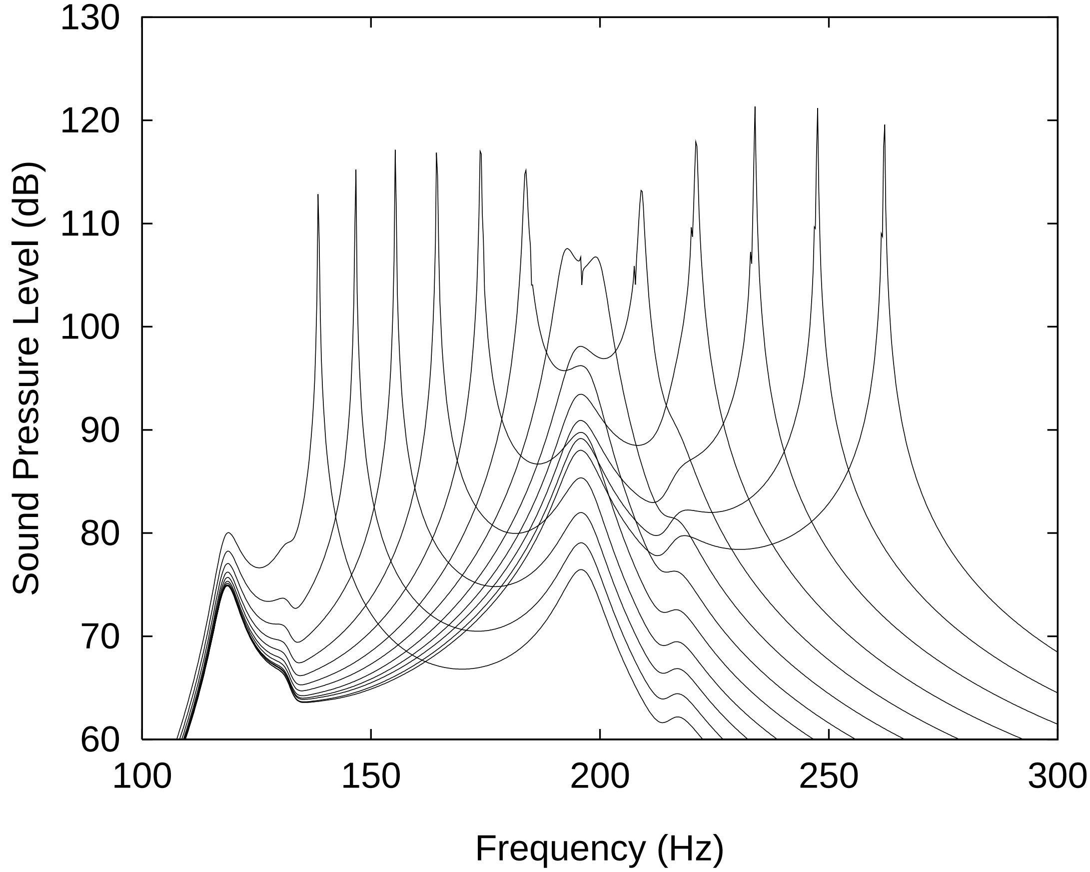


Figure 8.5: Sound pressure response of a number of string partials coupled to the $T(1,1)_2$ resonance showing the strongly coupled wolf-note. As the frequencies of the partials approach the body mode, their amplitudes increase. When the partials are close to the mode, their damping increases and their amplitudes decrease. For the coincident partial, the coupling results in a split resonance, which produces a slightly dissonant, rapidly decaying sound (the guitar wolf-note). Data for the body modes based on the TOP26 parameter set; effective mass of the T(1,1) is 80 g. Frequencies of string partials increased in semitone steps from 139 Hz to 277 Hz.

the $T(1,1)_1$ mode was found to be 97.4 Hz with the maximum and minimum 100 Hz and 93 Hz respectively. The $T(1,1)_2$ mode frequency was found to vary between 179.9 and 197 Hz with a mean of 190.4 Hz. The day-to-day variation in these mode frequencies due to atmospheric changes is therefore seen to be no more than $\pm 5\%$ (approximately $\pm$ one semitone). Any attempt by guitar makers to tune these modes so that they fall between adjacent semitones is therefore bound to fail, at least for a few days in every week! Meyer (1983a) attempted to correlate the perceived quality of several guitars with the frequency separation of the $T(1,1)_2$ mode from the closest note, but did not find a correlation between the two.

The effective mass of the $T(1,1)_2$ mode is the parameter which has the greatest influence on the strongly coupled wolf-like notes. A low effective mass is desirable because it gives good string-body coupling over a wide range of frequencies, resulting in a 'loud' and 'bright' tone, as discussed above. An effective mass that is too low, however, will result in notes that couple too strongly to the $T(1,1)_2$ mode, resulting in an unpleasant, rapidly decaying note. It is a question of achieving an acceptable compromise between the undesirable effects of strong coupling (poor sustain, inharmonicity) and the desirable effects (increased volume, good high-frequency radiation). Careful control of the effective mass of the $T(1,1)_2$ mode provides the means with this balance can be achieved.

Body transient

The $T(1,1)_2$ mode, having the greatest value of A/m, will make the greatest contribution to the body transient part of each plucked note. Notes with partials whose frequencies are close to the $T(1,1)_2$ may mask the body transient quite strongly, as highlighted by test pair number three in the third listening test (Table 7.7). In spite of this, the percussive sound of the body transient is likely to add a subjective impression of attack to the start of most plucked notes. A louder body transient, which could be achieved by increasing the value of A/m for the $T(1,1)_2$ mode, may give the impression of a stronger attack, or 'more punch', to the note. Different resonance frequencies or Q-values of the mode will give different subjective impressions of the transient portion of the notes. Some indication of these effects can be seen in the results of the listening tests, but it is usually not possible to make a clear distinction between changes in the body transient portion of the sound, and changes in the radiated spectrum of string partials. One exception is test pair number three in the third listening test, which involved a three-fold reduction of the Q-value of the $T(1,1)_2$ mode. Examination of the sound pressure responses indicates no significant change in amplitude of the string partials, the only visible change being the shape and height of the $T(1,1)_2$ peak. Careful listening to the difference in tone quality brought about by this change in Q-value reveals that the 'string parts' of the two sounds are indeed indistinguishable, but the body transient part of the second sound dies away more rapidly. Some of the descriptions used by the test subjects to describe this difference in tone include: 'softer attack', 'dry', 'less thud', 'more woody' and 'percussive part more muted'. The difference in tone quality is subtle, but distinct. This is reflected in the responses of different subjects, some evidently having a more discriminating sense of hearing than others.

The influence of the body transient on tone quality is less significant than the radiated spectrum of string partials. Variations in the amplitudes of the partials will give the greatest impressions of variation in tone. The body transient makes smaller but significant contributions to the perception of tone quality. The $T(1,1)_2$ mode has the greatest value of A/m and makes the greatest contribution to the body transient and so the characteristics of the body transient (pitch, loudness, decay) are most easily controlled by varying the frequency, amplitude and Q-value of the $T(1,1)_2$ resonance.

8.5.3 The T(1,2) mode

The T(1,2) mode is usually found at frequencies around 400 Hz (see Table 7.3). The nodal line for the mode runs across the top plate, often passing close to or through the bridge, resulting in large values of effective mass for most of the string driving positions. The lower antinodal region produces a smaller volume change than the larger upper antinodal region, and the mode thus produces a fairly large net volume displacement making it a relatively good radiator of sound, when adequately driven by the strings.

The T(1,2) mode may well have the strongest influence on tone quality after the T(1,1) mode triplet. Meyer (1983a) found that the amplitude of the T(1,2) in the guitar's sound pressure response, the rise in amplitude (i.e. the difference between the peak amplitude and the local minimum at a frequency just below the peak) and the Q-value of the T(1,2) mode could be correlated with assessments of subjective quality more strongly than any of the other physical quantities he measured. The amplitude of this mode in sound pressure response curves often shows it to be one of the strongest radiators after the $T(1,1)_2$. The frequency of the mode usually corresponds to notes fretted between the first and fifth frets on the top E string. The importance of the T(1,2) mode may be partly attributable to the stronger response of the body at frequencies in this region; a distinct change in tone for notes on the top string may be perceived as desirable.

A note whose fundamental coincides with the T(1,2) mode will couple more strongly and will be perceived as louder than neighbouring notes. Since the effective mass of the mode is usually rather high, over-strong coupling and wolf-like notes are unlikely to occur for coincident string partials. The frequency of the T(1,2) will determine the pitch of the louder note which coincides with the mode. The value of A/m of the mode (and to a smaller extent, the Q-value) will determine the change in loudness for the coincident note. Attempts by guitar makers to select the pitch of this louder note by producing a T(1,2) mode at a particular frequency may be complicated by the coupling between the plate mode and one of the air-cavity modes (see below). Results from the listening tests show that changes in the frequency of the T(1,2) mode have only a small influence on tone.

There is insufficient data available on the sign of the T(1,2) mode's effective area to completely justify any generalisations, but the weight of evidence shows that the T(1,2) usually radiates with opposite phase to that of the $T(1,1)_2$ (see Tables 8.1 and 8.2). The data presented by Christensen (1984) shows that, for the four guitars which show a prominent T(1,2) peak, the sign of its effective area is negative. This was also the case for guitars BR1 and BR2, the data for which is presented in Chapter 5. Results from finite element analysis of a fully strutted, spruce top-plate (Walker, 1991) also yielded negative values for the T(1,2) effective area. This situation corresponds to the nodal line of the mode lying between the bridge and the soundhole, so that the positions on the bridge at which the strings drive the top plate are situated in the smaller of the two antinodal regions. To obtain a T(1,2) mode with positive effective area, the nodal line of the mode would have to be below the bridge. There seems to be no particular reason why such a physical change could not be achieved by a guitar maker. Positioning of the bridge closer to the soundhole, or the addition of stiffening bars in the lower bout could conceivably produce such a change. The use of strongly asymmetrical strutting on the top plate might yield a T(1,2) mode whose nodal line ran diagonally through the bridge creating a positive radiating phase for one side of the bridge, and negative phase for the other.

It would certainly be interesting to know if any guitars exist which have a positive value for the T(1,2) mode's effective area, so that comparisons of the response and tone of the two classes of instruments (i.e. those having positive and negative effective areas for the T(1,2) mode) could be made.

The effect on the guitar's sound pressure response of a negative effective area for the T(1,2) is to produce a region of strong response below the T(1,2) due to the in-phase radiation of the T(1,2) and $T(1,1)_{2-3}$ modes. Above the T(1,2) resonance the radiation from the T(1,2) is out of phase with that of the T(1,1) and the sound pressure response is reduced. If the T(1,2) effective area is made positive, an antiresonance is created below the T(1,2) mode due to the out-of-phase radiation of the T(1,1) and T(1,2) modes, but the high-frequency response is increased as the two modes are now radiating in phase (see Figure 7.9). The former situation, in which the T(1,2) effective area is negative, will produce notes on the guitar's top E string with a fairly strong fundamental. The latter situation will cause the notes to have weaker fundamentals but stronger high-frequency partials.

By altering the value and sign of A/m for the T(1,2) mode we therefore have the opportunity to change the distribution of radiated energy. The response in the 250–450 Hz region can be strengthened at the expense of a lower response above 500 Hz, or vice versa. The effect of changes to the value of A/m of the T(1,2) mode will depend to some extent on the number of other low-frequency modes and their values and signs of A/m. The calculation of a residual value of A/m (Section 7.5.3) gives a measure of the net monopole radiation at high frequencies. If there are a number of modes with negative values of A/m it is possible that large changes to the T(1,2) mode can increase the guitar's response both below and above resonance (see Figure 7.12).

Some caution should be taken against applying these results directly to real instruments. I have mentioned above the need for a more detailed knowledge of the frequency dependence of the radiation fields associated with each body mode. At frequencies above 1 kHz it is still not clear how the directionality and amplitude of the radiation fields vary. The current model is able to predict the qualitative effect of the T(1,2) on tone quality, but the quantitative effects may be over-estimated.

Coupling between the T(1,2) and the air cavity

The resonance frequency and amplitude distribution of the T(1,2) mode are often similar to that of the first internal air-cavity mode. Evidence for coupling between the plate mode and the cavity mode has been presented, among others, by Richardson and Walker (1986). The coupling produces two T(1,2)-type resonances whose amplitudes and frequencies are determined by the properties of the uncoupled plate mode and cavity mode. Attempts by guitar makers to achieve particular mode frequencies for the T(1,2) or to move the mode's nodal line in order to reverse the sign of its effective area, may be complicated by the coupling between the plate mode and the cavity mode. Alterations to the properties of the uncoupled plate mode may not result in the desired mode properties in the complete instrument.

Coupling between the T(1,2) and longitudinal string forces

One other aspect of the T(1,2) mode's behaviour which may be significant is its coupling to the longitudinal forces exerted by the strings. Jansson (1973) found that forces applied in the longitudinal direction (along the length of the string) coupled strongly with the T(1,2) mode. The sound pressure response of guitar BR2 measured in the longitudinal direction by Richardson (1982, see Figure 2.8) shows a strong response in the 400–500 Hz region. This is largely due to the T(1,2) mode. Longitudinal vibrations of the string, arising due to changes in the string length during a cycle of vibration, will couple most strongly with the T(1,2) mode. During the transient part of the sound, when the amplitude of the longitudinal forces will be greatest, coupling to the T(1,2) mode may produce perceptually significant effects. Too low a value of the T(1,2) effective mass, measured in the longitudinal direction, may result in an undesirably fast drain on the string's longitudinal energy.

The existence of coupling between the transverse and longitudinal string vibrations may cause the interaction between the T(1,2) mode and the longitudinal string modes to have consequences for the transverse motion of the string. It is the transverse waves of the string, predominantly those polarised in the direction perpendicular to the top plate, which have the strongest influence on the radiated sound form the instrument. Coupling between longitudinal and transverse string modes may therefore provide an indirect mechanism by which the T(1,2) mode can affect the tone quality of the guitar.

The behaviour of the longitudinal string vibrations and their coupling to the transverse string modes and the modes of the body is, at present, poorly understood. More work is needed to clarify the physical and perceptual significance of such effects.

8.5.4 The T(3,1) mode

The T(3,1) mode is usually found above the T(1,2), at frequencies around 450–500 Hz. It is often characterised by a reasonably large value of A/m, making it an important sound radiator. Christensen (1983) made measurements of the acoustical energy radiated at frequencies below 4 kHz. As expected, the greatest contribution to the radiated sound energy came from the $T(1,1)_{1-2}$ modes, which accounted for between 45 and 50% of the radiated energy. However, the proportion of energy radiated by the T(1,2) and T(3,1) modes combined was found to be as much as 45%. In other words, the contribution to the radiated sound energy made by modes above the T(3,1) is very small, although this should not be interpreted to mean that such contributions are perceptually insignificant.

The T(3,1) mode, like the T(1,2), usually has a negative value for its effective area when the driving position is at the bridge (Christensen, 1984; Walker, 1991; see also Section 5.3). The reason for this is clear when the amplitude distribution of the mode (see Figures 2.1 and 2.9) and the position of the bridge is considered. The bridge is situated in the middle of the central antinodal region. When an inward force is applied to the bridge, the volume displacement produced by the central antinodal region is inward but smaller than the combined outward volume displacements of the two outer regions. An inward displacement of the bridge therefore produces a new outward volume displacement, causing the sign of the effective area to be negative. The position of the central part of the bridge at some distance away from the nodal lines also indicates that the effective mass of the T(3,1) will be rather smaller than that of the T(1,2), which often has a nodal line running through or close to the bridge.

The effect of the T(3,1) mode's out-of-phase radiation on the guitar's high-frequency sound pressure response is broadly the same as for the T(1,2) mode. Increases in the mode's value of A/m will cause the high-frequency response of the guitar to be reduced, while the response below the T(3,1) peak is strengthened. This will be true when the combined value of A/m for the positively radiating modes exceeds that for the negatively radiating modes (i.e when the residual value of A/m is positive). If the value of A/m for the T(3,1) mode becomes large enough, such that the residual value of A/m becomes negative, further increases in the

T(1,3) mode's ratio of A/m will increase the guitar's high-frequency sound pressure response while reducing the response at lower frequencies. The response in the vicinity of the T(3,1) resonance will, of course, be strengthened by increases in the mode's value of A/m, regardless of the sign of the effective area.

A reversal of the sign of the T(3,1) mode's effective area would be more difficult to achieve than for the T(1,2) mode since the nodal lines of the T(3,1) lie at some distance from the bridge. The T(3,1) mode is therefore likely to radiate high-frequency sound with opposite phase to the $T(1,1)_2$ for most, if not all, guitars.

8.5.5 The T(2,1) mode

Although the T(2,1) mode occurs at a frequency between that of the $T(1,1)_2$ and T(1,2), I have delayed discussion of its effects on tone quality until now because it more rarely produces a significant monopole component of sound. It's contribution to the guitar's sound pressure response and the directionality of its radiation field depend strongly on the strutting used on the top plate, and in particular, whether a symmetrical or asymmetrical design is favoured. A symmetrical strutting pattern results in two antinodal areas of the top plate which have equal size; the resulting radiation field is a pure dipole (see Figure 2.10). Asymmetrical strutting causes the size of one antinodal area to be greater than the other, and the mode thus produces a net volume change and hence a monopole component of sound radiation. The response of the guitar at frequencies close to the T(2,1) resonance can therefore be increased by the introduction of asymmetrical struts.

Measured values of effective mass and area of the T(2,1) mode depend strongly on the string position used to make the measurements because the mode's nodal line passes through the bridge. For symmetrically strutted top plates, the D and G string positions will lie close to the nodal line and will have relatively high values of effective mass. The top and bottom E strings, lying further from the nodal line, have lower values of effective mass. For guitars BR1 and BR2, T(2,1) effective masses were measured to be 9 and 3 kg respectively, when measured at the G string position. At the top and bottom E string positions, the effective mass was found to be 200–400 g.

The value of effective monopole area for a symmetrical T(2,1) mode will be zero at all of the string driving positions, since the volume displacement produced by each antinodal area of the mode is effectively cancelled by the anti-phase motion of the other antinodal region. It appears, however, that for symmetrically strutted guitars, the T(2,1) mode is not purely symmetrical. The measurements in Chapter 5 on guitars BR1 and BR2 show that the T(2,1) produces a small monopole component of sound. Guitars with deliberate asymmetrical strutting designs for the top plate cause the two antinodal areas to be of different sizes. The value of effective monopole area will therefore be non-zero, and may be quite large, its magnitude and sign depending on the string driving position. String termination points that are situated in the larger antinodal region will have a positive value of effective area, while those in the smaller antinodal region will have negative values.

The efficiency with which the T(2,1) mode is driven by the strings, determined by the mode's effective mass, and the efficiency of the sound radiation from the mode, determined by the effective area, are both seen to be strongly dependent on the string on which the notes are played. The influence of the T(2,1) mode on tone quality is therefore also dependent on the string used. Strings which have a termination point on the bridge with a positive value of effective area will tend to increase the sound pressure response of the instrument at frequencies above the T(2,1) mode. String terminations with a negative value of effective area will tend to reduce the guitar's high-frequency radiation, and produce a stronger response at low frequencies. This string-dependent effect could conceivably be used to strengthen the lower partials of the bass strings whilst reinforcing the high-frequency partials of the treble strings, but the perceptual significance of such a contribution by the T(2,1) mode would depend on its value of A/m relative to the other low-frequency modes.

Directionality of the symmetrical T(2,1) mode

A purely symmetrical T(2,1) mode, with its corresponding dipole radiation field, radiates reasonably strongly to the sides of the instrument but poorly to the front. From the audience's point of view, situated in front of the instrument, this makes the T(2,1) less important than modes such as the $T(1,1)_{1-3}$, T(1,2) and T(3,1), which radiate strongly in all directions. When the acoustics of the room are taken into account, the T(2,1) mode may have a somewhat stronger influence on the sound of the guitar because reflections from walls, ceilings and other surfaces will cause the sound radiated by the T(2,1) to be distributed more evenly in the three spatial dimensions. From the player's point of view, a symmetrical T(2,1) mode may have an important role to play since the sound which it radiates is strongest in the direction of the player's head. The tone quality perceived by the player may be substantially different from that perceived by a listener sitting directly in front of the guitar, due to the contributions of the T(2,1) mode and other symmetrical (2,n)-type modes.

8.5.6 Modes of the back plate

The coupling between the Helmholtz cavity mode and the fundamental top-plate mode gives rise to the $T(1,1)_1$ and $T(1,1)_2$ resonances. The B(1,1), or fundamental back-plate mode, may interact to create a third T(1,1) resonance, but a prominent $T(1,1)_3$ peak appears to be a relatively rare feature. Christensen (1982) showed that only two guitars from a group of nine had an identifiable $T(1,1)_3$ mode.

The listening tests have already shown that large changes to the B(1,1) mode produce virtually no change in tone quality. The B(1,1) mode parameters influence the frequency, damping and amplitude of all three T(1,1) modes, but it is the $T(1,1)_2$ and $T(1,1)_3$ modes that are most strongly affected. Plucked notes with string partials in the region close to the $T(1,1)_2$ and $T(1,1)_3$ modes (around 180–250 Hz) are therefore most likely to be affected by changes to the fundamental back-plate mode. The notes E_2, B_2 and $G\sharp_3$ used in the first listening test, and the notes B_2 and $F\sharp_3$ used in the second listening test, contain partials at frequencies close to the $T(1,1)_2$ and $T(1,1)_3$ modes. No significant change in tone was perceived for these notes, and it therefore seems likely that the B(1,1) mode has a small or negligible influence on the tone quality of all notes in the guitar's playing range.

Higher-frequency modes of the back plate undoubtedly make contributions to the sound pressure response of the guitar. The measurements made on guitars BR1 and BR2 in Chapter 5 both show the influence of the B(1,2) mode at around 300 Hz. The increasing density of modes observed at frequencies above 500 Hz may be due, in part, to contributions from the back plate. The relationship between the back plate's influence on the sound pressure response and the tone quality of the instrument is, however, open to question. The influence of changes to back-plate modes other than the B(1,1) was not examined in the listening tests. The B(1,1) mode will have the largest value of A/m of all the back-plate modes, and therefore has the greatest potential to influence the sound of the instrument. A change in the resonance frequency of the B(1,1) mode by 26%, or a three-fold change in its value of A/m resulted in little or no perceived change in tone. This suggests that the influence of other back-plate modes on the guitar's sound will be small.

In the current model, the back interacts with the top plate only via pressure changes in the air cavity. The sides (ribs) of the guitar are assumed to be perfectly rigid. Modes of vibration of the sides have been identified using holographic interferometry (Richardson, 1984). At high frequencies, a degree of mechanical coupling between modes of the top and back plates, via the sides, seems likely. This provides an additional mechanism by which the vibrational behaviour of the back can influence the radiated sound. The perceptual significance of such mechanical coupling is unknown and difficult to assess.

8.5.7 Mid-frequency top-plate modes

I have so far discussed the effect on tone quality of a number of low-frequency top-plate modes, namely the T(1,1)$_{1-2}$, T(1,2), T(3,1) and T(2,1). The influence on the guitar's sound pressure response of the T(2,1), as discussed above, depends strongly on the top-plate strutting. The other top-plate modes modes produce broadly similar features in the low-frequency sound-pressure response of different guitars, although the exact amplitudes and frequencies of the modes will depend on materials and construction.

Above the T(3,1) resonance, in the 500–1000 Hz 'mid-frequency' region, the response curves of different guitars begin to show much greater inter-instrument differences. In this region it is usually still possible to identify individual resonances, although their frequency separation becomes increasingly small at higher frequencies. The investigation of the responses of guitars BR1 and BR2 in Chapter 5 identifies more than fifteen body modes at frequencies below 1 kHz. The amplitudes of the peaks in the sound pressure response is usually, though not always, smaller in the mid-frequency region than at lower frequencies, indicating that the values of A/m for such modes are often lower than for modes such as the T(1,2) or T(3,1). The influence of the mid-frequency modes on tone quality is therefore likely to be smaller than modes such as the T(1,2), but they will add a degree of colouration to the sound.

Tables 8.1 and 8.2 show the measured values of A/m for a number of top-plate modes of different instruments, along with the mode frequencies and Q-values. Although the data is limited to a small number of modes for each instrument, there appears to be a general trend for modes with high resonance frequencies to have smaller values of A/m. The potential of individual top-plate modes to influence the sound of the guitar therefore seems to fall as the resonance frequencies of the modes increase. There will, of course, be exceptions (guitar 'Ibanez', Table 8.2). In cases where a mid-frequency top-plate mode has a reasonably large value of A/m, it may have a significant influence on tone.

The data in Tables 8.1 and 8.2 is the only published data which gives information on the relative values of A/m for a number of guitar top-plate modes. The data shows that the great majority of modes, excluding the T(1,1), have negative effective areas. This suggests that as the number of modes in the mid-frequency region increases, the radiated level of high-frequency string partials will be reduced. It would be useful to obtain measurements of the sound pressure responses of a larger number of guitars to confirm the observed finding that most top-plate modes above the T(1,1) have negative effective areas.

The number of nodal lines on the top plate increases as the resonance frequencies of the modes increase. Modes with higher resonance frequencies are therefore more likely to have one or more nodal lines passing through the bridge, causing values of effective mass and area to depend more strongly on driving position. In addition, the sign of the effective area is more likely to change for the different string positions, making an assessment of a guitar's sound-pressure response at a number of driving positions invaluable. The effect on tone of modes with resonance frequencies in the 500–1000 Hz region will therefore vary according to the string on which the notes are played.

By adding up the values of A/m for all modes of one guitar to obtain the residual value of A/m, the radiating strength of the T(1,1) mode (and other positively radiating modes) can be compared with the out-of-phase radiation produced by the other modes. For all eight guitars in Tables 8.1 and 8.2 the residual value of A/m is positive, showing that the radiation from the $T(1,1)_2$ mode, when driven at high frequencies, is greater than the net radiation from the other modes. This clearly depends on the number of top-plate modes considered. Other mid-frequency modes may contribute more to the 'negative-phase' radiation, but it seems that modes resonance frequencies above 600 Hz have small values of A/m. Their effect on the calculated residual value of A/m would therefore be small.

For guitars with positive residual values of A/m, increases in the value of A/m of the modes with negative effective areas will increase the sound radiation at low or middle frequencies at the expense of a loss of radiation at high frequencies. The situation described on page 183,

Guitar	Mode	Resonance frequency (Hz)	Ratio of A/m (m^2/kg)	Q-value
TOP26	T(1,1)	185	0.68	20
	T(1,2)	436	−0.04	47
	T(3,1)	507	−0.10	35
	T(1,3)	674	−0.12	28
	T(1,4)	864	0.04	25
	T(5,1)	879	0.02	40
BR1	T(1,1)	188	0.82	34
	T(2,1)	226	−0.02	55
	T(1,2)	430	−0.09	30
	T(3,1)	470	−0.16	45
	T(?,?)	528	−0.02	60
	T(?,?)	582	−0.02	60
BR2	T(1,1)	182	0.79	37
	T(2,1)	223	−0.07	64
	T(1,2)	418	−0.30	35
	T(3,1)	455	−0.12	55
	T(?,?)	485	−0.06	65

Table 8.1: Values of resonance frequency, Q-value, and the ratio of effective area to effective mass (A/m) for top-plate modes of three different guitars. Data for guitars BR1 and BR2 was determined by obtaining best-fit curves to experimentally determined velocity and sound pressure response curves (see Chapter 5). Data for TOP26 (except Q-values, for which 'typical' values chosen) taken from finite element analysis of a 2.6 mm spruce top-plate (Walker, 1991). Values of A/m are for a driving point at the top E string position on the bridge.

Guitar	Mode	Resonance frequency (Hz)	Ratio of A/m (m^2/kg)	Q-value
Ramirez	T(1,1)	200	1.00	25
	T(2,1)	257	0.40	25
	T(1,2)	410	−0.12	30
	T(3,1)	506	−0.10	40
	T(?,?)	627	−0.10	30
Ibanez	T(1,1)	208	0.70	25
	T(2,1)	282	0.15	35
	T(1,2)	405	−0.15	20
	T(3,1)	572	−0.10	50
	T(?,?)	770	−0.20	40
Taurus	T(1,1)	220	0.75	25
	T(2,1)	285	−0.08	15
	T(1,2)	415	−0.06	40
	T(3,1)	498	−0.03	70
	T(?,?)	559	−0.04	60
	T(?,?)	650	−0.08	20
Contreras	T(1,1)	216	0.90	12
	T(2,1)	310	−0.10	15
	T(1,2)	395	−0.15	25
	T(3,1)	495	−0.10	40
Romanillos	T(1,1)	187	1.40	15
	T(3,1)	522	−0.30	30
	T(?,?)	610	−0.05	50

Table 8.2: Values of resonance frequency, Q-value, and the ratio of effective area to effective mass (A/m) for top-plate modes of five different guitars. Data from Christensen (1984).

in which an increase in the sound output from one of the modes with negative effective area results in an increase in the high-frequency radiation, will only occur for relatively large changes to modes such as the T(1,2) which has a relatively large value of A/m. In general, it appears that the addition of modes with negative effective areas will reduce the high-frequency radiation from the guitar, and will tend to produce a sound which is less bright and/or more quiet.

The contribution of multipole radiation fields is overlooked by this data since the effective area A measures only a mode's monopole component of sound radiation. Multipole radiation produced by top-plate modes in the 'mid-frequency' region may give them greater influence on perceived tone quality than the data for A/m suggests. The effective masses measured for some of the top-plate modes in the 500–1000 Hz region (Section 5.2) remain fairly low (200–500 g), indicating that the modes can be efficiently driven by the strings. Modes which have strong multipole radiation fields could therefore have a reasonably large influence on tone quality. More measurements on the monopole and multipole radiation fields produced by the mid-frequency top-plate modes are needed to establish the degree of influence they have on the guitar's sound compared with that of lower frequency modes such as the T(1,2) and T(3,1).

8.5.8 High-frequency top-plate modes

The density of modes in the kilohertz region of the spectrum is high, and this part of the guitar's response has been referred to as a 'resonance continuum' (Caldersmith, 1981). In this region, it becomes extremely difficult to distinguish between separate body modes. The modelling scheme outlined in this thesis, in which the properties of individual top-plate modes are measured, and the coupling between string and top plate is calculated, cannot be successfully used to predict the contribution to the radiated sound of the high-frequency modes.

The vibrational behaviour of the top plate in this region is, at present, not well understood. The top-plate modes with high resonance frequencies are characterised by many antinodal regions. The modes produce small net volume changes, and since monopole radiation becomes inefficient at high frequencies, they radiate very small monopole components of sound. The radiation produced by the high-frequency modes is predominantly multipole in character.

The high density of modes in the region above 1 kHz, and the superposition of their multipole radiation fields, causes the strength and directionality of the sound radiated by the high-frequency modes to vary sharply with frequency. It is not yet clear how the strength of the multipole-type sound, radiated by modes with high resonance frequencies, compares with the strength of the monopole-type sound, radiated by the low-frequency modes when driven well above resonance in the kilohertz region of the spectrum. As the driving frequency increases, the fluid loading of the low-frequency modes becomes increasingly large and the efficiency of monopole radiation decreases. Multipole radiation, in contrast, becomes more efficient at high frequencies. It would be useful to know, for example, at what frequency the strengths of the two types of radiation become comparable. The dependence on driving frequency of the directionality of the radiation fields produced by the low-order modes is also poorly understood. The acoustical 'shadowing' effect, caused by the body of the instrument, is more pronounced at high frequencies where the instrument's dimensions become considerably greater than the wavelength of the radiated sound. Brooke (1992) modelled the radiation field of the $T(1,1)_2$ mode driven at nearly 2 kHz. The deviation from the low-frequency, monopole-type behaviour is clear; radiation to the front and back of the instrument is considerably stronger than the radiation to the sides.

One other phenomenon that may have an influence on the radiation of high-frequency sound is that of coincidence or critical frequencies. The existence of critical frequencies, at which the speed of sound in the plate equals the speed of sound in air, is highlighted by Fletcher and Rossing (1991). Radiation of sound from a plate at frequencies above the critical frequency is concentrated into two high-intensity, narrow beams which radiate in directions symmetrical about the normal to the plate. Using a value of 5000 m/s for the speed of sound along the grain in a spruce plate (Schleske, 1990), I estimate that for a 3 mm thick guitar top plate the critical frequency would be a little over 4000 Hz. No measurements of the high-frequency sound radiation from a guitar have yet been presented which indicate the existence of a critical frequency for the top plate.

The physical character of the high-frequency sound radiated by modes in the 'resonance continuum' is poorly understood. Similarly, the relative perceptual significance of the high-frequency, multipole-type radiation and the high-frequency radiation produced by the low-frequency modes is unclear. A greater understanding of both the physical and perceptual character of sound radiated at high frequencies is essential for a more complete model of the guitar.

8.6 Implications for the guitar maker

The use of numerical models to investigate the influence of the top-plate modes on tone quality allows the mode parameters to be varied independently. The guitar maker, dealing with pieces of wood, does not have this freedom. Removal of small amounts of wood from the plate may affect the properties of a number of modes. The maker must learn to identify areas on the plate which allow particular modes to be adjusted while leaving others relatively unaffected. The degree to which individual mode properties can be altered is limited not only by the inter-dependence of the mode parameters but also by the need for the finished instrument to conform to aesthetic and mechanical requirements. The top plate must be able to withstand the stresses imposed on it by string tension, and the appearance of the guitar must be judged to be attractive.

Another difficulty faced by the maker is that the properties of the modes change considerably when the plates are assembled to form a finished instrument. An understanding of the links between the modes of the uncoupled plates and those of the complete instrument is difficult to obtain. Experience is built up over many years which may help guitar makers to relate properties of the top plate to properties of the instrument in its complete form and will allow makers to build instruments with consistent tonal properties. Tap tones can be used to assess some properties of the plate during construction. The plate is held lightly between thumb and finger and tapped sharply at different points on the plate to excite some of the body modes. Information on the the frequencies and Q-values of two or three modes may be obtained in this way but information relating to the vibration amplitudes of the modes (i.e. data for the effective masses and areas) is more difficult to obtain. Without the use of specialised equipment, the properties of a small number of plate modes can only be partially assessed using tap tones. The knowledge that a certain combination of materials, plate thicknesses and top-plate strutting patterns has produced good instruments in the past probably serves as a stronger guide for the maker than an assessment of the mode properties of the plate during construction.

8.6.1 Mode frequencies

Many instrument makers, and much of the research relating to the acoustics of violins and guitars, places great emphasis on the resonance frequencies of the body modes. A variety of proposals has been made concerning ways in which 'good quality' instruments can be achieved by placing certain body modes at particular frequencies, but psychoacoustical tests are rarely performed to provide evidence linking the 'goodness' of tone with the frequencies of the modes. The psychoacoustical work presented in this thesis shows that the mode frequencies have a limited effect on tone quality and the properties of the modes which determine their amplitude of vibration (effective mass, effective area) have a more profound influence on tone.

Hutchins (1962) reports on work by Savart who removed the top and back plates from a number of Stradivarius and Guarnerius violins and measured the frequencies of the fundamental plate modes. He found that the frequencies of the back-plate modes were one or two semitones (6–12%) above those of the fundamental top-plate modes. This, of course, does not imply that violins with such a tuning of the top- and back-plate modes will give them a Stradivarius-like tone[1]. The tuning of the top and back-plate modes in this way is only one part of an overall construction scheme used by the great violin makers to produce violins of high quality. If this tuning of the body modes is used by other makers as part of a different overall scheme of construction, there is no reason to believe that it will offer particular advantages over an alternative tuning of the modes.

The variety of proposals made concerning the tuning of body modes emphasises that the success or failure of such schemes is highly dependent on the construction methods favoured by individual makers. There appear to be no general rules for the tuning of body modes which allows all makers to produce instruments with consistently 'good' tone qualities. Hutchins (1962), for example, reports that when modes of the top and back plate coincide, or are separated by more than a tone, a 'bad' violin is produced. When the modes are within a semitone of each other, a 'good' instrument is obtained. Some makers suggest that the fundamental back-plate mode should be tuned lower than the top, others that the back should be higher, and some suggest that the two should be tuned to the same frequency. The situation relating to mode frequencies in guitars is similar. After many years of research there is still

[1] We should also be careful in assuming, without proper psychoacoustical tests, that a Stradivarius violin (or one which claims to be a Stradivarius) *necessarily* has outstanding tonal qualities.

little consensus of opinion which universally relates the frequencies of the body modes to aspects of tone quality.

It is my belief that this cannot be achieved unless the properties of the body modes that determine their *amplitude* of vibration are examined. It is the relative amplitudes of the coupled string modes which have the greatest influence on perceived tone quality. It is therefore surprising that such a small proportion of the published work on stringed musical instruments investigates the properties that determine the instrument body's amplitude of vibration. It is, however, interesting to note that, of the 13 criteria for which Meyer (1983a) obtained correlations with tone quality, 11 involve measurements of the amplitudes of the modes in the sound pressure response curves.

For measurements of an instrument's velocity, admittance or acceleration response, it is the effective masses of the modes which largely determine the amplitude of the resonance peaks, large amplitude peaks corresponding to modes with low effective masses. For sound pressure responses, the peak amplitudes depend on the modes' effective masses as well as their radiation characteristics. The problems of measuring the radiation characteristics of a mode with a single parameter have already been discussed, but the definition of an effective monopole area provides an efficient way to model the sound radiation, at least for frequencies up to 1 kHz. The effective masses and areas of the modes therefore determine the amplitudes of the resonance peaks in measured response curves and have been shown (Chapter 7) to have a stronger and more widespread influence on tone quality than the mode frequencies.

The idea of an effective mass associated with each body mode, and a method for measuring it, was first published by Schelleng in his 1963 paper on violin acoustics. More recently, Gough (1980, 1981) has shown that the effective masses of the body modes provide the key to understanding the coupling between strings and body. The majority of published work on stringed musical instruments, however, still focusses on the frequencies of the modes. One likely reason for this is that the mode frequencies (and Q-values) can be measured easily, but accurate measurements of effective mass are more difficult. A sense of pitch allows instrument makers to make good estimates of the mode frequencies by using tap tones; with a small amount of relatively simple equipment, mode frequencies can be measured with great accuracy. It is more difficult for instrument makers to measure effective masses and effective areas but the use of Chladni patterns gives the maker a simple way to assess the amplitude distributions of the modes. The effective masses of the modes can be assessed by examining the proximity of nodal lines to the bridge. The values and signs of the effective areas are more difficult to estimate, but the relative sizes of the different antinodal regions will provide some clues.

8.6.2 Altering the body modes

The guitar maker has limited powers to vary the properties of the body modes. Once materials have been selected, the dimensions and thickness of the top plate will largely determine its vibrational response. The top plate is usually made rather thin (typically 2–3 mm) to enable it to couple more effectively with the air. Struts are added to the underside of the top plate to provide additional support. The tension transmitted through the bridge from the strings is sufficient to distort the top plate quite considerably if it is not made strong enough to withstand the forces. The pattern of struts used by guitar makers may vary considerably; three designs are illustrated in Figure 8.6. The potential that different strutting arrangements have to alter the mode properties is rather limited, at least for modes with low resonance frequencies. Holographic interferograms of guitars of different constructions show great similarity in the amplitude distributions of the low-frequency body modes. The struts will have a moderate influence on the mode frequencies; they will increase the stiffness of the plate and tend to raise the frequencies of the modes.

One way in which the modal properties can be more strongly influenced by strutting is with the use of strongly asymmetrical designs, one example of which is shown in Figure 8.6b. Top-plate modes which are normally symmetrical, and would not radiate strong monopole components of sound, may be turned into efficient sound radiators by using asymmetrical strutting. The T(2,1) mode, at around 250 Hz, is normally a poor radiator of monopole sound. With asymmetrical strutting designs, it will make a significant contribution to the radiation of sound.

The use of asymmetrical designs will not only change the sizes of the antinodal regions but will also move the nodal lines of some modes. The T(1,2) mode, for example, usually has a nodal line running across the top plate and through the bridge, causing it to be relatively weakly driven by the strings. By increasing the asymmetry of the mode, the nodal line could be moved so that it ran at an angle to the bridge. This would decrease the effective masses for many of the string positions, thus increasing the sound output from the mode. Nodal lines

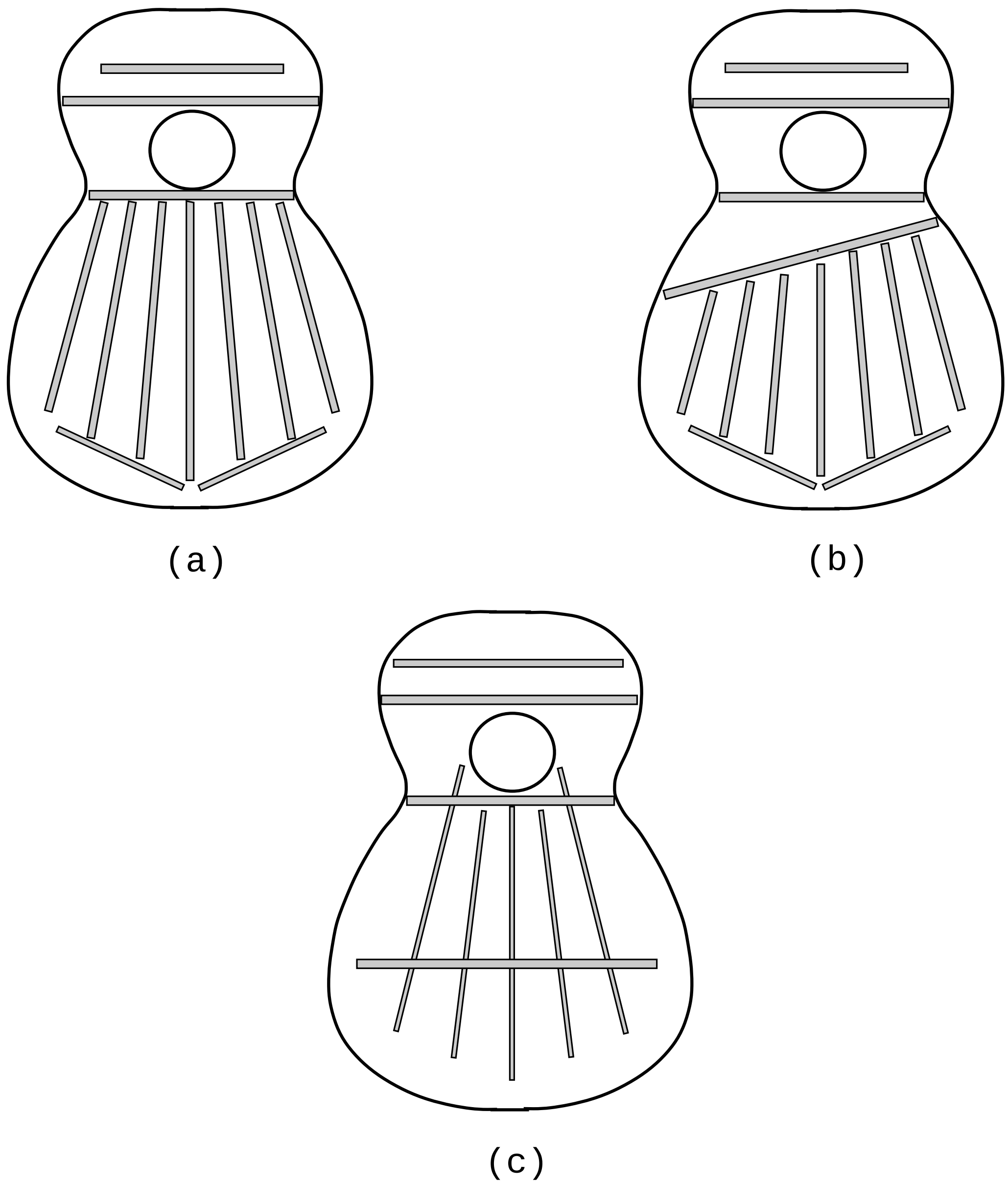


Figure 8.6: Arrangements of struts used on the top-plate: traditional Torres pattern (a), asymmetrical design (b), Bouchet design (c).

can also be moved by the positioning of extra struts and bars. Top-plate modes with high resonance frequencies have a larger number of nodal lines, and these will tend to fall along the positions of the struts because these are the stiffest parts of the plate. The position of the T(1,2) nodal line through the bridge is partly due to the stiffening effect of the bridge. Movement of the bridge, and the use of struts to add stiffness along particular parts of the top plate could be used to influence the nodal lines, and therefore the effective masses of modes such as the T(1,2) or T(3,1), although it is impossible to do this without having some influence (possibly unfavourable) on other top-plate modes.

The direction in which the asymmetrical bar in Figure 8.6b runs will also affect the tone. The sign of the effective areas of the modes is determined by the size of the antinodal region in which the string driving position is situated. With the T(2,1) mode, the larger antinodal region could be created on the 'bass' or 'treble' side of the bridge. The effects on tone of such changes were not examined in the listening tests, but it may be desirable to have the smaller antinodal region on the 'bass' side of the bridge. This would produce a negative effective area for the T(2,1) mode, and would help to strengthen the response between the T(1,1) and T(2,1) modes thus reinforcing low-frequency string partials in the region 200–250 Hz.

The effect of the bridge on the modes of the top-plate is often overlooked. The addition of the bridge adds significant mass and stiffness to the plate and will reduce the amplitudes of both the T(1,1) and T(1,2) modes. Meyer (1983b) performed experiments in which a number of different bridges were added to a guitar top plate, and their effect on the sound pressure response was measured. One of the most favourable bridge designs was one whose longest dimension was reduced considerably so that the wings of the bridge were completely removed. The effect of this reduction was to reduce both the mass and stiffness of the top plate. The output from the T(1,1) mode would certainly increase since its effective mass would be reduced. Results from the psychoacoustical listening tests show that such a change gives a sound which is perceived as 'louder' and 'brighter'. Meyer also found that the bridge with reduced dimensions increased the output from the T(1,2) mode. This may partly oppose the increase in 'brightness' achieved by the stronger T(1,1) mode because of the out-of-phase radiation produced by the T(1,2) at high-frequencies. The area of the bridge must be great enough to ensure that it can be firmly and permanently glued to the top plate. A reduction in the dimensions of the bridge may give an increased risk that the bridge will separate from the top plate after some time. If methods of securely attaching a smaller and lighter bridge to the top plate can be found, guitars with improved volume and high-frequency response might be produced.

The mode which the maker can control most easily is the fundamental top-plate mode or T(1,1). By making the top plate as light as possible, the effective mass of the T(1,1) can be minimised, and the output of the guitar over a wide range of frequencies would be maximised. A thinner top plate would reduce the effective mass of the T(1,1) mode but would tend to lower the mode frequency, the reduction in plate stiffness being more significant than the reduction in the plate mass. The use of extra struts on a thin top plate helps to maintain ahigh plate stiffness and would prevent the frequency of the T(1,1) mode being lowered, as well as providing the necessary strength for the plate to withstand the string forces. As mentioned above, the bridge adds significant mass to the top plate and affects the output of the T(1,1) mode. A light bridge would help to maintain a low effective mass for the T(1,1) mode and would maximise the sound output from the mode. Changes to the effective area of the T(1,1) mode are more difficult to achieve. An increase in the physical area of the plate brings additional risk of structural damage to the top plate, and the plate thickness would probably have to be increased to provide the necessary support. Some makers thin the plate slightly around the edges; this may increases the plate mobility and help to increase the effective area of the T(1,1).

The resonance frequency of the T(1,1) determines the pitch of the guitar wolf-note and some care may be needed to select an appropriate frequency. Guitar music often makes good use of the open bass strings, usually tuned to E_2 and A_2. Guitars with a strongly coupled wolf-note for one of these notes would be highly undesirable. The notes at which the guitar wolf-note is usually found are F♯, G or G♯. When the note is played at the bottom of the guitar's range (e.g. G_2) the second partial of the note will coincide with the coupled T(1,1) resonance. Notes played one octave higher (G_3) will suffer strong coupling between the string fundamental and the body mode. Adjustments to the frequency of the T(1,1) mode allow the maker to control the pitch of the wolf-notes although allowances must be made to account for the increases in mode frequency that occur when the free plate is coupled to the air cavity.

In an interesting article by Caldersmith and Williams (1986) the guitar maker Greg Smallman discusses the methods by which he created his 'lattice guitars', recently made popular by John Williams. His main idea was to make guitars louder, and his design is characterised by a very light top plate and bridge, supported by a lattice arrangement of struts. The use of struts which run at an angle to the wood grain will increase the stiffness of the plate both along and across the grain, and will prevent the mode frequencies of the light top plate from being too low as well as preventing the top plate being distorted by the string tension. Smallman describes his first light-weight guitars as 'ak-ak' guns; they were loud but the T(1,1) resonance was too high, making them unsuitable for classical music. After discussions with John Williams, he began to lower the resonance to around G$\sharp$, while maintaining the lightness of the top plate. These guitars were loud, but had the T(1,1) resonance at a frequency which was better suited to classical music, presumably because they had a stronger bass response. Smallman has also made guitars using a combination of cedar and carbon fibre to provide strength with low mass. The article shows Smallman's understanding of the importance of a top plate with low mass, as well as his knowledge of the connection between strong resonances and strongly coupled wolf-notes, but he says that it is not important for the luthier to understand the physics behind the art of guitar making. It is for the scientist, not the luthier, to understand the connections between physical changes to the top plate and its effects on tone quality. The more important skill for a luthier, according to Smallman, is to be able to hear the improvements in sound:

> *'You've got to be able to listen and say "That's too tight" or "That's too sloppy". It's important to be able to chop bits of wood out of them, hear the sound get better, and chop out a bit more, and to know when to stop. You've got to have good ears, that's what I've got.'*

Chapter 9

Conclusions

9.1 Linking the properties of the body modes with tone quality: a summary

Results from the psychoacoustical listening tests have made it possible to link objective, physical parameters relating to the guitar's body modes with subjective impressions of tone quality. Perceptions of the changes in tone that result from alterations to the mode parameters have yielded information on the way in which tone quality is affected by the vibrational response of the body. Changes to the effective masses and effective areas of the body modes have the most profound influence on perceived tone. Changes to the mode frequencies have a moderate influence, and changes to the Q-values have a small influence on tone quality.

The results from the listening tests have also highlighted a distinction between 'global' and 'local' effects on tone quality. Changes to the Q-value or resonance frequency of a mode affect the response of the guitar in a narrow range of frequencies that are local to the resonance peak. For example, if the resonance frequency of a mode is moved from 220 Hz to 240 Hz, string partials outside the range 200–260 Hz will be almost unaffected. If the effective mass of a mode at 220 Hz is changed, string partials up to 2 or 3 kHz may be significantly affected.

The value of A/m of a mode determines its amplitude in the sound pressure response and provides a measure of the potential that the mode has to influence the guitar's sound. Modes with the largest values of A/m have the potential to influence tone quality most strongly and over the widest range of frequencies. For all guitars, the fundamental top-plate mode, the T(1,1), will have the largest value of A/m and therefore has the strongest influence on tone quality. Changes to any one of the T(1,1) mode parameters will have a greater effect on tone than numerically equivalent changes to other top-plate modes. The results of the listening tests show that, for example, a change by a factor of 1.75 in the value of A/m of the T(1,1) produces a change in tone of roughly the same magnitude as a four-fold change in the A/m of the T(1,2). Similarly, changes in the frequency of the T(1,1) mode are more easily perceived than identical changes in the frequency of the T(1,2).

The modes which have the strongest influence on tone quality can be identified by the largest-amplitude peaks in the guitar's sound pressure response. Apart from the strongest radiator, the T(1,1) mode, the modes with large-amplitude resonance peaks usually include the T(1,2) and T(3,1) modes, which may occur as multiple peaks due to coupling to internal modes of the air cavity. For guitars with strongly asymmetrical strutting, the T(2,1) mode may also have a strong peak in the sound pressure response measured in front of the instrument. It is these low-frequency modes with large values of A/m which have the greatest influence on the tone quality of the guitar. Top-plate modes with resonance frequencies above 600 Hz tend to have relatively small peak amplitudes in the sound pressure response and will have rather low values of A/m. The potential of the top-plate modes to influence tone quality appears to diminish with increasing frequency, although this overlooks the effects of multipole radiation, the perceptual significance of which is unknown.

Loudness

The listening tests show that changes to the amplitude of a mode in the sound pressure response strongly affect the perceived 'loudness' of the guitar. Increasing a mode's value of A/m strengthens the radiation of string partials not only in the vicinity of the mode itself, but also at frequencies above the mode's own resonance. Changes to the effective masses and areas of the T(1,1) and T(1,2) modes affect the perceived 'loudness' of notes with fundamental frequencies up to at least 500 Hz. The frequencies of the body modes will determine the notes at which local changes in loudness occur when string fundamentals coincide with the body mode, but the effective masses and effective areas of the modes affect not only the magnitude of these local changes in loudness but also the global characteristics of loudness.

Evenness

Local changes in loudness, if too extreme, may give an impression of an uneven tone quality. The change in loudness for notes whose fundamentals coincide with a body mode is determined by the rise in amplitude of the resonance peak, which is in turn affected by the value of A/m and the Q-value of the mode. The rise in amplitude will also be affected by the properties of the other body modes. In particular, the signs of the effective areas of the modes determine whether antiresonances, with large local changes in amplitude, or ‘plateau-like’ features, with smoother changes in amplitude, are created at frequencies above and below the body resonance (see Figure 2.5). Low Q-values for the body modes may help to produce more gradual changes in loudness for sequences of notes whose fundamentals pass through a body mode. If two body modes have resonances that are narrowly separated, there may be two sudden changes in loudness, causing a more extreme perception of ‘unevenness’.

Sustain

The effective masses of the body modes determine the strength of the string-body coupling, which affects not only the loudness of notes but also their sustain. A reduction in the effective masses results in stronger string-body coupling, which gives a louder but less sustained sound. A balance between the two effects is needed. The T(1,1) mode has the lowest effective mass and couples strongly with notes over a wide frequency range, therefore having a strong influence on the sustain of many notes. On occasions when string partials coincide with other body modes, the coupling to these modes may be stronger than the coupling to the T(1,1), and the effective masses of the coincident mode will have a greater effect on sustain.

Brightness

The strongly radiating modes (those with large values of A/m) contribute most to the radiation of high-frequency sound. Perceptions of ‘brightness’ are therefore associated with the modes’ values of A/m. The calculation of a residual value of A/m provides a way of monitoring the effect on the high-frequency radiation of changes to modes with positive or negative effective areas. Increases in the output from positively radiating modes, such as the T(1,1), will usually increase the residual value of A/m and will increase the perceived 'brightness' for most notes in the guitar's playing range. Increases in the output of negatively radiating modes, such as the T(1,2) or T(3,1), will usually have the opposite effect, but these results depend on the number, strength and radiating phase of the other low-frequency body modes. One way in which a strong high-frequency response can be obtained is by creating a large-amplitude peak (large A/m) for the positively radiating T(1,1) mode, and much smaller values of A/m for the modes with negative effective areas. This gives a large residual value of A/m and will produce a 'bright' tone. Multipole radiation from high-frequency top-plate modes will also affect the perception of the high-frequency radiated sound. The relationships between the strength and directionality of multipole radiation and perceptions of 'brightness' are not known.

Concluding remarks

The influence of the body modes on tone quality has been shown to be strongly dependent on their values of effective mass and effective area. A greater awareness of the importance of the effective masses and effective areas of top-plate modes, and more measurements of these quantities on a variety of instruments, would undoubtedly make a significant contribution to the understanding of the relationships between a guitar's tone quality and the properties of its modes of vibration.

9.2 Future work

The guitar body

The behaviour of the low-frequency body modes when driven at high frequencies is, at present, not clearly understood. The radiation from these modes, for driving frequencies up to 1 kHz, can be well approximated by monopole and dipole sources (Brooke 1992). At higher frequencies, the changes in the directionality and strength of the radiation from the low-frequency modes are not clear. Measurements or modelling of the radiation from the low-frequency modes, when driven at frequencies above 1 kHz, would yield valuable information on the contributions of the low-order modes to the high-frequency sound pressure response of the guitar. Fluid loading will have a significant effect on the modes which produce large net volume changes. At high frequencies the influence of fluid loading on the top plate becomes more significant and its effect on the radiation of high-frequency sound also needs to be investigated.

The radiation characteristics of the top-plate modes with high resonance frequencies are poorly understood. They will produce small monopole components and large multipole components of sound, but the amplitudes and directionality of their radiation is not known. At high driving frequencies, the relative radiating strengths of the multipole and monopole-type modes is not known. Further work is needed on the physical and perceptual character of the two types of sound radiation.

The current model calculates the coupling between the plate modes and the fundamental cavity mode. Coupling between the top and back plate, via the sides of the instrument, may significantly affect the response of the guitar. There is also strong evidence of coupling between modes of the top plate, particularly the T(1,2) mode, and internal air-cavity modes. Models of these effects should be developed so that their perceptual significance can be assessed.

The strings

Accurate modelling of the vibrations of a real guitar string presents many challenges. The current model simplifies the motion of the string greatly and assumes small-amplitude motion in one transverse plane only. The inclusion of two transverse polarisations of string motion, coupled to the guitar body, would provide a more realistic model of the guitar. Large-amplitude motion of the string results in non-linear coupling between the two transverse modes of vibration, causing precession of the string orbit (Gough, 1984). This undoubtedly causes perceptually significant effects, but the variations in tone that result from such non-linearities can only be assessed if suitable models are built. Such models should include the motion of the end support so that coupling between the two transverse string modes and the top plate can be included. The effects of coupling between different strings, via the motion of the bridge, may have a significant influence on the string vibrations, and the coupling between the body and the longitudinal string modes may also be important, and should be investigated.

Appendix A

Theory detail

A.1 Solution for the motion of coupled top-plate and air-cavity pistons

This section derives expressions for the displacements of the top-plate and air-cavity pistons, following the Newtonian model presented by Christensen and Vistisen (1980). For the two-oscillator model of the guitar (top-plate piston coupled to the air cavity) the equations of motion, already quoted in Chapter 4, are:

$$m_t \frac{\partial^2 x_t}{\partial t^2} = F - k_t x_t - R_t \frac{\partial x_t}{\partial t} + A_t \Delta P \ , \tag{A.1}$$

and

$$m_h \frac{\partial^2 x_h}{\partial t^2} = A_h \Delta P - R_h \frac{\partial x_h}{\partial t} \ , \tag{A.2}$$

where F is the sinusoidal driving force applied to the top plate, ΔP is the change in cavity pressure, subscript t refers to the top plate and subscript h to the air cavity. If the small changes in equilibrium cavity pressure are assumed to constitute an adiabatic process, that is, one in which no heat energy is gained or lost, then

$$PV^{\gamma} = constant \ , \tag{A.3}$$

where V is the volume of the cavity and γ is the ratio of specific heats. Differentiating with respect to V gives

$$\frac{\partial P}{\partial V} = -\frac{P\gamma}{V} \ . \tag{A.4}$$

The bulk modulus of a material is given by $B = -V\frac{dP}{dV}$ and the phase speed of sound in air by $c = \left(\frac{B}{\rho}\right)^{1/2}$. Using these two expressions with Equation A.4 leads to the following expression for the change in pressure in the cavity:

$$\Delta P = -\mu \Delta V \ , \tag{A.5}$$

where $\mu = \frac{c^2 \rho}{V}$. The change in volume in the cavity caused by motion of the top plate and air pistons is given by

$$\Delta V = A_t x_t + A_h x_h \ , \tag{A.6}$$

where positive displacements of either piston are associated with movement out of the cavity. The pressure change in the cavity can now be written as

$$\Delta P = -\mu(A_t x_t + A_h x_h) \ . \tag{A.7}$$

After substituting this into Equations A.1 and A.2 the following are obtained:

$$m_t \frac{\partial^2 x_t}{\partial t^2} = F - x_t(k_t + \mu A_t^2) - R_t \frac{\partial x_t}{\partial t} - \mu A_t A_h x_h \ , \tag{A.8}$$

and

$$m_h \frac{\partial^2 x_h}{\partial t^2} = -\mu A_h^2 x_h - R_h \frac{\partial x_h}{\partial t} - \mu A_h A_t x_t \ . \tag{A.9}$$

As before, the displacements are assumed to be sinusoidal for a sinusoidal driving force, so expressions for the derivatives of x_t and x_h are easily found. These can be substituted into Equations A.8 and A.9 giving

$$-m_t \omega^2 x_t = F - x_t(k_t + \mu A_t^2) - R_t i\omega x_t - \mu A_h A_t x_h \ , \tag{A.10}$$

and

$$-m_h \omega^2 x_h = -\mu A_h^2 x_h - R_h i\omega x_h - \mu A_h A_t x_t \ . \tag{A.11}$$

The natural resonance frequency ω_t of the top-plate piston without coupling to the air cavity (obtained by setting $A_h = 0$ in Equation A.10) is found to be

$$\omega_t^2 = \frac{k_t + \mu A_t^2}{m_t} \ . \tag{A.12}$$

Substituting this result back into Equation A.10 and defining $\gamma_t = \frac{R_t}{m_t}$ an expression for x_t is obtained:

$$x_t = \frac{F - \mu A_h A_t x_h}{m_t(\omega_t^2 - \omega^2 + \gamma_t i\omega)} \ . \tag{A.13}$$

The corresponding equation for x_h is found by treating Equation A.11 similarly. The natural resonance frequency of the undamped, uncoupled oscillator is found to be:

$$\omega_h^2 = \frac{\mu A_h^2}{m_h} \ . \tag{A.14}$$

Using this result we obtain:

$$x_h = \frac{-\mu A_h A_t x_t}{m_h(\omega_h^2 - \omega^2 + i\omega\gamma_h)} \ , \tag{A.15}$$

where $\gamma_h = \frac{R_t}{m_t}$. The coupling term $\mu A_h A_t$ appears in both equations and involves the areas of both the top plate and air pistons. By setting $\alpha_{ht} = \mu A_h A_t$ and also defining $D_t = m_t(\omega_t^2 - \omega^2 + i\omega\gamma_t)$ and $D_h = m_h(\omega_h^2 - \omega^2 + i\omega\gamma_h)$, the solutions for x_t and x_h are obtained by substituting Equation A.15 into A.13:

$$x_t = \frac{F D_h}{D_h D_t - {\alpha_{ht}}^2} \ , \tag{A.16}$$

and
$$x_h = \frac{-\alpha_{ht} F}{D_h D_t - {\alpha_{ht}}^2} \ . \tag{A.17}$$

A.2 Solution for the system of coupled top-plate, back-plate and air-cavity pistons

The oscillator model of the guitar can be extended to include motion of the back plate by adding a third oscillator with mass m_b working against a spring of stiffness k_b and driving a piston with effective area A_b (Christensen, 1982). The equations of motion for the coupled three-oscillator system are:

$$m_t \frac{\partial^2 x_t}{\partial t^2} = F - k_t x_t - R_t \frac{\partial x_t}{\partial t} + \Delta P A_t \ , \tag{A.18}$$

$$m_h \frac{\partial^2 x_h}{\partial t^2} = \Delta P A_h - R_h \frac{\partial x_h}{\partial t} \ , \tag{A.19}$$

and
$$m_b \frac{\partial^2 x_b}{\partial t^2} = \Delta P A_b - k_b x_b - R_b \frac{\partial x_b}{\partial t} \ , \tag{A.20}$$

where R_b is the resistance to motion felt by the back-plate piston and ΔP is, as before, the pressure change in the cavity resulting from movement of the pistons. The volume change resulting from this movement is

$$\Delta V = A_t x_t + A_h x_h + A_b x_b \ , \tag{A.21}$$

where all positive piston displacements are outward. Substituting this expression into Equation A.5 gives an expression for ΔP which can be used in the three equations of motion to give:

$$m_t \frac{\partial^2 x_t}{\partial t^2} = F - k_t x_t - R_t \frac{\partial x_t}{\partial t} - \mu {A_t}^2 x_t - \alpha_{ht} x_h - \alpha_{bt} x_b \ , \tag{A.22}$$

$$m_h \frac{\partial^2 x_h}{\partial t^2} = -\mu {A_h}^2 x_h - \alpha_{ht} x_t - \alpha_{hb} x_b - R_h \frac{\partial x_h}{\partial t} \ , \tag{A.23}$$

and $$m_b \frac{\partial^2 x_b}{\partial t^2} = -\mu {A_b}^2 x_b - \alpha_{bt} x_t - \alpha_{hb} x_b - R_b \frac{\partial x_b}{\partial t} \ , \tag{A.24}$$

where the coupling constants are defined as

$$\alpha_{ht} = \mu A_h A_t \ , \tag{A.25}$$

$$\alpha_{hb} = \mu A_h A_b \ , \tag{A.26}$$

and $$\alpha_{bt} = \mu A_b A_t \ . \tag{A.27}$$

The natural resonance frequency of the lightly damped, uncoupled back-plate oscillator is:

$${\omega_b}^2 = \frac{k_b + \mu {A_b}^2}{m_b} \ . \tag{A.28}$$

Substituting this result, along with Equations A.12 and A.14 into the equations of motion above leads to the following three equations:

$$\frac{\partial^2 x_t}{\partial t^2} = \frac{F}{m_t} - {\omega_t}^2 x_t - \gamma_t \frac{\partial x_t}{\partial t} - \frac{\alpha_{ht}}{m_t} x_h - \frac{\alpha_{bt}}{m_t} x_b \ , \tag{A.29}$$

$$\frac{\partial^2 x_h}{\partial t^2} = -{\omega_h}^2 x_h - \gamma_h \frac{\partial x_h}{\partial t} - \frac{\alpha_{ht}}{m_h} x_t - \frac{\alpha_{hb}}{m_h} x_b \ , \tag{A.30}$$

and $$\frac{\partial^2 x_b}{\partial t^2} = -{\omega_b}^2 x_b - \gamma_b \frac{\partial x_b}{\partial t} - \frac{\alpha_{bt}}{m_b} x_t - \frac{\alpha_{hb}}{m_b} x_h \ , \tag{A.31}$$

where the back-plate damping factor has been defined as $\gamma_b = \frac{R_b}{m_b}$. Assuming the displacements to be sinusoidal for a sinusoidal driving force, expressions for the derivatives of x_t , x_h and x_b can be substituted into the above (see Equations 4.4 and 4.5) leading to

$$x_t = \frac{F - \alpha_{ht}x_h - \alpha_{bt}x_b}{m_t({\omega_t}^2 - \omega^2 + i\gamma_t\omega)} , \tag{A.32}$$

$$x_h = \frac{-\alpha_{ht}x_t - \alpha_{hb}x_b}{m_h({\omega_h}^2 - \omega^2 + i\gamma_h\omega)} , \tag{A.33}$$

and
$$x_b = \frac{-\alpha_{bt}x_t - \alpha_{hb}x_h}{m_b({\omega_b}^2 - \omega^2 + i\gamma_b\omega)} . \tag{A.34}$$

To simplify matters, the previous definitions of D_t and D_h are used as well as the equivalent definition of $D_b = m_b({\omega_b}^2 - \omega^2 + i\gamma_b\omega)$. The above equations are rewritten as

$$x_tD_t + \alpha_{ht}x_h + \alpha_{bt}x_b = F , \tag{A.35}$$

$$x_hD_h + \alpha_{ht}x_t + \alpha_{hb}x_b = 0 , \tag{A.36}$$

and
$$x_bD_b + \alpha_{bt}x_t + \alpha_{hb}x_h = 0 . \tag{A.37}$$

To solve for the three displacements x_t , x_h and x_b the three equations are arranged in matrix form:

$$\begin{bmatrix} D_t & \alpha_{ht} & \alpha_{bt} \\ \alpha_{ht} & D_h & \alpha_{hb} \\ \alpha_{bt} & \alpha_{hb} & D_b \end{bmatrix} \begin{bmatrix} x_t \\ x_h \\ x_b \end{bmatrix} = \begin{bmatrix} F \\ 0 \\ 0 \end{bmatrix} . \tag{A.38}$$

Cramer's rule is applied to obtain the solutions for the three displacements. The solution for the top-plate displacement is found to be

$$x_t = \frac{det \begin{bmatrix} F & \alpha_{ht} & \alpha_{bt} \\ 0 & D_h & \alpha_{hb} \\ 0 & \alpha_{hb} & D_b \end{bmatrix}}{det \begin{bmatrix} D_t & \alpha_{ht} & \alpha_{bt} \\ \alpha_{ht} & D_h & \alpha_{hb} \\ \alpha_{bt} & \alpha_{hb} & D_b \end{bmatrix}} , \tag{A.39}$$

where *det* [ ] means the determinant of the matrix. After calculating the determinants of the two matrices, the solution is found to be

$$x_t = \frac{F(D_b D_h - {\alpha_{hb}}^2)}{D_t D_h D_b + 2\alpha_{ht}\alpha_{hb}\alpha_{bt} - [D_t(\alpha_{hb})^2 + D_h(\alpha_{bt})^2 + D_b(\alpha_{ht})^2]} \ . \tag{A.40}$$

Following the same procedure, the displacements of the air-cavity and back-plate pistons are found to be

$$x_h = \frac{-F(D_b\alpha_{ht} - \alpha_{hb}\alpha_{bt})}{D_t D_h D_b + 2\alpha_{ht}\alpha_{hb}\alpha_{bt} - [D_t(\alpha_{hb})^2 + D_h(\alpha_{bt})^2 + D_b(\alpha_{ht})^2]} \ , \tag{A.41}$$

and

$$x_b = \frac{F(\alpha_{ht}\alpha_{hb} - D_h\alpha_{bt})}{D_t D_h D_b + 2\alpha_{ht}\alpha_{hb}\alpha_{bt} - [D_t(\alpha_{hb})^2 + D_h(\alpha_{bt})^2 + D_b(\alpha_{ht})^2]} \ . \tag{A.42}$$

Appendix B

Equipment specifications

B.1 Equipment used for acoustic measurements

Brüel & Kjær equipment

Heterodyne Analyser, type 2010

Input section:

Frequency response (linear): 10 Hz to 50 kHz, ± 0.2 dB.

Sensitivity: 10 μV to 330 V f.s.d. in 10 dB steps.

Constant bandwidth filter:

3.5 dB bandwidth: 3.16, 10, 31.6, 100, 316 and 1000 Hz.

Distortion (input section and filter): $< 0.03\%$, 20 Hz–100 kHz.

BFO section:

Frequency range: 2 Hz to 200 kHz.

Amplifier linearity: ± 0.2 dB.

Distortion: $< 0.015\%$, 20 Hz–50 kHz.

Noise: < -70 dB.

Accelerometer, type 4374

Frequency response: 1 Hz to 26 kHz.

Charge sensitivity: 0.122 pC/ms^{-2}.

Maximum transverse sensitivity: 3.8%.

Force transducer, type 8200

Charge sensitivity: 4pC/N.

Linearity: < ± 1% of maximum force.

Charge amplifier, type 2635

Frequency range: 0.1 Hz to 200 kHz.

Acceleration sensitivity: 0.1 mV/pC–10 V/pC.

Noise: < 2.5 μV.

Typical rise time: 2.5 V/μs.

Conditioning amplifier, type 2626

Frequency range: 0.3 Hz to 100 kHz.

Sensitivity: 0.1–1000 mV/pC.

Noise: < 2.5 μV.

Rise time: 2.5 V/μs.

X-Y recorder, type 2308

Sweep rates: 0.2–100 mm/s.

Slewing speed: 1000 mm/s.

Accuracy: 0.2% of f.s.d.

Condenser microphone, type 4144

Frequency response: 2 Hz to 7.5 kHz, ± 1.5 dB.

Microphone power supply, type 2807

Frequency response: 2 Hz to 200 kHz, ± 0.2 dB.

Distortion < 1% at maximum output.

Microphones were callibrated with a Brüel & Kjær pistonphone, type 4220, which produces a sound pressure level of 124.0 dB ± 0.2 dB.

Other equipment

Quad power amplifier, type 303

Distortion: 20 Hz to 70 Hz, < 0.03%; 700 Hz to 10 kHz, < 0.1%.

Frequency response: 30 Hz to 35 kHz, ± 1 dB.

B.2 Equipment used for psychoacoustical listening tests

Sennheiser HD 330 headphones

Frequency response: 18–22000 Hz (no level variation within this range quoted).

Total harmonic distortion: $< 0.8\%$.

Sensitivity at 1 kHz: 94 dB $\pm$ 2 dB.

Denon PMA - 350Z amplifier

Output power: 27 W per channel.

Frequency response: 8 Hz to 150 kHz, $\pm$ 0.5 dB.

Distortion: < 0.5 mV r.m.s.

Rockland filter, type 852

Frequency range: 0.01 Hz to 111 kHz, $\pm$ 2%.

Roll-off (2 channels connected in series): 96 dB/octave.

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